Free Convection Heat Transfer near Leading Edge of
Semi-infinite Vertical Flat Plate with Finite Thickness*
(1st Report, Isothermal Flat Plate, Pr = 0.72)

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Free convection heat transfer near the leading edge of an isothermal semi-
infinite vertical flat plate with finite thickness has been analyzed numerically
by the finite difference method for Pr = 0.72. The effects of the adiabatic
horizontal floor under the leading edge and of the slope of the leading edge on
free convection heat transfer have been made clear. When the dimensionless
height, \( \theta \), of the leading edge from the floor is larger than \( \frac{1}{4} \), Nusselt number
distributions near the leading edge are hardly influenced by the floor, taking
lower values than any previous perturbation solutions and being correlated by
the following equation:

\[
\frac{NuC}{X^{1/4}} = 0.3608 + 0.1209 X^{-1.318}, \quad X \gtrsim 1.5
\]

The Nusselt numbers near the leading edge of flat plates with finite thickness
are correlated by the Grashof number based on the plate thickness, and the
present results for a triangular leading edge agreed well with previous experi-
mental results.

1. Introduction

It is well known from the experimental and theoretical analysis, that the heat
transfer coefficient for laminar free convection near the leading edge of a uniformly
heated semi-infinite vertical flat plate is really higher than the predicted value from
the classical boundary layer solution. Since Yang and Jergel obtained a perturbation
solution of the two-dimensional Navier Stokes, continuity and energy equation,
utilizing the classical boundary layer solution as the zeroth-order approximation, many analytical studies of such higher
order boundary layer effects have been published as shown in Table 1. These results
in Table 1 agree with each other in that the heat transfer coefficient near the
leading edge is really higher than the predicted value from the classical boundary
layer solution. But also the difference between the results are evident. Stated
exactly, the difference between local Nusselt numbers increases as the leading
edge is approached. The reason is that the leading term of each higher order perturbation
solution is a classical boundary layer solution which is singular at the leading
edge.

On the other hand, Surtanto et al. obtained the perturbation solution for lami-
nar free convection along a vertical finite flat plate, utilizing the solution of the
steady heat conduction as the zeroth-order

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approximation. But the obtained results
were limited to very small Grashof numbers.

In his experiments, Brodovicek investigated the temperature and velocity fields
around the leading edges with various geometrical configurations on the vertical
flat plate. Aihara measured the distributions of the local heat transfer coefficients at the

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
Spawar, Guine (2), (1968), Pr = 0.733 & \\
\hline
\frac{NuC}{X^{1/4}} = 0.3591 (1 + 0.4616X^{-1/2}) & \\
\hline
Hieber (3), (1974), Pr = 0.72 & \\
\hline
\frac{NuC}{X^{1/4}} = 0.3588 (1 - 0.19671X^{-1/2} + 0.4866X^{-1/2}) & \\
\hline
\frac{NuC}{X^{1/4}} = 0.478 + 0.686X^{-3/8} & \\
\hline
Riley, Drake (4), (1975), Pr = 0.72 & \\
\frac{NuC}{X^{1/4}} = 0.3568 (1 + 0.0866X^{-3/2}) & \\
\hline
Messiter, Liski (5), (1976), Pr = 0.72 & \\
\frac{NuC}{X^{1/4}} = 0.476 + 0.633X^{-3/8} & \\
\hline
Bereznovskii, Martynenko, Sokovishin (6), (1978), Pr = 0.72 & \\
\frac{NuC}{X^{1/4}} = 0.478 + 0.238ln(X^{1/4})X^{-3/8} + 0.606X^{-3/8} & \\
\hline
Bereznovskii, Sokovishin (7), (1978), Pr = 0.72 & \\
\frac{NuC}{X^{1/4}} = 0.557 (1 + 0.501X^{-1/4} + 0.501X^{-1} + 0.12X^{-3/2}) & \\
\hline
\end{tabular}
\caption{Comparison of Nusselt numbers between previous perturbation solutions}
\end{table}
lower and vertical ends of a vertical fin array, using a thermocouple probe. He investigated the effects of the fin thickness and the temperature difference between the fin surface and ambient air on the local heat transfer coefficient at the lower and vertical ends of fin.

Gryzogoridis measured the distributions of the local Nusselt numbers near the leading edges with various shapes on the vertical flat plate, using a Mach-Zehnder interferometers. The leading edges used by him were a round semi-circular edge, a triangular edge with a 90° apex and a flat edge. He found that the local Nusselt number distributions near the leading edges depended not only on Grashof number but also on the temperature difference between the flat plate and the ambient air.

In this paper, the free convection heat transfer near the leading edges with various shapes on the vertical flat plate was analyzed numerically by the use of the finite-difference method, utilizing the two dimensional complete Navier-Stokes, energy and continuity equation. A difficulty in such numerical analysis was that the position of the outer boundary of the numerical calculation region corresponding to the physical infinity must be finite. The calculated region in the present investigation was magnified within the possible limit, using a variable size of meshes of the finite-difference. The finite-difference meshes were fine near the leading edge and coarse away from it. The horizontal floor was placed under the leading edge as a lower boundary with physical meaning. Local Nusselt number in the present solution at the downstream boundary was compared with the similarity solution to the boundary layer equation. At a sufficiently large local Grashof number, the present numerical solution gave an equivalent local Nusselt number to the classical boundary layer solution.

When the Prandtl number was 0.72 and the thickness of the flat plate was infinitesimally small, numerical solutions were obtained for various heights of the leading edge from the horizontal floor. When the dimensionless height \(\hat{H}\) was larger than 40, the effects of the horizontal floor on the free convection heat transfer near the leading edge were insignificant.

For \(\hat{H} = 40\), geometrical effects of the leading edge were investigated for a triangular edge with a 90° apex, a flat edge and a vertical leading edge connected to a vertical adiabatic wall extended to the floor. For the triangular leading edge, the distribution of the local Nusselt numbers was found to be correlated by the Grashof number based on the plate thickness instead of the temperature difference used by Gryzogoridis. The present solution agreed well with the experimental results obtained by Gryzogoridis.

The calculated velocity distributions around the leading edges agreed well with the experimental results obtained by Brodowicz.

2. Nomenclature

\(a, a_2, a_3, b_2, b_3\) : thermal diffusivity
\(c_1, c_2, c_3, d_2\) : constants defined by Eq.(32)
\(d\) : thickness of a flat plate
\(D\) : dimensionless thickness, Eq.(9)
\(e_{i-i+1,j}^{i} \) : third order derivatives of 
\(e_{i-i+1,j}^{i} \) : the differentiated function
\(g\) : acceleration due to gravity
\(h\) : height of the leading edge from a horizontal floor
\(H\) : dimensionless height, Eq.(9)
\(i, j\) : finite difference mesh numbers in \(X\) and \(Y\) directions
\(i_0\) : value of mesh number \(i\) at \(X = 0\)
\(j_n\) : maximum value of mesh number \(j\)
\(j_0\) : value of mesh number \(j\) at \(Y = D/2\)
\(j_n\) : maximum value of mesh number \(j\)
\(N\) : dimensionless normal distance from surface of the solid
\(N_t\) : local Nusselt number, \(= a_x/\lambda\)
\(N_e\) : average Nusselt number, \(= Q/\lambda A\)
\(N_t\) : average Nusselt number at the bottom of flat plate, \(= (60/30)D\)
\(p\) : pressure
\(Pr\) : Prandtl number
\(Q_x\) : total heat loss from the plate surface for \(0 < x\), including the base of the plate
\(T\) : temperature
\(u, v\) : velocities in \(x\) and \(y\) directions
\(U, V\) : dimensionless velocities in \(x\) and \(y\) directions, Eq.(10)
\(x\) : coordinate along the vertical plate from the leading edge
\(y\) : horizontal coordinate from the center of the vertical flat plate
\(X, Y\) : dimensionless coordinates of \(x\) and \(y\), Eq.(9)
\(X_{max}, Y_{max}\) : maximum values of \(X\) and \(Y\) in the calculated region
\(\alpha, \sigma\) : local and average heat transfer coefficients
\(\alpha_{x}, \alpha_{y}, \alpha_{y}\) : parameters in Eq. (22), (23) and (24) for modification of time derivative
\(h\) : volume expansion
\(\Delta T\) : temperature difference, \(= T_0 - T_w\)
\(\Delta X, \Delta Y\) : mesh sizes of \(X\) and \(Y\) directions
\(\Delta X_{min}, \Delta Y_{min}\) : minimum values of \(\Delta X\) and \(\Delta Y\)
\(\Delta t\) : time step
\(\delta X, \delta Y\) : \(= \Delta X_i \Delta X_{i+1} + \Delta Y_i \Delta Y_{i+1}\)
\(\varepsilon\) : vorticity
\(\theta\) : dimensionless temperature difference, Eq.(11)
\(30/30, 30/30\) : dimensionless normal gradient of temperature and its average value at the base of the plate
\(\lambda\) : thermal conductivity
\(\nu\) : kinematic viscosity
\(\xi\) : dimensionless vorticity, Eq. (12)
\(\rho\) : density

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\( \tau \) : dimensionless time, Eq. (8)
\( \phi \) : stream function, Eq. (7)
\( \psi \) : dimensionless stream function, Eq. (13)

\( \nabla^2 \) : Laplace operator

subscripts
\( \omega \) : heating surface
\( \infty \) : ambient condition
superscript
\( \bar{\cdot} \) : average value

3. Basic Equations and Numerical Solutions

3.1 Basic equations and boundary conditions

Laminar free convection from an isothermal semi-infinite vertical flat plate with four different leading edge configurations above a horizontal floor, as shown in Fig. 1, was numerically analyzed. In Fig. 1, each coordinate system is also shown. The four cases of the leading edges in Fig. 1 are:

Case 1- A triangular edge with a 90° apex
Case 2- An infinitesimally thin flat plate
Case 3- A vertical edge connected with a vertical adiabatic wall
Case 4- A flat edge

To examine the problem described in the Introduction, the basic equations were made two-dimensional and were simplified by adopting the well-known Boussinesq approximation for the gravitational term, assuming physical properties otherwise constant and neglecting the dissipation and thermal radiation effect. A continuity equation, two momentum equations and an energy equation are then given by:

\[ \frac{3 \mu}{3x} + \frac{3v}{3y} = 0 \]  
\[ \frac{3u}{3x} + \frac{3v}{3y} + g \beta (T - T_0) + \nabla^2 \psi = 0 \]  
\[ \frac{3u}{3y} + \frac{3v}{3y} + \frac{3u}{3y} = 0 \]  
\[ \frac{3T}{3x} + \frac{3T}{3y} + \frac{3T}{3y} = \rho \nabla^2 T \]

Although only the steady state free convection was considered in this study, unsteady terms were introduced in equations (2), (3) and (4) for convenience of numerical solution. Eliminating the pressure between the two momentum equations and introducing a stream function and a vorticity, we rewrite Eqs. (1), (2) and (3) as:

\[ \frac{3 \xi}{3x} + \frac{3 \xi}{3y} + \frac{3 \xi}{3y} = g \beta (T - T_0) + \nabla^2 \zeta \]
\[ \zeta + \nabla^2 \psi = 0 \]  
\[ \frac{\dot{\xi}}{\dot{\xi}} = \frac{\nabla^2 \zeta}{\nabla^2 \psi} \]

Eqs. (4), (5), (6) and (7) were transformed into a dimensionless form, introducing the following dimensionless variables:

\[ \tau = \frac{(g \beta T_0)^{2/3}}{v} t \]
\[ X = \left( \frac{g \beta T_0}{v^2} \right)^{1/3} x, \quad Y = \left( \frac{g \beta T_0}{v^2} \right)^{1/3} y \]
\[ H = \left( \frac{g \beta T_0}{v^2} \right)^{1/3} \frac{1}{h}, \quad D = \left( \frac{g \beta T_0}{v^2} \right)^{1/3} d \]
\[ U = \frac{u}{(v g \beta T_0)^{1/3}}, \quad V = \frac{v}{(v g \beta T_0)^{1/3}} \]
\[ \theta = \frac{T - T_0}{\Delta T} \]
\[ \psi = \frac{\psi}{\Delta x} \]

Dimensionless basic equations are given by:

\[ \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{1}{\gamma^2} \dot{\psi} \]
\[ \frac{\partial \psi}{\partial x} = - \frac{\partial \psi}{\partial x} + 3 \frac{\partial \psi}{\partial x} + \psi \frac{\partial \psi}{\partial x} - 3 \frac{\partial \psi}{\partial y} \]
\[ \xi + v^2 \psi = 0 \]

For example, the boundary conditions in case 2 are:

\[ X = -H, \quad 0 < Y < \infty; \quad \psi = 0, \quad \xi = -\frac{3 \psi}{\Delta x}, \quad \theta = 0 \]

\[ Y = 0; \quad \psi = 0, \quad \xi = 0, \quad \theta = 1 \]
$0 < X < \infty , \; Y = \infty ;$

$$\frac{\partial \psi}{\partial Y} = 0 , \; \xi = 0 , \; \theta = 0 \quad (21)$$

$36 \zeta \alpha = 0$ in Eq.(18) indicates the adiabatic floor, and $36 \psi \alpha = 0$ in Eq.(21) corresponds to $U = 0$ at $Y = \infty$. Eq.(19) is the condition for symmetry.

In other cases, the boundary conditions at the exposed surfaces of the flat plate are given by:

$$\psi = 0 , \; \xi = -3 \psi / 3 N^2 , \; \theta = 1 ,$$

where $N$ is a dimensionless normal distance from the exposed surface.

### 3.2 Finite-difference equations

Eqs.(14), (15), (16) and (17) were solved numerically by the finite difference technique using the ADI method proposed by Mallinson and Davillier for the three-dimensional problems. A variable mesh size was used in the present study. An example of mesh arrangement near the leading edge is shown in Fig. 2. The dimensionless basic equations were transformed into the following conservative form:

$$\frac{1}{\alpha} \frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial X} - \frac{\partial \psi}{\partial Y} + \frac{1}{\nu} \frac{\partial \psi}{\partial N} \quad (22)$$

$$\frac{1}{\alpha} \frac{\partial \xi}{\partial t} = -\frac{\partial \xi}{\partial X} - \frac{\partial \xi}{\partial Y} + \frac{\partial \psi}{\partial N} \quad (23)$$

$$\frac{1}{\alpha} \frac{\partial \theta}{\partial t} = \frac{\partial \psi}{\partial N} \quad (24)$$

Each term of the partial differential equations is transformed into the following finite-difference form:

$$\frac{\partial \psi}{\partial N} = \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right) + \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right)$$

$$\frac{\partial \xi}{\partial N} = \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right) - \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right)$$

$$\frac{\partial \theta}{\partial N} = \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right) - \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right)$$

$$\frac{\partial \psi}{\partial N} = \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right) - \frac{\Delta X \Delta Y}{\Delta X \Delta Y - 1} \left( \xi_{i,j}^{n+1,j} - \xi_{i,j}^{n+1,j} \right)$$

The fourth terms in the right hand of Eqs. (25) and (26) correspond to the truncation error terms.

For example, the boundary conditions of case 2 in finite difference form are given by:

$$X = -H , \; 0 \leq Y \leq \text{Ymax} i$$

$$\psi_{i,j} = 0 , \; \xi_{i,j} = -\frac{3 \psi_{i,j}}{\Delta Y} - \frac{1}{2} \xi_{i,j} \quad (27)$$

$$\theta_{i,j} = a \theta_{i,j} + a \theta_{i,j} \quad (28)$$

$$0 \leq X \leq \text{Xmax} , \; Y = 0 ;$$

$$\psi_{i,j} = 0 , \; \xi_{i,j} = -\frac{3 \psi_{i,j}}{\Delta Y} - \frac{1}{2} \xi_{i,j} \quad (29)$$

$$\theta_{i,j} = 1 \quad (30)$$

$$0 < X < \text{Xmax} , \; Y = \text{Ymax} j$$

$$\psi_{i,j} = a \psi_{i,j} + a \psi_{i,j} , \; \xi_{i,j} = 0 , \; \theta_{i,j} = 0 \quad (31)$$

$$\psi_{i,j} = a \psi_{i,j} + a \psi_{i,j} , \; \xi_{i,j} = a \xi_{i,j} + a \xi_{i,j}$$

$$\theta_{i,j} = 1$$

$$X = \text{Xmax} , \; Y < \text{Ymax} j$$

$$\psi_{i,j} = a \psi_{i,j} + a \psi_{i,j} , \; \xi_{i,j} = a \xi_{i,j} + a \xi_{i,j}$$

$$\theta_{i,j} = 1$$

The mesh size at the point of origin corresponding to the leading edge is minimum and given by $\Delta X_{min} \times \Delta Y_{min}$. $\Delta X$ and $\Delta Y$ at larger values of $|X|$ and $Y$ are gradually magnified. The rate of magnification $(\Delta X_{max} - \Delta X_{j-1}) / \Delta X_{j-1}$ and $(\Delta Y_{max} - \Delta Y_{j-1}) / \Delta Y_{j-1}$ must be chosen less than 0.5, and $(\Delta X_{max} - \Delta X_{j-1}) / \Delta X_{j-1}$ must be larger than $(\Delta Y_{max} - \Delta Y_{j-1}) / \Delta Y_{j-1}$ for a rapid convergence of the numerical solutions. An example of the mesh arrangement in Fig. 2 is based on this rule.

Initial values of $\xi_{i,j}, \theta_{i,j}$ and $\psi_{i,j}$ are zero at all mesh points excepting $\theta_{i,j}$ at the heating surface. $\theta_{i,j}$ is always zero at the heating surface. The iterative calculation using the ADI method is continued until convergence is reached. The sums of the absolute value of the changes from one iteration to the next of all mesh point values of $\xi, \theta$ and $\psi$ excepting boundary mesh points values are calculated.

* When fluid is air at normal temperature and $\Delta T$ is 10°, all dimensionless lengths $(X, Y, D, H, N)$ roughly correspond to the actual length being read in scale of millimeters. The cube of $X$ is equal to the conventional Grashof number.

** Vorticity was considered as divergent at the corner of the flat plate with finite thickness.
Fig. 2 An example of the finite difference mesh arrangement near the leading edge.

These three numbers are then normalized using maximum values of the respective quantities at each iteration. Convergence was declared to have been reached when the three normalized numbers are less than $10^{-6}$ in the present calculation.

Local Nusselt number may be evaluated by:

$$
\text{Nu}_x = \frac{\alpha x}{\lambda} = \frac{\Delta x}{2\gamma} \gamma \theta_i = 0
$$

$$
= x \left[ \frac{\Delta x + \Delta y}{\Delta x + \Delta y} \left( 1 - \frac{\partial \psi}{\partial x} \right) \right]
$$

$$
- \Delta x \left[ \frac{\Delta x + \Delta y}{\Delta x + \Delta y} \left( 1 - \frac{\partial \psi}{\partial x} \right) \right]
$$

(33)

4. Results and discussions

Numerical solutions were obtained on a FACOM M-190 in Kyushu University and for $Pr = 0.72$, $x_{max}$ was larger than about 50 and $x_{max}$ larger than about 40. The maximum value of $x_{max}$ was 30x20. Number of iterative cycles required for convergence depended on the values of $x_{max}$, $H$, $\Delta x_{min}$ and $\Delta y_{min}$. For reference, a standard example is given by:

case 2; $x_{max} = 40$, $H = 39$, $\gamma = 40$, $x_{max} = 30$ x 18, $\Delta x_{min} = \Delta y_{min} = 0.5$, $\Delta t = 5$, $\alpha = 1$, $\alpha = 0.04$. Number of iterations was 1120 and solution time was about 1 minute.

The effect of mesh size on the accuracy of the solution was examined by comparing 0.5 with 0.3 of $\Delta x_{min}$ and $\Delta y_{min}$. For case 2 this difference of $\Delta x_{min}$ and $\Delta y_{min}$ caused an disagreement in the solution only near the leading edge. This discrepancy expressed by the temperature gradient $(\partial \psi/\partial x)$ on the flat plate was about 20% at $x = 0$ and was negligibly small for $x \geq 0.5$. Throughout the following analysis 0.5 of $\Delta x_{min}$ and $\Delta y_{min}$ was used.

4.1 Effects of the leading edge height from the floor on Nusselt number

Fig. 3 Comparison of Nusselt number distribution between various heights of the leading edge from the floor, case 2.

Numerical solutions were obtained for $0, 2.25, 7.75, 16.25, 39.75$ and 88.75 of dimensionless height, $H$, and case 2. Fig. 3 shows the local (solid lines) and the average (dotted lines) Nusselt number distributions on flat plate with various leading edge heights mentioned above. The difference of the Nusselt numbers between 88.75 and 39.75 of dimensionless height were negligibly small. Therefore the results for $H = 88.75$ are not shown in Fig. 3, but the results from the similarity solution have shown for comparison. These results may give the following conclusions.

1) The lower values of $H$ give the smaller value of local Nusselt number near the leading edge. The local Nusselt number near the leading edge for lower $H$ than 7.75 is smaller than one of the similarity solution.

2) The effects of the floor upon the Nusselt number distribution on the flat plate can be neglected for larger $H$ than about $40$.

3) The local Nusselt numbers at the upper location $X$ than 15 on the flat plate are not influenced practically by the floor for any $H$.

4) For 88.75 of $H$, the local Nusselt number distribution can be approximated accurately by the following equation.

$$
\frac{\text{Nu}_x}{\text{Nu}_x} = 0.5688 + 0.1809 X^{-1.128}, \text{for } X \leq 1.5
$$

(34)

The Eq. (34) can be compared with the previous perturbation solution in an infinite fluid. The present solution is different from the results of any perturbation solutions in Table 1 and gives the smallest local Nusselt number near the leading edge.

4.2 Effects of the leading edge configuration on Nusselt number

Throughout the following analysis was about 40. Nusselt number distributions on flat plates with a triangular (case 1) and a flat leading edge (case 4) are shown in Figs. 4 and 5, respectively. The local Nusselt number near the leading edge with larger dimensionless thickness $D$ of case 4 (on Fig. 5) can be seen to be lower than
that of the similarity solution. Average Nusselt number in both figures indicates one corresponding to the total heat flow, $Q_{in}$, from the flat plate from 0 to X including the base areas. In the left side of Fig. 5, the dotted horizontal lines indicate the average temperature gradients, $\delta T/\delta x$, at the base. Average temperature gradients at a triangular base are nearly equivalent to the corresponding values of case 4 with the same dimensionless thickness.

In Fig. 6, the present local Nusselt numbers for case 4 are compared with the corresponding experimental data from Gryazgoridis [11]. The experimental results were obtained using a plate with 8.5 mm thickness and altering the temperature difference between the flat plate and the ambient air. An alteration of the temperature difference with a constant thickness of the plate amounts to an alteration of the dimensionless thickness in the present analysis. Experimental results agree with the present analysis. Furthermore, Gryazgoridis measured the local Nusselt numbers on the flat plate with a semi-circular leading edge. These experimental results for local Nusselt number are correlated simultaneously by taking $Nu/\delta x$ against $X/\delta x$, in spite of the variance of the temperature difference and the leading edge shape.

In Fig. 7, average Nusselt numbers at the base of cases 1 and 4 are compared with that on the horizontal cylinder and on the lower end of the vertical fin array. The results of the horizontal cylinder indicate Senftleben's correlation equation recommended by Fujii [14] and the numerical solutions by Fujii et al. [15], substituting a thickness $d$ for diameter of cylinder. The exponential data obtained by Aihara are the results using a vertical fin array which has 10 mm of fin pitch and 55 mm of vertical length. The latter Nusselt numbers are generally higher and they vary from the value near the fin root to the

![Diagram](image-url)
value near the fin tip, under the effects of the finite horizontal fin length and a rising air in the fin array. The present results indicate that the differences between cases 1 and 4 are relatively small, and the present average Nusselt numbers are lower about 20% at \( D = 1 \) and about 6% at \( D = 8 \) than Bentletmen's correlation. But it seems that a rigorous comparison is impossible, considering the additional error in the present numerical analysis near the base.

### 4.3 Velocity field around the leading edge

Brodovicz measured velocity distributions around the leading edges of vertical flat plates. His leading edges were a semi-circular edge (No.1), a triangular edge with 60° apex formed by a vertical and an oblique surface (No.2) and the same configuration as in the present case 3 (No.3). He found that velocity distributions around the leading edge were not always symmetrical about the flow axis and fluctuated with time, even if the shape of the leading edge was symmetrical about the axis. For these reasons, the measured velocity distributions were scattered. In Figs. 8 and 9, the calculated velocity distributions at \( X = 0 \) and \( X = 1/4 \) are compared with the experiment. The experimental data obtained from the figures in Brodovicz studies include a slight variance in transpiration and a distinction between symmetry (\( \theta \), \( \phi \), \( \xi \), \( \eta \)) and un-symmetry (\( \delta \), \( \varphi \), \( \psi \), \( \chi \)) is made by the symbols in Figs. 8 and 9.

It can be seen that the present results of case 1 in Fig. 8 are relatively well coincident with the symmetric velocity distributions of the experiment No.1, in spite of a little difference of the leading edge shape. The present results of case 3 agree well with the experiment of No.3. On the other hand, the velocity distributions at \( X = 1/4 \) in Fig. 9 are very low and may be influenced by the floor in the present analysis and by the fluctuations of the flow in the experiment. Then, it can be said that the numerical results agree well with the experiment. But the present results in case 3 correspond relatively well to the experiment in No.3. Particularly the horizontal velocities \( \nu \) can be seen to agree well with each other.

### 5. Conclusions

The characteristics of free convection heat transfer near the leading edge of an isothermal semi-infinite vertical flat plate have been investigated numerically by the finite-difference method for \( Pr = 0.72 \). An adiabatic horizontal floor was placed under the leading edge as the lower boundary of the numerical calculation region. The effects of the floor, the thickness of the flat plate and the shape of the leading edge upon free convection heat transfer were investigated. The obtained results give the following conclusions:

1. When the dimensionless height \( H \) is larger than about \( 4/0 \), the effects of the floor upon the Nusselt number distribution along the thin flat plate are negligibly small.

2. The lower value of \( H \) gives the lower Nusselt numbers near the leading edge. But the Nusselt numbers at the higher vertical location on the flat plate than 15 of the dimensionless length \( X \) from the leading edge are hardly influenced by the floor for any \( H \).

3. For a larger \( H \) than \( 4/0 \), the local Nusselt number distribution is approximated accurately by the following equation.

\[
\frac{Nu}{X^{1/4}} = 0.8688 + 0.1809 X^{1.238}, \quad X \geq 1.5
\]

4. The average temperature gradient, \( 38/3H \), on the base of the flat plate with finite thickness hardly depends on the difference of the shape of the base (leading edge). The larger dimensionless thickness \( D \) gives the lower value of \( 38/3H \). The present results give about 20% lower value of \( 38/3H \) than the corresponding one of the horizontal cylinder with the same diameter with \( D \) between 1 and 8.

5. Present local Nusselt number on the flat plate with the triangular leading edge agrees with the experimental results.

6. The calculated velocity distributions at \( X = 0 \) agree with the corresponding experimental results.
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References