Fundamental Studies on the Self-locking Characteristics of Oneway Clutch*

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In the locked condition of a oneway clutch, high stress will be generated at the contact surfaces between rollers or sprags and the races of the inner and outer wheelings. A study on the characteristics of self-locking can not go without regard for the effect of the deformation caused by these stresses in each part of the clutch. In this study a unique way to derive mapping functions has been developed and each part of the clutch has been analysed by using the two-dimensional theory of elasticity. Furthermore, on the reflection of the contact surfaces, the behavior of the sprag which rotates under the locked condition is analysed here by using the finite element method.

1. Introduction

Oneway clutches are examples of machine elements applying the mechanism of self-locking and are widely used in various power transmission systems such as over-running, back-stopping and intermittent indexing devices by using the feature to transmit a torque in only one direction. The torque transmitting mechanism of a oneway clutch utilizes the self-locking effect of rollers or sprags, so high stresses occur in a region where the rollers or sprags are in contact with the races of outer wheeling and inner wheeling. The self-locking characteristics of oneway clutches can not be explained without regard for the effect of the deformations caused by these stresses in each part of the clutch. Also, there are many unsolved problems concerning the discrepancy between actual magnitude of contact forces and the statically calculated results, and the slip regions in contact faces.

In customary designs of oneway clutches, the contact forces are estimated by using the geometrically obtained angle of wedge (strut angle), then the constant pressures in the wheelings are calculated by the Hertz formula and the elementary formulae of strength materials respectively for these estimated contact forces.⁽¹⁾⁻⁽³⁾ However, the elementary calculation like this makes clear neither a detailed distribution of stresses and deformations caused in a oneway clutch nor the behavior of the oneway clutch in the locked condition. Oneway clutches are one

of the useful machine elements, so various experiments, developments and improvements have been performed on them.⁽⁴⁾⁻⁽⁶⁾ But there are relatively few theoretical analyses of oneway clutches.⁽⁶⁾

It goes without saying that the stresses and deformations of simple shaped structures such as the rollers of roller-clutches or the inner and outer wheelings of sprag-clutches are easily analysed by using the theory of elasticity. Furthermore, complex shaped structures such as the sprags and inner wheelings of roller-clutches are analysed in a similar way when the mapping functions can be obtained for them.

On the other hand, the theory of elasticity is found inadequate to give information about the behavior of sprags regarding the surface of contact, because the sprags are compelled to rotate in the locked condition. But, in this case, a solution can be easily obtained by using a numerical method like the finite element method.

2. Analyses of Oneway Clutches using the Two-dimensional Theory of Elasticity

When a unit circle of the τ -plane is mapped into the region of z -plane by a mapping function $\omega(\tau)$, a stress function $\phi(\tau)$ is obtained from the boundary condition specifying the plane stresses:⁽⁷⁾

$$\varphi(\tau) + \frac{w(\tau)}{w'(\tau)} \overline{\varphi'(\tau)} + \overline{\phi(\tau)} = P(\tau) \dots\dots\dots(1)$$

and other supplementary conditions. Using these functions $\phi(\tau)$ and $\psi(\tau)$, the displacements u and v are given by

$$u + iv = (1 + \nu) \left[\{ (3 - \nu) / (1 + \nu) \} \varphi(\tau) - \frac{w(\tau)}{w'(\tau)} \overline{\varphi'(\tau)} - \overline{\phi(\tau)} \right] \frac{1}{E} \dots\dots\dots(2)$$

Stress functions $\phi(\tau)$ and $\psi(\tau)$ have the representations

$$\left. \begin{aligned} \varphi(\tau) &= \sum_{k=-\infty}^{\infty} a_k \tau^k + a \log \tau \\ \phi(\tau) &= \sum_{k=-\infty}^{\infty} b_k \tau^k + b \log \tau \end{aligned} \right\} \dots\dots\dots(3)$$

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If the given region is a simply connected domain, the coefficients a , b , a_k and b_k vanish.

2.1 Analyses of Rollers and Wheelings

Bodies having a circular boundary like the rollers or the inner and outer wheelings of sprag clutches may be treated as the basic problems for a circular region. In this study, analyses were made by assuming that the contact surfaces have the pressure distributions provided by H. Hertz. For the rollers, the coefficients in Eq.3 becomes

$$\left. \begin{aligned} a_{n+1} &= q_a \left\{ \sum_{k=1}^m \cos K' \beta \frac{J_1(K' \theta_0)}{K'} + C_a \right\} \\ b_n &= -q_b \left\{ (n-2) \sum_{k=1}^m \cos K' \beta \frac{J_1(K' \theta_0)}{K'} + (n-1) \cos n \beta \frac{J_1(n \theta_0)}{n} + C_b \right\} \end{aligned} \right\} \quad (4)$$

Here, $a_1 = -N/2\pi r_0 L$, $a_0 = b_0 = b_1 = 0$, $q_a = 2\sigma_{\max}/(n+1)r_0^n$ and $q_b = 2\sigma_{\max}/n(n+1)r_0^n$. Where r_0 is the radius of the roller, L is the length of the rollers, N and $\sigma_{\max}$ are the normal forces and the maximum pressure in the area of contact. The coefficients K' , C_a and C_b are chosen as follows:

$$K' = 2k, C_a = \theta_0/4, C_b = (n-2)\theta_0/4, l=1 \quad \dots \quad n=2m$$

$$K' = 2k-1, C_a = C_b = 0, l=2 \quad \dots \quad n=2m-1$$

The deformations of the inner and outer wheelings of sprag clutches are investigated by K. Timmer, but there seems to be still unclear points. As a result of more work using different approaches with the theory of elasticity, the authors obtained the change in length of radii of wheelings. The amount of the contraction of the inner wheeling: U_i and the expansion of the outer wheelings: U_o are given as follows at the contact points.

$$\left. \begin{aligned} U_{i,0} &= \frac{z_0 N}{2\pi E L_{i,0}} \left\{ \frac{C_{i,0}}{1-\beta_{i,0}^2} + 4 \sum_{h=1}^{\infty} [-m(1+\nu) \bar{C}_{i,0m1} + m(1+\nu) \bar{C}_{i,0m2} \right. \\ &\quad \left. - ((m-2) + \nu(m+2)) \bar{C}_{i,0m3} + ((m+2) + \nu(m-2)) \bar{C}_{i,0m4} \right] \frac{J_1(\alpha_{i,0n})}{\alpha_{i,0n}} \right\} \\ m &= n z_0, \quad \alpha_{i,0n} = \frac{n z_0}{R_{i,0}} \sqrt{\frac{8 r_{i,0} R_{i,0} N}{\pi E L_{i,0} (R_{i,0} + r_{i,0})}}, \quad \beta_{i,0} = R_{i,0} / R_{i,00} \\ C_0 &= (1-\nu) + (1+\nu) \beta_0^2, \quad C_i = -[(1+\nu) + (1-\nu) \beta_i^2] \end{aligned} \right\} \quad (5)$$

Fig. 1 Roller and sprag clutch

where z_0 is the number of sprags, L_i and L_o are lengths of wheelings, J_1 is a Bessel function of the first kind of first order. The expressions for C_{imk} , C_{omk} , K_i and K_o used in Fig.5 are given in Table 1. From the analytical results, the deformations of the roller and the inner and outer wheelings are shown in Fig.3 and Fig.4 respectively.

2-2. Analyses of Sprag and Inner Wheeling of Roller Clutch

Generally, there is no way to find the mapping functions of every simply- or doubly connected regions easily. In electro-static field, there is a graphical method to construct equipotential surfaces schematically and approximately. But this method seems to have defects in that it depends completely on the sense of sight and requires a skill from the constructor.

In this study, developing a unique way different from this, the mapping function of each part of the clutch has been found out concretely. Then the deformation and stress distributions of the parts have been analysed by using these mapping functions. The mapping function that maps the interior of the unit circle $|\tau| < 1$ on the regions of z -plane is represented in the power series $z = \sum a_n \tau^n$. τ is the interior of the unit circle and has the representation $\tau = \rho e^{i\theta}$. In this section, citing an example of the inner wheeling of roller clutch, we show a method*

Fig.2 Pressure distribution

Fig.3 Deformation of roller

* This iterative method may be an improved one called the method of osculation. But the method of osculation has not found practical use.

for actually determining the mapping functions.

At first we find the mapping function

$$z = e^{i\theta} + e^{ik\theta} \quad (i = \sqrt{-1}) \quad (6)$$

whose mapped region contains closely the

Table 2 Coefficients in Eq. (5)

k	C_{kmk}
1	$\frac{-(m-1)+m\beta_1^2-\beta_1^{2m}}{2(m-1)K_1}$
2	$\frac{-(m+1)+m\beta_1^2+\beta_1^{2m}}{2(m+1)K_1}$
3	$\frac{-(m+1)+m\beta_1^2+\beta_1^{2m}}{2(m+1)K_1}$
4	$\frac{-(m-1)+m\beta_1^2-\beta_1^{2m}}{2(m-1)K_1}$
k	C_{0mk}
1	$\frac{-(m-1)+m\beta_0^2-\beta_0^{2m}}{2(m-1)K_0}$
2	$\frac{-(m+1)+m\beta_0^2+\beta_0^{2m}}{2(m+1)K_0}$
3	$\frac{-(m+1)+m\beta_0^2+\beta_0^{2m}}{2(m+1)K_0}$
4	$\frac{-(m-1)+m\beta_0^2-\beta_0^{2m}}{2(m-1)K_0}$

$K_1 = 2(m^2-1) - m^2(\beta_1^2 + \beta_1^{-2}) + (\beta_1^{2m} + \beta_1^{-2m})$
 $K_0 = 2(m^2-1) - m^2(\beta_0^2 + \beta_0^{-2}) + (\beta_0^{2m} + \beta_0^{-2m})$

Fig.4 Deformations of the inner and outer wheelings

figure of the inner wheeling as shown in Fig.5-a. This time, the image under the mapping of the inner wheeling by the inverse function of Eq.6 will be as shown in Fig.5-b.

Next, in a similar manner, we find the mapping function of this figure shown in the interior of the circle in the form of Eq(6). Repeating this procedure and composing the functions obtained in each step successively, successive images will converge to the given region as in Fig.6. In this study, the individual steps were carried out by graphical methods or the computer, then after some iterations the mapping functions of the sprag and the inner wheeling are obtained as shown in Fig.7. Figures 8 and 9 show the deformation and stress distributions calculated by using these mapping functions. These results are discussed and used in the sections that follow.

2-3. Wind-up of the Oneway Clutch

The elastic deformations caused in each part of the oneway clutch induce a relative angular movement between the inner and the outer wheeling called wind-up. The deformations of the sprag and the inner wheeling of the roller clutch that are calculated in the preceding sections agree approximately with those obtained from Föpple's formula. For the sprag clutch, the larger the number of sprags, the more noticeable the effects of bending become. And the deformations of the inner and outer wheelings become considerably different from Föpple's solution.

Fig.5 Graphical method to find the mapping function

Fig.6 Successive images obtained in each step

The wind-up X_w is due to the contact deformations $\Delta R_o, \Delta R_i, \Delta r_o, \Delta r_i$ that are written in the forms from Eq.(1) to Eq.(5), then

$$X_w = -\Delta\alpha + \sqrt{\Delta\zeta_o^2 r_o^2 + \Delta r_o^2} - \Delta R_o^2 / R_o + \sqrt{\Delta\zeta_i^2 r_i^2 + \Delta r_i^2} - \Delta R_i^2 / R_i \dots\dots\dots (7)$$

Figures 10 and 11 show the wind-ups of the roller clutch and the sprag clutch respectively in the cases of different sizes of the clutch. For the sprag clutch analysed in this study, the inner and outer wheelings

Fig.7 Conformal mapping

Fig.8 Deformation and stress distribution in sprag

Fig.9 Deformation and stress distribution in inner wheeling

have doubly-connected regions and the centers of curvature of the contact surface of the sprag are supposed to be fixed at points in the sprag. So the calculated amounts of the wind-up are large in spite of the small deformations of the contact.

3. Finite Element Analysis of the Oneway Clutch

The components of the oneway clutch can be analysed too by using the finite element method. In the case of the analysis of the component having periodicity such as a point-symmetric sprag or an inner wheeling of the roller clutch, it is not necessary to analyse the whole region of the component. In this case, it is enough to analyse the periodical portion appearing at regular intervals (Fig.12). Fig.12-b, for example, is drawn to locate the nodal points along the ξ_2 -axis at the same distance of the corresponding nodal points in the ξ_2 -axis. Then the stiffness matrix is transformed so that the displacement and force components of the nodes along the ξ_1 -axis and the ξ_2 -axis are referred to the ξ_1 and η_1 coordinates and the ξ_2 and η_2 coordinates respectively. Hence the boundary conditions become

$$F_1 + F_2 = 0, \quad \delta_1 = \delta_2$$

where F_1 and δ_1 are the nodal force vector and the nodal displacement vector with respect to the nodes along the ξ_1 -axis; F_2 and δ_2 are those along the ξ_2 -axis. Naturally, the results obtained by the two-dimensional finite element analysis agree approximately with those shown in Fig.8.

In a similar manner, the sprag shown in Fig.12-b can be analysed about the upper half of it. Assuming that the inner and

Fig.10 Wind-up of the roller clutch

Fig.11 Wind-up of the sprag clutch

outer wheelings are rigid and the contact surfaces are regarded to have the distribution provided by Hertz, the coefficients of static friction can be calculated by using a method of iteration and convergence as shown in Fig.13. It is clear that slipping occurs at both sides of the surfaces of contact. And, although the angle of wedge has a value of 3° and satisfies the statical condition of self-locking; $\tan \alpha < \mu_s$, the finite element analysis assuming $\mu_s = 0.2$ shows that the whole surface of contact becomes a slipping regions. Hence, it is seen that a statically simple relation does not exist in the actual self-locking condition. Furthermore, the theory of elasticity seems not to be directly applicable to the analysis of the oneway clutch, because the inner and outer wheelings of the oneway clutch have a thickness larger than the width of the sprag and a very complicated shape. In what follows, the behaviors of the oneway clutch are discussed by using the finite element method.

3-1. Analysis of the Inner and Outer Wheelings of the Sprag Clutch

The inner and outer wheelings may be analysed by either one of the two different finite element procedure; the three dimensional analysis making use of symmetry (Fig.14) or the axi-symmetric analysis for non-symmetrical load (Fig.15).

For the latter case, also with the analysis using the theory of elasticity, the loads acting on the inner and outer wheelings were represented in the power series and

Fig.12 Structure having periodicity

Fig.13 Coefficients of friction

the coefficients of power were used as the stress amplitudes in each harmonic. The responses can be obtained as the sum of the partial solutions in each harmonic, so the elements are arranged in a coarse mesh in order to save the computational time and the core storage of the computer. Table 2 shows a comparison of the results of the finite element analysis and the analysis of elasticity problem in the case when the widths of contact of the sprags are equal to the thickness of the inner and outer wheelings. Although a slight difference in the analytical conditions does not allow casual comparison of these results, it seems to be right to consider that these results shown in Tab. 2 approximately agree.

And, if the width of the sprag is reduced to half of the thickness of the inner and outer wheeling without altering the size of the wheelings and the magnitude of the load, the obtained results will be as shown in Tab.3 and Fig.16. Figure 16 shows the deformed state of a periodical part of the wheeling analysed by the axi-symmetric finite element method. Hence, it is clear that the localized deformations near the surfaces in contact with the sprags become more uniform at the position nearer to the end plane. And, in the case when the width of contact is a half of the thickness of the wheelings, the magnitude of the deformation is about the same for the case when these are equal.

3-2. Contact Problem of the Sprag

The solution of the contact problem using the finite element method was already given by Tsuta et al. But the contact problem of the oneway clutch considering the characteristics of self-locking is very

Fig.14 Element division of inner and outer wheelings

Fig.15 Element division of inner and outer wheelings

Table 2 Comparison between the theory of elasticity and the finite element method

		Axi-symmetric analysis (F.E.M)	Theory of elasticity	Three-dimensional analysis (F.E.M)
Inner wheeling	U _{max}	1.302	1.239	1.128
	U _{min}	0.733	0.719	0.698
Outer wheeling	U _{max}	2.324	2.239	2.121
	U _{min}	1.582	1.565	1.493

Table 3 Comparison between axi-symmetric analysis and three-dimensional analysis

		Axi-symmetric analysis	Three-dimensional analysis
Inner wheeling	U _{max}	1.680	1.297
	U _{min}	0.743	0.705
Outer wheeling	U _{max}	2.670	2.267
	U _{min}	1.590	1.510

U_{max} : Radial displacement at the contact point [μm]
 U_{min} : Radial displacement at the midpoint of contacts [μm]

difficult in comparison with those of balls or cylinders that have been discussed. For this reason, the inner and outer wheelings are supposed to be rigid in order to simplify the analysis. The curvatures of the inner and outer races are so great in comparison with those of the races of the sprag that the races of the inner and outer wheelings may be supposed to run in parallel in the vicinities of the contact points as shown in Fig.17.

Rotational rigid body displacements as well as elastic deformations are produced in the self-locking process of the sprags, so the iteration and convergences are used together with the method of infinitesimal increment of load (or displacement). In this way the displacement magnifications for all nodal points near the contact surfaces are calculated to find the minimum value $r_{\min}$ for them. A node having this minimum value $r_{\min}$ becomes the point in contact with either of the inner wheeling or the outer wheeling in an incremental

step. The directions of the forces acting on the contact surfaces and the magnitude of the friction forces are used as the criterion to judge whether the nodal points in contact with the inner and outer wheeling at the previous stage keep contact during an increment of displacement. The displacement increment for next stage is given to system, after convergence can be considered achieved by repeating the process described above. In this study, with the assumption that the coefficient of the maximum static friction μ_s has a value of 0.4, the self-locking characteristics of the sprag are investigated by using the element division shown in Fig.18. The results are shown in Figs.19,20 and 21. The relations of friction force to normal force at the contact surfaces between the sprags and the inner and outer wheeling show a deviation from the statically obtained expression $F = N \tan \alpha$ more notably with an

Fig.16 Deformed state of the wheelings

Fig.17 Displacement magnification

intensifying engagement. For the sprag in locking contact under the engagement, slipping takes place over the part of the contact where the sprag is initially in contact with the wheeling (Fig.20). Then a detailed investigation for the parts near the surfaces shows that the distribution of the stresses is unsymmetric with respect to the contact point, and that the contours of contact equivalent stress are extended into the deep interior of the sprags as shown in Fig.21.

Also, the same analysis assuming $\mu_s=0.5$ shows that slipping occurs over whole regions of contact surfaces at the normal force of 150 Kgf. And it is found that the system satisfying the statically obtained condition of self-locking does not always meet the actual condition of self-locking. The above results are obtained regarding the inner and outer wheelings as two parallel planes. Also, in a similar way of thinking, the wind-up of the oneway clutch can be calculated with regard to the curvatures of the inner and outer wheelings.

4. Conclusions

In this paper, considering the situation that there are relatively few theoretical analyses of oneway clutch, the self-locking characteristics of oneway clutch are studied by using the two-dimensional theory of elasticity and finite element method.

Results obtained are summarized as follows:

- (1) A unique way to derive mapping functions has been developed and applied to the two-dimensional analysis of a oneway clutch.
- (2) A statically simple relation does not exist in the actual self-locking condition.

Fig.18 Element division of sprag

Fig.19 Relation of friction force to normal force

Fig.20 Detailed investigation for the contact surface

Fig.21 Contours of contact equivalent stress

(3) Even for the oneway clutch that satisfies broadly the self-locking condition, slipping takes place over some part of the contact surface.

So far, fundamental studies on the method to analyse a oneway clutch and the characteristics of self-locking by using the two-dimensional theory of elasticity and the finite element method have been performed. Although, there are many unsolved problems concerning the oneway clutches such as dynamical behavior and the characteristics of self-locking of the oneway clutch whose analysis accounts for the lubricated contact surfaces and the flexibilities of the inner and outer wheelings, these are the subjects for future studies along with experiments. The authors of this paper are grateful to Technical Official K.Kobayashi of Shinshu University for his help. The calculation presented in this paper was carried out by using HITAC 8700/8800 in Computer Center of the University of Tokyo.

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