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Study on Spherical Foil Bearing*

(2nd report)

(Analysis including foil bending stiffness and its experimental verification)

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Three dimensional numerical analysis of a spherical foil bearing used in a rotating head recording device is presented. Reynolds equation is solved simultaneously with the representative Green function of plane or cylindrical foil deformation, which includes bending stiffness. Calculated pressure, spacing and foil deformation are illustrated and typical trends are discussed for small bearing penetration. The calculated results are found to be in good agreement with the experimental results under submicron spacing conditions.

1. Introduction

In digital magnetic recording with a rotating head, noncontact head scan on a magnetic tape is one of the most important techniques used to avoid head and tape wear, and to achieve reliable high-density recording. Efforts to gain a better understanding of the air-lubricating phenomena in the head-to-tape interface and to control the spacing have been made by many researchers in so-called foil bearing studies since the early 1960s. However, most of the previous reports on the foil bearing have been confined to a simple two dimensional model, in which a perfectly flexible foil is wrapped around a cylinder.

Since a rotating head generally has a spherical bearing surface, a three dimensional numerical analysis is essential for revealing its spacing characteristics. In a previous paper, the authors presented a three dimensional numerical method to analyze the perfectly flexible spherical foil bearing, and clarified the general bearing characteristics of a plane foil, on which the spherical bearing scans in an arbitrary direction.⁽¹⁾ However, the calculated results did not agree quantitatively with the experimental results, since the bending stiffness was not included in a foil deformation equation.

On the other hand, one of the authors has derived a foil deformation Green function, including the bending stiffness, and has shown that the calculated foil deformation agrees well with that of the actual magnetic tape within linear deformation limits.⁽²⁾ When this foil deformation equation is

coupled with the Reynolds equation to calculate the head-to-tape spacing, a problem of excessive calculation time and memory requirements arises. This paper presents a practical three dimensional numerical analysis method which overcomes this difficulty. This method is successfully applied to plane and cylindrical foils on which a spherical bearing scans longitudinally with small penetration. The calculated results are discussed in comparison with experimental results.

2. Nomenclature

O-xyz, O- $\xi\eta\zeta$: rectangular coordinates a : half foil width mm b : bearing surface height from origin mm b_z : bearing penetrating depth into foil mm h : film thickness mm l : half foil length mm p : pressure on bearing kg/mm² p_a : ambient pressure kg/mm² q : effective pressure on foil deformation kg/mm² r : spherical bearing radius mm t : foil thickness mm w : foil deformation mm w_g : Green function of foil deformation mm/kg $A = a/r\epsilon^{1/3}$ $B = b/r\epsilon^{2/3}$ $B_z = b_z/r\epsilon^{2/3}$ $D_1 = Eh^3/12(1-\nu^2)(T-\rho tU^2)r^2\epsilon^{2/3}$ $D_2 = Etr^2\epsilon^{2/3}/R^2(T-\rho tU^2)$ E : Young's modulus kg/mm² $H = h/r\epsilon^{2/3}$ $L = l/r\epsilon^{1/3}$ $P = pr/(T-\rho tU^2)$ $Q = qr/(T-\rho tU^2)$ R : cylinder radius mm T : foil tension kg/mm U : bearing scanning velocity mm/s $W = w/r\epsilon^{2/3}$

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$$\begin{aligned}
X &= x/r_s^{1/3} \\
Y &= y/r_s^{1/3} \\
\beta &: \text{length to width ratio for bearing area} \\
\rho &: \text{foil density kg}\cdot\text{s}^2/\text{mm}^4 \\
\lambda_a &: \text{molecular mean free path at ambient pressure mm} \\
\lambda &= 6\lambda_a/r_s^{2/3} \\
\nu &: \text{Poisson's ratio} \\
\mu &: \text{lubricant viscosity kg}\cdot\text{s}/\text{mm}^2 \\
\varepsilon &= 6\mu U/(T-\rho t U^2) \\
\delta &= (T-\rho t U^2)/r p_a \\
\kappa &= r/R \\
\Sigma &= \xi/r \\
H &= \eta/r
\end{aligned}$$

3. Theory

3.1 Basic equations

Consider a spherical bearing, which moves at constant velocity U in the tension direction penetrating into a foil by b , as shown in Fig. 1. Foil length is $2l$ and width is $2a$. Foil is stretched between two cylindrical guides or wrapped around a cylinder, whose radius is R .

For analytical simplicity, consider a rectangular coordinate system whose origin is fixed on the top of the moving spherical bearing, when penetrating depth b_z is zero. Let x and y axes coincide with longitudinal and transverse directions of an undeflected foil. Though the following equations treat the cylindrical foil shown in Fig. 1(b), these equations coincide with those for the plane foil, when R is infinite.

Foil deformation w , caused by pressure q , is governed by the equation

$$w = \int \int w_0(\xi, \eta, x, y) q d\xi d\eta \quad (1)$$

where w_0 is the Green function of foil, which is the deformation caused by the unit force acting on an arbitrary point (ξ, η) . The Green function, which can describe the deformation in an actual magnetic tape, whose boundaries are simply supported in the longitudinal direction and free in the transverse direction, is as follows:

$$\begin{aligned}
w_0(\xi, \eta, x, y) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cdot \\
&\frac{\sin \frac{m\pi(l_1+\xi)}{2l} \sin \frac{n\pi(a_1+\eta)}{2a} \sin \frac{m\pi(l_1+x)}{2l} \sin \frac{n\pi(a_1+y)}{2a}}{al \left[\frac{Et^3}{12(1-\nu^2)} \left\{ \left(\frac{m\pi}{2l} \right)^2 + \left(\frac{n\pi}{2a} \right)^2 \right\}^2 + (T-\rho t U^2) \left(\frac{m\pi}{2l} \right)^2 + \frac{Et}{R^2(1-\nu^2)} \right]} \\
&+ \sum_{m=1}^{\infty} f_m(\xi, \eta, y) \sin \frac{m\pi(l_1+x)}{2l} \quad (2)
\end{aligned}$$

Pressure q in Eq. (1), which is effective pressure to make foil deflect from the equilibrium position of the cylindrical foil, is given by

$$q = p - p_a - (T - \rho t U^2)/R \quad (3)$$

The pressure distribution in the lubricant film between foil and bearing surface, on the other hand, is governed by Reynolds equation in the form

$$\begin{aligned}
\frac{\partial}{\partial x} \left\{ (ph^3 + 6\lambda_a p_a h^2) \frac{\partial p}{\partial x} \right\} \\
+ \frac{\partial}{\partial y} \left\{ (ph^3 + 6\lambda_a p_a h^2) \frac{\partial p}{\partial y} \right\} = 6\mu U \frac{\partial (ph)}{\partial x} \quad (4)
\end{aligned}$$

(a) Plane foil

(b) Cylindrical foil

Fig. 1 Physical model for analysis

Using bearing surface height b , film thickness h is also given by

$$h = w - b \quad (5)$$

and b can be approximated as

$$b = b_s - (x^2 + y^2)/2r + x^2/2R \quad (6)$$

To examine the general characteristics of the spherical foil bearing, let the nondimensional quantities used in the previous report be introduced here. By substituting Eqs. (3) and (6) into Eqs. (4) and (5), respectively, and normalizing Eqs. (1), (2), (4) and (5), we can finally obtain the basic equations for a spherical foil bearing in the form

$$W(X, Y) = \int \int W_0(\Sigma, H, X, Y) Q(\Sigma, H) d\Sigma dH \quad (7)$$

$$\begin{aligned}
W_0 &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cdot \\
&\frac{\sin \frac{\pi m(L_1+\Sigma)}{2L} \sin \frac{\pi n(A_1+H)}{2A} \sin \frac{\pi m(L_1+X)}{2L} \sin \frac{\pi n(A_1+Y)}{2A}}{AL \left[D_1 \left\{ \left(\frac{\pi m}{2L} \right)^2 + \left(\frac{\pi n}{2A} \right)^2 \right\}^2 + \left(\frac{\pi m}{2L} \right)^2 + D_2 \right]} \\
&+ \sum_{m=1}^{\infty} f_m(\Sigma, H, Y) \sin \frac{\pi m(L_1+X)}{2L} \quad (8)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial X} \left[\{ (1 + \delta\kappa + \delta Q) H^3 + \lambda H^2 \} \frac{\partial Q}{\partial X} \right] + \frac{\partial}{\partial Y} \left[\{ (1 + \delta\kappa + \delta Q) H^3 + \lambda H^2 \} \frac{\partial Q}{\partial Y} \right] \\
= \frac{\partial}{\partial X} \{ (1 + \delta\kappa + \delta Q) H \} \quad (9)
\end{aligned}$$

$$H = W - B_s + (X^2 + Y^2)/2 - \kappa X^2/2 \quad (10)$$

The above equations are to be solved using the boundary condition that Q is zero along the bearing boundary.

3.2 Basic equation discretization and numerical solution method

Let the bearing region be divided by the distances of ΔX and ΔY into M and N parts, and grid points be indexed by i and j along X and Y axes, respectively. Using variables at the grid points represented by index numbers i and j , Eq. (7) is approximated by

$$W_{ij} = \sum_{K=1}^{M+1} \sum_{L=1}^{N+1} W_g(X_K, Y_L, X_i, Y_j) Q_{K,L} \Delta X_K \Delta Y_L \quad (11)$$

where the discretized Green function W_g represents an influence coefficient that is foil deformation at (X_i, Y_j) caused by unit concentrated force acting on the position (X_K, Y_L) . Since each grid point has $(M+1)(N+1)$ influence coefficients W_g 's, there are $(M+1)^2(N+1)^2$ influence coefficients in all. Though these W_g 's can be calculated independently of bearing specification, too much storage memory capacity would be required. Moreover, their calculation time is excessively large, because Eq. (8) converges very slowly.

To overcome these difficulties, let the representative influence coefficient $W_g(0, 0, X_i - X_K, Y_j - Y_L)$, which is the influence coefficient with respect to the origin, be substituted for the influence coefficient $W_g(X_K, Y_L, X_i, Y_j)$ at position (X_i, Y_j) with respect to the arbitrary position (X_K, Y_L) . Within a small area, such as a spherical bearing surface, deformation characteristics for actual magnetic tape, which is generally wide and long enough, is almost the same wherever the concentrated force acts except at its edge, and is approximately expressed as a function of the distance from the concentrated force. As shown in the previous paper (2), this assumption is satisfied, except for 2~3 mm near the edge of the cylindrical tape and 10 mm near the edge of the plane tape. Numerical analysis of three dimensional spherical foil bearing including the foil bending stiffness using the existing computers becomes feasible by the introduction of a representative influence coefficient.

The number of influence coefficients $W_g(0, 0, X_i - X_K, Y_j - Y_L)$, which should be calculated first, is reduced to $(2M+1)(2N+1)$ if the bearing surface is divided by the constant steps, as explained previously. Further, the number of coefficients can be reduced to $(M+1)(N+1)$ by the symmetric condition, if the bearing locates on the foil center.

Thus, the representative influence coefficient W_g is expressed in the form

$$W_g = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cdot \frac{\sin \frac{\pi m L_1}{2L} \sin \frac{\pi n A_1}{2A} \sin \frac{\pi m (L_1 + X_i - X_K)}{2L} \sin \frac{\pi n (A_1 + Y_j - Y_L)}{2A}}{AL \left[D_1 \left\{ \left(\frac{\pi m}{2L} \right)^2 + \left(\frac{\pi n}{2A} \right)^2 \right\}^2 + \left(\frac{\pi m}{2L} \right)^2 + D_2 \right]} + \sum_{m=1}^{\infty} f_m (Y_j - Y_L) \sin \frac{\pi m (L_1 + X_i - X_K)}{2L} \quad (12)$$

Applying the finite difference method, Eq. (9) is approximated by the following algebraic equation

in the form

$$F_{ij}(Q_{K,L}) = A_{ij} Q_{i,j+1} + B_{1ij} Q_{i+1,j} + B_{2ij} Q_{ij} + B_{3ij} Q_{i-1,j} + C_{ij} Q_{i,j-1} + D_{ij} = 0 \quad (i=2 \sim M, j=2 \sim N) \quad (13)$$

where $A_{ij} \sim D_{ij}$ are functions of film thickness $H_{K,L}$ and pressure $Q_{K,L}$. In the same manner, Eq. (10) is expressed in the discretized form

$$H_{ij} = W_{ij} - B_z + (X_i^2 + Y_j^2)/2 - \kappa X_i^2/2 \quad (14)$$

Now, let us solve the fifth order nonlinear equation, Eq. (13), with respect to unknown variables $Q_{K,L}$, using the iteration procedure. Since $(M-1) \cdot (N-1)$ equations of Eq. (13) are related to $(M-1) \cdot (N-1)$ unknown variables $Q_{K,L}$ through the influence coefficient, $(M-1)^2(N-1)^2$ matrix coefficients should be calculated in every iteration step and an excessive amount of memory capacity and calculation time would be required if the Newton-Raphson iteration method were applied here, as in the previous report.

To overcome this difficulty, it was tried to pick up only the nearest Y terms, such as $J-1, J, J+1$, from the Newton-Raphson linearized algebraic equations, neglecting other terms $Q_{K,L}$ ($L=2 \sim J-2, J+2 \sim N-1$).

Thus, the column method⁽³⁾, which could save calculation time and memory capacity, was applied to these simplified equations. This method was found to improve calculation performance in the cylindrical foil in which the deformation is locally restricted. However, direct iteration was more effective for the plane foil.

The calculation procedure is the same as that used in the previous report. The results explained later are for the typical case where foil and bearing dimensions are symmetrical, that is $L_1=L, A_1=A$ and $\beta=1$, and are obtained by dividing the bearing surface into 30 and 20 parts along X and Y axes, respectively.

4. Calculation results

4.1 Effective bearing area

Figure 2 shows a half bearing length X_s effect on the minimum spacing $H_{\min}$ and bearing force F , when bearing penetrating depth B_z is zero. It can be seen from this figure that X_s must be greater than 2 to account for the whole effective bearing area. The effective bearing area is almost the same when B_z takes a small value, as treated in this paper. A slight decrease in $H_{\min}$ and F for large X_s is due to the increase in numerical calculation error.

4.2 Distributions of pressures, foil deformations and spacings

Figure 3(a) shows a pressure and a foil deformation distribution for the plane foil when foil stiffness is considered, while Fig. 3(b) shows those when foil stiffness is ignored. As seen from this figure, a foil is hard to deform and the pressure peak value is large when foil stiffness is considered. Further, deformation variation in a direction orthogonal to foil tension

Fig. 2 Effective bearing area

decreases.

Figure 4 shows the contour maps for pressure, foil deformation and spacing distributions, when B_z is 1.5 for the plane foil, while Fig. 5 and Fig. 6 show those for the cylindrical foil, when B_z is 0 and 0.6, respectively. As seen from Figs. 3(a) and 4(b), the foil deforms concavely in the negative pressure region, when B_z is near zero in the case of plane as well as cylindrical foils.

When the bearing penetrates into the plane foil by $B_z = 1.5$, the minimum spacing is still on the bearing apex. On the other hand, since the effective wrap angle in the cylindrical foil is larger than that in the plane foil, its positive pressure region and its magnitude abruptly increase as B_z increases. Therefore, the minimum spacing moves behind the apex when B_z increases in the cylindrical foil.

4.3 Minimum spacing and bearing force

Effects of bending stiffness D_1 and foil length L on the minimum spacing H_{min} and bearing force F in the plane foil are shown in Fig. 7. Figure 8 shows

the effects of bending stiffness D_1 and D_2 on them. It can be seen from these figures that H_{min} decreases and F increases when L is small or D_1 and D_2 are large, because the foil becomes hard to deform. Especially, when D_1 and D_2 are greater than unity, the F/H_{min} increase rate is greater than that when D_1 and D_2 are less than unity. The reason why the bearing force becomes hard to decrease when D_1 and D_2 are small, is considered to lie in the foil flexibility which increases the positive pressure area and decreases the negative pressure area.

Since the deformed foil area is limited to several times the bearing area in the cylindrical foil, L and A do not affect H_{min} and F , when L and A are larger than several times X_s . The curves in Fig. 8 come close to those in Fig. 7, when D_2 and κ approach zero and if the values of L and A are the same as those in Fig. 7. The curves in Fig. 7 also approach those of the perfectly flexible spherical foil bearing shown in the previous report, when D_1 approaches zero. In the actual rotating head mechanism, D_1 and D_2 are about 0.5 and 2, respectively.

Figure 9 shows the effect of the bearing penetrating depth on H_{min} and F . Since the concentrated reaction force is almost proportional to the foil displacement at its acting point, the bearing force F increases proportionally to B_z , except in the neighborhood of zero. The decreasing rate for the minimum spacing, however, becomes small when B_z increases, since the effective bearing area increases and the bearing force reaches equilibrium with the foil reaction force for a slight H_{min} decrease. When B_z is greater than 1.5 in the plane foil and 0.6 in the cylindrical foil, the calculation often diverges due to a sharp change in the pressure in the bearing center region, if 30×20 division mesh is used.

Figure 10 shows the dimensional characteristics of the minimum spacing vs. the bearing scanning speed. If the foil stiffness is not considered in the plane foil, the minimum spacing is about 5 times larger than that when the stiffness is considered. Due to the wrapping effect of the cylindrical foil, the minimum spacing is about a half of that in the plane foil, when the cylinder radius R is 40 mm.

Fig. 3 Pressure and foil deformation distributions in the plane foil

(a) Pressure (b) Foil deformation (c) Spacing

($B_z = 1.5$, $X_s = 3$, $L = 100$, $A = 50$, $D_1 = 1$, $D_2 = 0$, $\kappa = 0$, $\delta = 0.1$, $\lambda = 0.01$)

Fig. 4 Contour maps of bearing characteristics in the plane foil

(a) Pressure (b) Foil deformation (c) Spacing

($B_z = 0$, $X_s = 3$, $L = 25$, $A = 25$, $D_1 = 1$, $D_2 = 1$, $\kappa = 0.5$, $\delta = 0.1$, $\lambda = 0.01$)

Fig. 5 Contour maps of bearing characteristics in the cylindrical foil

(a) Pressure (b) Foil deformation (c) Spacing

($B_z = 0.6$, $X_s = 3$, $L = 25$, $A = 25$, $D_1 = 1$, $D_2 = 1$, $\kappa = 0.5$, $\delta = 0.1$, $\lambda = 0.01$)

Fig. 6 Contour maps of bearing characteristics in the cylindrical foil

5. Experiment

5.1 Description of apparatus and measuring instruments

The experimental apparatus and measuring instruments for the plane foil are illustrated in Fig. 11. A spherical bearing 1, whose radius is 20 mm, is fixed to a micrometer head 7 which moves parallel to the base plate and penetrates into a half inch magnetic tape 2. The magnetic tape is supported by an externally pressurized air bearing 4 and driven by a DC motor capstan 3 whose surface is coated with neoprane rubber to prevent slip when tape speed is high. Tape speed can be varied from zero to 27 m/s. The foil tension can be varied from zero to 50 g/mm by controlling the pressure in the vacuum column 5.

The bearing-to-tape spacing is measured by the stroboscopic white light interferometry method.⁽⁴⁾

Fig. 7 Stiffness D_1 and length L effect on minimum spacing and bearing force in the plane foil

Fig. 8 Stiffness D_1 , D_2 effect on minimum spacing and bearing force in the cylindrical foil

Fig. 9 Bearing penetrating depth B_z effect on spacing and bearing force

Fig. 10 Bearing velocity U effect on minimum spacing

Fig. 11 Experimental apparatus outline

When the window on the magnetic tape, where the magnetic coating is removed, comes just in front of the spherical bearing, a xenon lamp 11 is triggered by a photo coupler 9. Through this window, white light interferometry fringe pattern is measured by a microscope 8. When the bearing-to-tape spacing is less than $1\text{ }\mu\text{m}$, interferometry fringe colors correspond to absolute spacing and measuring accuracy is within $0.05\text{ }\mu\text{m}$.

5.2 Comparison of calculation with experiment

In Fig. 12, the results obtained by the numerical analysis are compared with experimental results obtained using the above mentioned apparatus and instruments when bearing penetrating depth b_z values are zero and $10\text{ }\mu\text{m}$. The reason why the experimental data are scattered widely when b_z is zero is that the tape is apt to vibrate because the reacting force to keep the tape at the equilibrium position is very small. It can be said, from this figure, that the calculated results agree well with the experimental results in both cases.

In this figure, the minimum spacing increases more rapidly than the $U^{2/3}$ rate, when b_z is zero. The reason is considered to be that the molecular mean free path becomes dominant on the minimum spacing, because the normalized molecular mean free path becomes greater than 0.1, when tape velocity is less than 10 m/s .

On the other hand, the minimum spacing curve has a larger exponent than unity with respect to U when b_z is $10\text{ }\mu\text{m}$. It is considered that this phenomenon is caused by not only the molecular mean free path effect but also by the bearing penetrating depth effect, seeing that the normalized bearing penetrating depth B_z increases when U decreases.

Fig. 12 Comparison between calculation and experiment

6. Conclusions

A three dimensional numerical method for a spherical foil bearing, in which the foil has bending stiffness, is presented. The calculated results are illustrated and discussed generally when the bearing penetrating depth is small. The calculated results are found to be quantitatively in good agreement with the experimental results under submicron spacing conditions.

Spherical foil bearing is one of the three dimensional elasto-hydrodynamic lubrication (EHL) problems⁽⁶⁾, but its analysis has a difficulty due to the complexity of foil deformation equation. Since computing time for a Green function is large and the Green function takes different values at all points, excessive time and memory capacity are required to calculate the foil deformation. To overcome these difficulties, the assumption that the Green function on the bearing surface can be represented by one at the bearing center is adopted, because the bearing area is relatively small, compared with the foil area.

Greenberg⁽⁶⁾ also reported on a spherical foil bearing study at the same time when this report was written in Japanese. Though the dynamic characteristics in foil deformation were included in his report, the boundary condition, where the foil edges were simply supported, was employed and the calculated results were not verified quantitatively by the experiment. Further, the method to overcome excessive computing time and memory capacity that would arise in foil deformation calculation was not described.

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