A Displacement Analysis of Overconstrained Mechanisms*

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The coupler motions without any rotations of single-loop overconstrained mechanisms with degrees of mobility 1 have been analyzed considering the figures described by the open chains composing the mechanisms.

The statical characteristics of overconstrained mechanisms and general mechanisms have been compared considering the joint clearances, and the errors of the output displacements of the above mechanisms have been derived by modeling the joint configurations.

1. Introduction

It has been known that there exist fourteen kinds of single-loop overconstrained spatial chains with mobility 1 by a number synthesis of such chains⁽¹⁾, and a dimensional analysis and synthesis of one of the mechanisms derived from the above chains have been carried out⁽²⁾.

In general, the spatial motion of a body may be described instantaneously by a screw displacement, and the screw axis changes its direction with the time. But the couplers of some kinds of overconstrained mechanisms move without changing the direction of those screw axes or without any rotations.

In this paper, the single-loop overconstrained mechanisms whose coupler motions have no rotation are studied: the types of such mechanisms are determined by the kinematic inversion of the above 14 kinds of chains, and the statical characteristics are analyzed by considering the joint clearances.

2. Displacement analysis of coupler motions without any rotations

2.1 Classification of mechanisms considering coupler motions

A coupler motion may be described with use of the spherical indicatrix which corresponds one-to-one to the closure equations of rotation⁽³⁾, that is, when the arc of great circle corresponding to the coupler does not move against the arc corresponding to the fixed link, the coupler has no rotation, and when it turns around a fixed point, the coupler moves without changing the direction of the screw axis. The mechanisms whose coupler motions of rotation are constrained are obtained from 14 kinds

of single-loop overconstrained chains with use of the above method, and classified as shown in Table 1.

In the table, the symbols "-" connecting two open chains mean the couplers which satisfy the motions in the left column, and both ends of each mechanism mean the fixed joints. Three kinds of mechanisms bearing the symbol "*" in the table, for instance, are shown in Fig.1. The mechanisms of Fig.1(a) and (b) may be applied to the conveying machines of the straight line motion and the spatial motion without any rotations respectively. And the mechanism of Fig.1(c) is applicable to the conveying machine, the screw axis of motion of which does not change its direction.

2.2 Displacements of mechanisms whose couplers do not change their directions

We shall analyze the displacements of the mechanisms which have no screw axis in Table 1.

The open chains composing the mechanisms and the generating surfaces described by them are shown in Table 2, in which the right extreme column represents the mechanisms obtained by combining two open chains in the same number.

Now put a moving coordinate system $O_i-x_iy_iz_i$ on the i -th link of an open chain as shown in Fig.2, and express the point X_j on the generating surfaces shown in Table 2 by the algebraic method.

(1) Plane (PP open chain)

$$\begin{pmatrix} X_j \\ Y_j \\ Z_j \end{pmatrix} = \begin{pmatrix} s_2 \sin \theta_1 \sin \alpha_1 + C_{1x} \\ -s_2 \cos \theta_1 \sin \alpha_1 + C_{1y} \\ s_1 + s_2 \cos \alpha_1 + C_{1z} \end{pmatrix} \dots (1)$$

(2) Elliptical cylinder (R_1R_1P open chain)

Conditions for rotation: $\theta_1 + \theta_2 = \text{const.}$, $\alpha_1 = \alpha_2 = \alpha$

$$\begin{pmatrix} X_j \\ Y_j \\ Z_j \end{pmatrix} = \begin{pmatrix} a_1 \cos \theta_1 + s_2 \sin (\theta_1 + \theta_2) \sin \alpha + C_{1x} \\ a_1 \sin \theta_1 - s_2 \cos (\theta_1 + \theta_2) \sin \alpha + C_{1y} \\ s_2 \cos \alpha + C_{1z} \end{pmatrix} \dots (2)$$

(3) Hyperboloid of one sheet (R_1PR_1 open chain) Conditions for rotation: $\theta_1 + \theta_2 = \text{const.}$, $\theta_2 = \pi$, $\alpha_1 = \alpha_2 = \alpha$

$$\begin{pmatrix} X_j \\ Y_j \\ Z_j \end{pmatrix} = \begin{pmatrix} (a_1 - a_2) \cos \theta_1 + s_2 \sin \theta_1 \sin \alpha + C_{1x} \\ (a_1 - a_2) \sin \theta_1 - s_2 \cos \theta_1 \sin \alpha + C_{1y} \\ s_2 \cos \alpha + C_{1z} \end{pmatrix} \dots (3)$$

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Table 1 Classification of mechanisms whose coupler motions have no rotation

Coupler motions		Mechanisms (Open chain - Open chain)
No screw axis	1-dimension	$R_1R_1R_1-R_2R_2R_2^*$
	2-dimension	$PP \begin{cases} PP \\ PR_1R_1, R_1PR_1, R_1R_1P \\ R_1R_1R_1, R_1R_1R_2R_2, R_1R_2R_2R_1 \end{cases}$ $R_1R_1 \begin{cases} PPP \\ PPR_2R_2, PR_2PR_2, PR_2R_2P, R_2PPR_2, R_2PR_2P, R_2R_2PP \\ PR_2P_2R_2, R_2PR_2R_2, R_2R_2PR_2, R_2R_2R_2P \end{cases}$ $R_1R_1R_1-PR_2R_2, R_2PR_2, R_2R_2P$
	3-dimension	$PR_1R_1-PR_2R_2, R_2PR_2, R_2R_2P$ $R_1PR_1-PR_2R_2, R_2PR_2$ $R_1R_1P-PR_2R_2^*$
Direction of rotational axis is constant	2-dimension	$R_1R_1 \begin{cases} PPR_1, PR_1P, R_1PP \\ PR_1R_1, R_1PR_1, R_1R_1P \\ PR_1R_2R_2, R_1PR_2R_2, R_1R_2PR_2, R_1R_2R_2P \\ R_1R_2R_2R_2 \end{cases}$
Direction of screw axis is constant	3-dimension	$PR_1^* \begin{cases} PPR_1, PR_1P, R_1PP \\ PR_1R_1^*, R_1PR_1, R_1R_1P \\ PR_1R_2R_2, R_1PR_2R_2, R_1R_2PR_2, R_1R_2R_2P \\ R_1R_2R_2R_2 \end{cases}$ $R_1R_1R_2 \begin{cases} PPR_2, PR_2P, R_2PP \\ PR_2R_2, R_2PR_2, R_2R_2P \end{cases}$

Note; Axes of pairs having same suffix are parallel with each other

(4) Torus ($R_1R_2R_2R_1$ open chain) Conditions for rotation; $\theta_2+\theta_3=\pi$, $\theta_1+\theta_4=\text{const.}$, $\alpha_1=\alpha_2=\alpha$

$$X_j = \begin{pmatrix} a_2(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2 \cos \alpha) \\ a_2(\sin \theta_1 \cos \theta_2 + \cos \theta_1 \sin \theta_2 \cos \alpha) \\ a_2 \sin \theta_2 \sin \alpha \\ + (a_1 - a_2) \cos \theta_1 + (s_2 + s_3) \sin \theta_1 \sin \alpha + C_4x \\ + (a_1 - a_2) \sin \theta_1 - (s_2 + s_3) \cos \theta_1 \sin \alpha + C_4y \\ + C_4z \end{pmatrix} \quad \dots\dots\dots(4)$$

(5) Surface of the fourth order ($R_1R_1R_2R_2$ open chain) Conditions for rotation; $\theta_1+\theta_2=\text{const.}$, $\theta_3+\theta_4=\text{const.}$

$$X_j = \begin{pmatrix} a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \cos \theta_3 \\ a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2) \cos \theta_3 \\ a_2 \sin \theta_3 \sin \alpha_2 \\ - a_2 \sin(\theta_1 + \theta_2) \sin \theta_3 \cos \alpha_2 + C_5x \\ + a_2 \cos(\theta_1 + \theta_2) \sin \theta_3 \cos \alpha_2 + C_5y \\ + C_5z \end{pmatrix} \quad \dots\dots\dots(5)$$

Where C_{jx}, C_{jy}, C_{js} ($j=1\sim 5$) are constants.

Since the variables of the above open chains are 2, the mechanisms obtained by combining those two chains have 4 variables. On the other hand, the number of closure equations is three, then three variables may be obtained by giving the input value.

3. Degrees of mobility and transmissibility of mechanisms with clearances

3.1 Relationships between clearances and degrees of mobility of mechanisms

When there exist clearances, degrees of freedoms of pairs are increased. Considering this increase f_c , redundant freedoms $f_R^{(s)}$ and passive freedoms $f_P^{(s)}$, the increase of degrees of mobility of mechanisms caused by the clearances may be expressed as follows;

(a) $R_1R_1R_1R_2R_2R_2$ mech.

(b) $R_1R_1P_1P_2R_2R_2$ mech.

(c) $PR_1PR_1R_1$ mech.

Fig.1 Conveying machines with use of overconstrained mechanisms

Table 2 Open chains and their generating surfaces

No.	Open chains	Generating surfaces	Open chains	Generating surfaces	Mechanisms
1	PP	Plane	PP	Plane	PPPP
2	PP		$R_1R_1R_1$		$PPR_1R_1R_1$
3	$R_1R_1R_1$		$R_2R_2R_2$		$R_1R_1R_1R_2R_2R_2$
4	PP	Plane	R_1R_1P	Elliptical cylinder	$PPPR_1R_1$
5	PP		PR_1R_1		PPR_1R_1P
6	$R_1R_1R_1$		R_2R_2P		$R_1R_1R_1PR_2P_2$
7	PP	Plane	R_1PR_1	Hyperboloid of one sheet	PPR_1PR_1
8	$R_1R_1R_1$		R_2PR_2		$R_1R_1R_1R_2PR_2$
9	PP	Plane	$R_1R_2R_2R_1$	Torus	$PPR_1R_2R_2R_1$
10	PP	Plane	$R_2R_2R_1R_1$	Surface of the forth order	$PPR_1R_1R_2R_2$
11	R_1R_1P	Elliptical cylinder	R_2R_2P	Elliptical cylinder	$R_1R_1PPR_2R_2$
12	PR_1R_1		PR_2R_2		$PR_1R_1R_2R_2P$
13	R_1R_1P		PR_2R_2		$R_1R_1PR_2R_2P$
14	R_1R_1P	Elliptical cylinder	R_2PR_2	Hyperboloid of one sheet	$R_1R_1PR_2PR_2$
15	PR_1R_1		R_2PR_2		$PR_1R_1R_2PR_2$
16	R_1PR_1	Hyperboloid of one sheet	R_2PR_2	Hyperboloid of one sheet	$R_1PR_1R_2PR_2$

Note; PP chain generates the total plane and $R_iR_iR_i$ ($i=1, 2$) chain generates a part of plane

$$\Delta M = f_c - f_r - f_p \dots\dots\dots(6)$$

When $\Delta M \geq 1$, the mechanism has multi-degrees of mobility and displacement errors. Then, we shall clarify the relationships between the clearances and the displacement errors with use of Eq.(6) about the general and overconstrained mechanisms.

3.1.1 General mechanisms In general mechanisms, we can put $f_p=0$, and Eq.(6) becomes as follows:

$$\Delta M = f_c - f_r \dots\dots\dots(7)$$

Therefore, when the increase of degrees of freedom of pairs is smaller than redundant freedoms, the mechanisms have no displacement errors.

3.1.2 Overconstrained mechanisms

(i) When the mechanism has no passive freedoms Putting $f_p=0$, similarly to the case of general mechanism, we have Eq.(7). For example, the following mechanisms apply to this case.

(a) A plane mechanism with only radial clearances.

(b) A spherical mechanism with clearances on a spherical surface.

(c) Sarrus mechanism with only thrust clearances.

(ii) When the mechanism has passive freedoms In this case, pairs with clearances have passive freedoms. Now we denote the sum of freedoms of pairs considering clearances by ΣF_i . When

$$(1) \Sigma F_i - f_r \leq 7 \dots\dots\dots(8)$$

the mechanism has neither increase of degrees of mobility nor displacement errors even though there exist clearances. And

Fig.2 Coordinate systems of a spatial mechanism

when

$$(2) \Sigma F_i - f_r \geq 8 \dots\dots\dots(9)$$

the mechanism has the displacement errors similarly to the general mechanism.

The condition (1) may be satisfied by the following mechanisms.

(a) A plane (or spherical) mechanism with thrust clearance at a pair or a plane (or spherical) mechanism with radial and thrust clearances at a pair.

(b) Sarrus mechanism with radial and thrust clearances at two pairs.

And the condition (2) may be satisfied by the following mechanism.

(a) Sarrus mechanism with radial clearances at two or more pairs.

From the above Eqs.(6) ~ (9), we know whether the mechanism with clearances has

displacement errors or not.

3.2 Influences of clearances upon transmissibility and displacement errors

When there exist clearances at pairs, the force and moment acting on the output link differ from those when the mechanism has no clearance, and then the transmissibility⁽⁴⁾ also changes.

Now, we denote the force and moment acting from the coupler link to the output link at a pair J, the velocity and angular velocity of the output link by F , M , V and Ω respectively as shown in Fig.3. In the figure, $J-xyz$ is a moving coordinate system fixed on the link, similarly to Fig.2. The components of F , M , V and Ω in each direction of the moving coordinate axis are represented by bearing the suffix x , y or z . For example, when there exists a clearance in the z direction, the pairing elements may move in the direction relatively with each other. Therefore the conditions for the velocity and force are given by $V_z \neq 0$ and $F_z = 0$ respectively. Similarly, if the pairing elements can turn relatively around the z axis, the conditions for the angular

velocity and moment are given by $\Omega_z \neq 0$ and $M_z = 0$ respectively. As mentioned above, the relationships between the clearances and the conditions for the velocity, angular velocity, force and moment of a revolute or a prismatic pair are obtained as shown in Table 3.

As a result, the transmissibility of the overconstrained mechanism is clarified as follows:

(i) When the mechanism has no passive freedoms It makes a little change of the transmissibility if the clearance is small, because the force and moment exist regardless of the clearances, and the transmission index may change continuously from the state with no clearance.

(ii) When the mechanism has passive freedoms This state of Nos.2~5 for the revolute pair in the table correspond to (1) in 3.1.2(ii), and the transmission index may change largely from the state with no clearance even though the clearance is small.

4. Displacement analysis of mechanisms with clearances

Fig.3 Force and moment acting from the coupler link to the output link

(a) Revolute pair

(b) Prismatic pair

Fig.4 Models of the joint configurations

Table 3 Conditions for velocity, force, angular velocity and moment

Joint	No.	Direction of clearances	Velocity	Force	Angular velocity	Moment
R	1	Nothing	$V_x=0, V_y=0, V_z=0$	$F_x \neq 0, F_y \neq 0, F_z \neq 0$	$\Omega_z \neq 0$	$M_z = 0$
	2	x (y)	$V_x \neq 0$ ($V_y \neq 0$)	$F_x = 0$ ($F_y = 0$)	$\Omega_z \neq 0$	$M_z = 0$
	3	z	$V_z \neq 0$	$F_z = 0$	$\Omega_z \neq 0$	$M_z = 0$
	4	x, z (y, z)	$V_x \neq 0, V_z \neq 0$ ($V_y \neq 0, V_z \neq 0$)	$F_x = 0, F_z = 0$ ($F_y = 0, F_z = 0$)	$\Omega_y \neq 0, \Omega_z \neq 0$ ($\Omega_x \neq 0, \Omega_z \neq 0$)	$M_y = 0, M_z = 0$ ($M_x = 0, M_z = 0$)
	5	x, y	$V_x \neq 0, V_y \neq 0$	$F_x = 0, F_y = 0$	$\Omega_z \neq 0$	$M_z = 0$
	6	x, y, z	$V_x \neq 0, V_y \neq 0, V_z \neq 0$	$F_x = 0, F_y = 0, F_z = 0$	$\Omega_x \neq 0, \Omega_y \neq 0, \Omega_z \neq 0$	$M_x = 0, M_y = 0, M_z = 0$
P	1	Nothing	$V_z \neq 0$	$F_z = 0$	$\Omega_x = 0, \Omega_y = 0, \Omega_z = 0$	$M_x = 0, M_y = 0, M_z = 0$
	2	x (y)	$V_x \neq 0, V_z \neq 0$ ($V_y \neq 0, V_z \neq 0$)	$F_x = 0, F_z = 0$ ($F_y = 0, F_z = 0$)	$\Omega_y \neq 0$ ($\Omega_x \neq 0$)	$M_y = 0$ ($M_x = 0$)
	3	x, y, z	$V_x \neq 0, V_y \neq 0, V_z \neq 0$	$F_x = 0, F_y = 0, F_z = 0$	$\Omega_x \neq 0, \Omega_y \neq 0, \Omega_z \neq 0$	$M_x = 0, M_y = 0, M_z = 0$

(R: Revolute pair, P: Prismatic pair)

We shall investigate the displacement errors of mechanisms by modeling the joint configurations.

4.1 Modeling the joint configurations

we shall model the configurations of a revolute and prismatic pairs as shown in Fig.4. Let C_{ri} , C_{ri} , $2L_i$ and β_{imax} be the thrust and radial clearances, the axial length of the bearing and the relative angle between the pin and bearing respectively in Fig.(a), then we have

$$C_{ri} = \sqrt{L_i^2 + C_{ri}^2} - L_i \quad (10)$$

$$\beta_{imax} = \tan^{-1}(C_{ri}/L_i) \quad (11)$$

Similarly, let C_{ri} , $2L_i$ and τ_i be the radial clearance, the axial length of bearing and the relative swinging angle between the pin and bearing respectively in Fig.(b), then β_{imax} is given by Eq.(11).

4.2 Displacement errors of mechanisms with clearances

we shall analyze the displacement errors of mechanisms satisfying Eq.(9) in Table 2 by using the modeled joints in Section 4.1. In Table 1, the overconstrained mechanisms are composed of following six kinds of basic open chains:

- (i) PP, (ii) R₁R₁R₁, (iii) R₁R₁P
(iv) R₁PR₁, (v) R₁R₁R₁R₁, (vi) R₁R₁R₁R₁

Where there is no clearance at the fixed joint of the right or left extreme end of each chain.

Now, fixing the input link in an arbitrary position and applying a force or torque to the output link, the displacement errors of each basic open chain are obtained as follows:

Fig.5 Output error of the PP chain

Fig.6 Output error of the R₁R₁R₁ chain

(i) PP chain Figure 5 shows a dyad with two prismatic pairs J_i and J_{i+1} , and the fixed joint J_{i+1} has no clearance. The output error ΔP_i caused by the clearance C_{ri} at the pair J_i is

$$\Delta P_i = C_{ri} / \sin \alpha_i \quad (12)$$

Where α_i is the twist angle between the joint axes.

(ii) R₁R₁R₁ chain Figure 6 shows a triad in which the pairs J_i and J_{i+1} have clearances and J_{i-1} has no clearance. Applying counterclockwise and clockwise torques to the link $i+1$, the maximum and minimum values of the angular displacement φ of a link $i+1$ become as follows:

$$\begin{bmatrix} \varphi_{\max} \\ \varphi_{\min} \end{bmatrix} = \begin{bmatrix} \tan^{-1}(B_i/A_i) - \cos^{-1} \left\{ \frac{A_i^2 + B_i^2 + a_{i+1}^2}{(2a_{i+1}\sqrt{A_i^2 + B_i^2})} \right\} \\ \tan^{-1}(B_i/A_i) - \cos^{-1} \left\{ \frac{A_i^2 + B_i^2 + a_{i+1}^2}{(2a_{i+1}\sqrt{A_i^2 + B_i^2})} \right\} \end{bmatrix} \quad (13)$$

Where $A_i = X_{i+1} - X_{i-1}$, $B_i = Y_{i+1} - Y_{i-1}$, and $(X_{i-1}, Y_{i-1}, Z_{i-1})$ and $(X_{i+1}, Y_{i+1}, Z_{i+1})$ are the centers of the pairing elements J_{i-1} and J_{i+1} , respectively, and r_i is the relative displacement to the radial direction between the pairing elements of J_i . In Eq.(13), since the joint axes are parallel one to another when φ becomes maximum or minimum, the relative displacements between pairing elements are given by $r_{i\max} = r_{i\min} = 2C_{ri}$.

(iii) R₁R₁P chain In Fig.7(a), the prismatic pair is fixed and O-xy is a moving coordinate system on the link connecting two revolute pairs. The existing domain of the pin center when the pin moves on this

(a)

(b)

Fig.7 Output error of the R₁R₁P chain

plane is shown in Fig.7(b) by the oblique lines, which are formulated as

$$\left. \begin{aligned} x &= \frac{a_i}{2} \cos \beta_i \pm (C_{r1} \cos \beta_i - L_i \sin \beta_i) \\ y &= \frac{a_i}{2} \sin \beta_i \pm (C_{r1} + L_i - C_{r1} \sin \beta_i - L_i \cos \beta_i) \end{aligned} \right\} \dots\dots\dots(14)$$

Moreover, changing β_i , the existing domain may become a domain enclosed by the curves and straight line segments in the figure. Seeking for a point of intersection of the limit line with the displacement vector drawn parallel from the point $(a_i/2, 0)$ to the axis of a prismatic pair, the displacement errors of open chains are obtained. When the displacement vector is not coplanar with the joint axis, the displacement errors are obtained by transforming the coordinate systems. The displacement errors caused by the clearances about the other basic open chains are also obtained by the similar method.

5. Example

Figure 8 shows a $R_1C_1R_2$ mechanism⁽⁵⁾ with number of general constraint 3, which transforms a turning motion of the crank 2 into that of the rocker 4. This is a special case of the mechanism No.11 in Table 2, and is expressed fundamentally as the $R_1R_1P_1P_2R_2R_2$ mechanism. Now, given the kinematic parameters as shown in Table 4 and the input angle $\theta (= \theta_2)$, we obtain the output angle $\varphi (= 2\pi - \theta_1)$ and the displacements s_4 and s_5 at the pairs J_2 and J_3 respectively. This mechanism consists of two open chains $R_1R_1P_1$

Table 4 Parameters of mechanism

a_{12} mm	100.0	a_{41} mm	22.5
a_{23} mm	52.0	C_{r2} mm	0.01
a_{34} mm	200.0	C_{r3} mm	0.01

Fig.8 $R_1R_1P_1P_2R_2R_2$ mechanism

and $R_2R_2P_2$, then the displacements may be obtained with use of the closure equations (2) properly transformed. Considering the conditions for the twist angles $\alpha_1 = \alpha_4 = 90^\circ$, $\alpha_2 = \alpha_3 = \alpha_5 = \alpha_6 = 0^\circ$, the distances between joint axes $a_2 = 0$, $a_3 = 0$, the translational displacements $s_1 = \text{const.}$, $s_2 = \text{const.}$, $s_3 = 0$, $s_4 (= \text{unknown})$, $s_5 (= \text{unknown})$, $s_6 = 0$, the angular displacements $\theta_1 + \theta_4 = \pi$, $\theta_2 = \text{the input angular displacement (=known)}$, $\theta_2 + \theta_3 = \pi$, $\theta_3 = 0$, $\theta_4 = 0$ and $\theta_6 = \text{the output angular displacement (=unknown)}$, we have

$$\begin{bmatrix} a_1 + a_2 \cos \theta_2 - a_6 \cos (\pi - \theta_6) - a_4 \\ -a_6 \sin (\pi - \theta_6) - s_2 - s_4 \\ -a_2 \sin \theta_2 - s_1 - s_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \dots\dots\dots(15)$$

Solving the above equations about s_4 , s_5 and θ_6 , and putting $a_1 = a_{41}$, $a_2 = a_{12}$, $a_4 = a_{23}$, $a_6 = a_{34}$ and $\pi - \theta_6 = \theta_1$, we have

$$\left. \begin{aligned} \theta_1 &= \cos^{-1} \left(\frac{a_{41} - a_{23} + a_{12} \cos \theta_2}{a_{34}} \right) \\ s_4 &= -a_{34} \sin \theta_1 - s_2 \\ s_5 &= -a_{12} \sin \theta_2 - s_1 \end{aligned} \right\} \dots\dots\dots(16)$$

Considering the relation $\varphi = 2\pi - \theta_1$ the displacements φ , s_4 and s_5 are obtained as shown in Fig.9(a) and (b).

Moreover, the maximum and minimum output errors $\Delta\varphi_{\max}$ and $\Delta\varphi_{\min}$ are given by

$$\begin{bmatrix} \Delta\varphi_{\max} \\ \Delta\varphi_{\min} \end{bmatrix} = \begin{bmatrix} \cos^{-1} \left\{ \frac{-(a_{41} + a_{12} \cos \theta_2 - (a_{23} - C_{r2} - C_{r3}))}{a_{34}} \right\} \\ \cos^{-1} \left\{ \frac{-(a_{41} + a_{12} \cos \theta_2 - (a_{23} + C_{r2} + C_{r3}))}{a_{34}} \right\} \end{bmatrix}$$

(a) $\theta - \varphi$ curve

(b) $\theta - s_4, s_5$ curves

(c) $\theta - \Delta\varphi$ curve

Fig.9 Output displacements and errors of $R_1R_1P_1P_2R_2R_2$ mechanism

$$\left. \begin{aligned} &-\cos^{-1}\left\{\frac{-(a_{41}+a_{12}\cos\theta_2-a_{23})}{a_{34}}\right\} \\ &-\cos^{-1}\left\{\frac{-(a_{41}+a_{12}\cos\theta_2-a_{23})}{a_{34}}\right\} \end{aligned} \right] \dots\dots\dots(17)$$

and they are shown in Fig.9(c).

6. Conclusions

The results obtained in this paper are summarized as follows.

(1) The determination of the types single-loop overconstrained path generators has been made easy by classifying the types of the coupler motions of such mechanisms.

(2) The displacement analyses of the coupler motions without any rotations have been made easy by formulating the coupler surfaces of second and fourth order described by the open chains consisting the mechanisms.

(3) The discriminant of degrees of mobility which is applicable to both the overconstrained mechanisms and the general mechanisms has been derived in consideration of the joint clearances, and it has been

clarified the relationships between the joint clearances and the degrees of mobility, the transmissibility, the displacement errors of mechanisms.

(4) A numerical analysis of the displacement errors of the overconstrained mechanisms has been made possible by formulating the displacement errors caused by the joint clearances of the open chains consisting the mechanisms.

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