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Reattachment Flow Issuing from a Finite Width Nozzle*

(Report 3. Effects of Inclinations of Reattachment Wall)

By Tsutomu NOZAKI**, Keiji HATTA***

and Masahiro NAKASHIMA****

By using the results for a two-dimensional jet flow issuing from a finite width nozzle, an approximate calculation of the reattachment flow having various offsets, inclinations of the reattachment wall and initial turbulence intensities was done and compared with experiments. The calculated results of a reattachment distance agree well with the experimental ones in a wide range of offsets, except very small ones. The approximate calculation reveals that the maximum inclination which maintains the reattachment is independent of both the initial turbulence intensity and the offset. The experimental results show that there exist three distinct regions, that is, a natural reattachment region, a forced reattachment region and a non-reattachment region, according to the inclination. Both the results of the calculation and the experiment reveal that the mean bubble pressure shows the minimum value at a certain inclination.

1. Introduction

In order to study a generalized reattachment flow, an approximate calculation of the flow having the reattachment wall with an arbitrary inclination is carried out, and at the same time, the experiments are also done, and then the results are compared.

Both the reattachment point model and the control volume model have been used for the approximate calculations of the reattachment flow. In the case of the flow having a reattachment wall with no inclination, the calculated results of the reattachment distance by the authors⁽¹⁾ using the reattachment point model agree well with the experimental ones. But in the case of the flow having a reattachment wall with any inclinations, the calculation using the reattachment point model is not available, because the inclination is not taken into consideration in this model. In fact, in such a flow, as the inclination becomes large, discrepancy between the results of the experiment and the calculation increases. Therefore, a control volume model in which the inclination of the wall is taken into consideration is used, and the coefficient of base pressure, which is de-

defined as the ratio of the mean bubble pressure to the pressure on the nozzle plane, is chosen so that the results from both models may be the same expression at no inclination. This value of the coefficient of base pressure coincides with the one proposed by Perry⁽²⁾.

About the experiments of the flow having various inclinations of the reattachment wall, Bourque and Newman⁽³⁾ reported on the case of no offset, and also McRee and Edwards⁽⁴⁾ and Rodrigue⁽⁵⁾ reported on a flow having arbitrary offsets. But since none of their experiments prescribed the initial turbulence intensity of the flow, they can not be compared with the results of the approximate calculation by the authors considering the effects of the initial turbulence intensity of the flow. Therefore, the experiments prescribing the initial turbulence intensity have been done by the authors.

The experiments by Bourque and Newman⁽³⁾ with a flow having no offset revealed that there exist three regions, where the jet is always reattached, the jet is reattached or separated depending on how the flow is started, and the jet is always separated as a free jet. In authors' experiments having arbitrary offsets, it will be discussed that these regions are influenced by the length of reattachment wall and the initial turbulence intensity. Furthermore, for a flow having any inclinations, it is found from the experimental and calculated results that a mean bubble pressure shows the minimum value at a certain inclination for each offset.

2. Flow model and approximate calculation

The approximate calculation of the re-

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** Associate Professor, Faculty of Engineering, Kagoshima University, Koorimoto, Kagoshima.

*** Professor, Chubu Institute of Technology, Matsumoto-cho, Kasugai.

**** Lecturer, Kagoshima Technical College, Shinko, Hayato-cho, Aira-gun, Kagoshima.

attachment flow having any inclinations of the reattachment wall is based on the following assumptions:

- (i) The flow is two-dimensional and steady everywhere.
- (ii) The velocity profile at the nozzle exit is uniform.
- (iii) The control volume model is used and the base pressure is introduced, but a curvature of the jet is constant taking a mean bubble pressure.
- (iv) The flow has the same structure as a free jet along the curved jet center line.
- (v) The wall friction is neglected.

In addition to these assumptions, two new assumptions are set:

- (vi) The jet consists of two regions, that is, the zone of flow establishment and the zone of established flow.
- (vii) The angle between a perpendicular to the reattachment wall and the line joining the reattachment point on to the center of curvature of the jet center line, say, the center angle of reattachment, differs from the angle between the curved jet center line and the reattachment wall.

A schema of the reattachment flow model having an arbitrary inclination of the reattachment wall and the symbols are shown in Fig.1. Let the axis x' be taken along the curved jet center line and the axis y' perpendicular to it, and u and v denote the velocity components in each direction. Let ρ denote the density of fluid, p_{mean} the mean bubble pressure and p_a the atmospheric pressure, then the radius of curvature of the jet center line is given by

$$\frac{R}{b_0} = \frac{2\rho U_0^2}{p_a - p_{mean}}, \quad (1)$$

and after introducing the coefficient of mean bubble pressure

$$Cp_{mean} = \frac{p_{mean} - p_a}{\rho U_0^2 / 2}, \quad (2)$$

Eq.(1) reduces to

$$Cp_{mean} = -4 \frac{b_0}{R}. \quad (3)^{(6)}$$

For the flow having the reattachment wall with no inclination, the equation for the reattachment point model

$$J_s \cos \theta = J_{s1} - J_{s2} \quad (4)$$

is used, where J_s denote the momentum of the fluid issuing from the nozzle exit per unit time, J_{s1} that of the fluid flowing downstream along the wall and J_{s2} that of the fluid returning into the bubble. Then behaviours of the reattachment flow are well expressed as reported previously⁽¹⁾. But in the case of the flow having an arbitrary inclination, the method using the above model revealed that the value of reattachment distance, for example, is smaller than that of experiment as the inclinations and the offsets increase, because the inclination of the reattachment wall is not taken into consideration in

Fig.1 Schema of a reattachment flow model with an arbitrary inclination of the wall

that model. Then, in such a flow, the control volume model is used, since in this model the inclination appears. By considering the base pressure p_b at the nozzle plane in the bubble, the equation for this model is given by

$$J_s \cos \alpha - J_{s1} = (p_a - p_b)(D + b_0) \cos \alpha, \quad (5)$$

where α is an inclination of the reattachment wall. In Eq.(5), J_s and J_{s1} are given as follows:

$$J_s = 2\rho U_0^2 b_0 \quad (6)$$

and

$$J_{s1} = \int_{-b}^{y'_c} \rho u^2 dy', \quad (7)$$

where $y'_c = A'C'$ in Fig.1. The distance y'_r between the jet center line and the reattachment streamline has to satisfy the following continuity equation:

$$\int_0^{y'_r} \rho u dy' = \rho U_0 b_0. \quad (8)$$

In addition to the above conditions, from the geometrical relations, the length x'_c from the nozzle exit to the reattachment wall measured along the jet center line is given by

$$\frac{x'_c}{b_0} = \frac{R}{b_0} (\theta + \alpha), \quad (9)$$

and the length x_c from the point of intersection of the nozzle plane and the reattachment wall to the point of intersection of the jet center line and the reattachment wall is given as follows:

$$\frac{x_c}{b_0} = \frac{R}{b_0} \frac{\sin(\theta + \alpha)}{\cos \alpha}. \quad (10)$$

Furthermore, the relation

$$\frac{R}{b_0} \left(1 - \frac{\cos \theta}{\cos \alpha}\right) = \frac{D}{b_0} + 1 \quad (11)$$

is used. Let the center angle of reattachment be ζ , then the length x_a' from the nozzle exit to the point C measured along the jet center line is given as follows:

$$\frac{x_a'}{b_0} = \frac{R}{b_0}(\zeta + \alpha). \quad (12)$$

By denoting $\overline{AC} = y_a'$ in Fig.1, the reattachment distance x_a is given by

$$\frac{x_a}{b_0} = \left(\frac{R}{b_0} - \frac{y_a'}{b_0} \right) \frac{\sin(\zeta + \alpha)}{\cos \alpha} \quad (13)$$

and we obtain

$$\frac{R}{b_0} - \frac{D}{b_0} - 1 = \left(\frac{R}{b_0} - \frac{y_a'}{b_0} \right) \frac{\cos \zeta}{\cos \alpha}. \quad (14)$$

Now, the solutions of a two-dimensional jet issuing from a finite width nozzle⁽⁷⁾, which will be used in the following calculations, are stated briefly. In the zone of flow establishment, a parameter

$$\eta' = (y' - b_c) / (b - b_c) \quad (15)$$

is used, where $2b_c$ and $2b$ are the core width and the width of the jet, respectively, and the velocity in the region $b_c \leq y' \leq b$ is given as follows:

$$u = U_0 f(\eta'). \quad (16)$$

In the zone of established flow, using the parameter

$$\eta' = y' / b \quad (17)$$

and with the velocity U on the jet center line, the velocity is also given by

$$u = U f(\eta') \quad (18)$$

for $y' \leq b$. The function $f(\eta')$ contained in Eqs.(16) and (18) is given as a polynomial of η'

$$f(\eta') = 1 - 6\eta'^2 + 8\eta'^3 - 3\eta'^4 \quad (19)$$

by considering the boundary conditions.

In the zone of flow establishment, the half jet width and the half core width are given as follows, respectively:

$$\frac{b}{b_0} = 1 + \frac{6864}{469} \kappa \frac{x'}{b_0} \quad (20)$$

and

$$\frac{b_c}{b_0} = 1 - \frac{13728}{2345} \kappa \frac{x'}{b_0}, \quad (21)$$

where κ is an empirical constant contained in the eddy kinematic viscosity. From Eq.(21), the length of the zone of flow establishment x_1' is obtained as

$$\frac{x_1'}{b_0} = \frac{2345}{13728} \frac{1}{\kappa} \quad (22)$$

from the condition that the core vanishes, namely, $b_c = 0$.

In the zone of established flow, the half jet width and the velocity on the jet center line are given as follows:

$$\frac{b}{b_0} = \frac{27456}{2905} \kappa \left(\frac{x'}{b_0} - \frac{x_1'}{b_0} \right) + \frac{7}{2} \quad (23)$$

and

$$\frac{U}{U_0} = \left\{ \frac{54912}{20335} \kappa \left(\frac{x'}{b_0} - \frac{x_1'}{b_0} \right) + 1 \right\}^{-1/2}. \quad (24)$$

In order to carry out the approximate calculation mentioned above, the value of κ which is contained in the eddy kinematic viscosity is expressed by using the initial

turbulence intensity T_0 . If we denote κ for the zone of flow establishment with the subscript 1 and that for the zone of established flow with 2, these are given by

$$\kappa_1 = 0.0102 + 0.393T_0 - 1.61T_0^2 \quad (25)$$

and

$$\kappa_2 = 0.0283 + 0.264T_0 - 1.24T_0^2, \quad (26)$$

respectively⁽⁸⁾.

2.1 Reattachment in the zone of flow establishment

In the zone of flow establishment, Eq.(7) becomes

$$J_{S1} = \int_{-b}^{-b_c} \rho u^2 dy' + 2\rho U_0^2 b_c + \int_{b_c}^{y_c'} \rho u^2 dy'. \quad (27)$$

Using Eqs.(6),(15),(16),(20),(21) and (27), Eq.(5) based on the control volume model becomes

$$\begin{aligned} \cos \alpha - \left(1 - \frac{3432}{335} \kappa \frac{x_c'}{b_0} \int_{\eta_c'}^{\eta_c'} r^2 d\eta' \right) \\ = \frac{p_a - p_b}{2\rho U_0^2} \left(\frac{D}{b_0} + 1 \right) \cos \alpha. \end{aligned} \quad (28)$$

The coefficient of base pressure

$$Cp_b = \frac{p_a - p_{mean}}{p_a - p_b} \quad (29)$$

is introduced, and using Eqs.(2),(3) and (29), Eq.(28) becomes

$$\begin{aligned} \cos \alpha - \left(1 - \frac{3432}{335} \kappa \frac{x_c'}{b_0} \int_{\eta_c'}^{\eta_c'} r^2 d\eta' \right) \\ = \left(\frac{R}{b_0} Cp_b \right)^{-1} \left(\frac{D}{b_0} + 1 \right) \cos \alpha. \end{aligned} \quad (30)$$

Substituting Eqs.(9) and (11) into Eq.(30), we obtain

$$\begin{aligned} (\cos \alpha - \cos \theta)^2 = Cp_b \left\{ (\cos \alpha - \cos \theta)(\cos \alpha - 1) \right. \\ \left. + \frac{3432}{335} \kappa \left(\frac{D}{b_0} + 1 \right) (\theta + \alpha) \cos \alpha \int_{\eta_c'}^{\eta_c'} r^2 d\eta' \right\}. \end{aligned} \quad (31)$$

The value of Cp_b becomes 2 from the condition that the result deduced by Eq.(4) based on the reattachment point model coincides with Eq.(31) in the case of no inclination ($\alpha = 0^\circ$). This is the same result as used by Perry⁽²⁾. He pointed out that the value of Cp_b depends on the offsets and inclinations, but Cp_b remains almost constant from the experimental results by the authors. The value of η_c' contained in Eq.(31) is obtained as 0.341 regardless of x_c' by using Eq.(8). By using the values of Cp_b and η_c' , Eq.(31) determines the value of θ for each offset ratio and inclination of the reattachment wall at given initial turbulence intensity. Therefore, the radius of curvature of a jet center line R is determined from Eq.(11), and x_c' and the coefficient of mean bubble pressure Cp_{mean} are determined from Eqs.(9) and (3), respectively.

Next the reattachment distance is to be determined. Putting $\eta' = \eta_a'$ for $y' = y_a'$ in Eq.(15), we obtain

$$\frac{y_a'}{b_0} = \frac{b_c}{b_0} + \eta_a' \left(\frac{b}{b_0} - \frac{b_c}{b_0} \right), \quad (32)$$

and substituting Eqs.(20) and (21) into Eq.

(32), we obtain

$$\frac{y_{a'}}{b_0} = 1 + \frac{6864}{335} \kappa \frac{x_{a'}}{b_0} \left(\eta_{a'} - \frac{2}{7} \right), \quad (33)$$

since $x' = x_{a'}$ for the reattachment point. Substituting Eqs.(33) and (12) into Eq.(14), we obtain

$$\frac{R}{b_0} - \frac{D}{b_0} - 1 = \left\{ \frac{R}{b_0} - 1 - \frac{6864}{335} \kappa \frac{R}{b_0} \times (\zeta + \alpha) \left(\eta_{a'} - \frac{2}{7} \right) \right\} \frac{\cos \zeta}{\cos \alpha}, \quad (34)$$

and using the value of R just determined above, Eq.(34) gives the center angle of reattachment ζ . Then $x_{a'}$ is determined from Eq.(12), $y_{a'}$ is determined from Eq.(33) and the reattachment distance x_a is determined from Eq.(13).

2.2 Reattachment in the zone of established flow

In the zone of established flow, Eq.(7) becomes

$$J_{s1} = \int_{-b}^0 \rho u^2 dy' + \int_0^{\eta_c'} \rho u^2 dy'. \quad (35)$$

Using Eqs.(2),(3),(11) and (29),

$$C_{pb} (J_s \cos \alpha - J_{s1}) = 2 \rho U_0^2 b_0 (\cos \alpha - \cos \theta) \quad (36)$$

is obtained from Eq.(5). Furthermore, substituting Eqs.(6) and (35) into Eq.(36), we obtain

$$\cos \theta = \cos \alpha - C_{pb} \left(\cos \alpha - \frac{1}{2} - \frac{7}{4} \int_0^{\eta_c'} f^2 d\eta' \right). \quad (37)$$

The value of C_{pb} is equal to 2 as well as in the zone of flow establishment. The value of η_c' in Eq.(37) is determined in the following manner. Substituting Eqs.(17) and (18) into Eq.(8), we obtain

$$\frac{U}{U_0} \frac{b}{b_0} \int_0^{\eta_c'} f d\eta' = 1. \quad (38)$$

Substituting Eqs.(23) and (24) into b and U contained in Eq.(38), and equating x' to x_c' , we obtain

$$\left\{ \frac{54912}{20335} \kappa \left(\frac{x_c'}{b_0} - \frac{x_1'}{b_0} \right) + 1 \right\}^{1/2} \int_0^{\eta_c'} f d\eta' = \frac{2}{7}. \quad (39)$$

Substituting Eqs.(9),(11) and (37) into Eq.(39), we also obtain

$$\left\{ \frac{54912}{20335} \kappa \left[\left\{ \alpha + \cos^{-1}(\cos \alpha - C_{pb} A) \right\} \left(\frac{D}{b_0} + 1 \right) \times \frac{\cos \alpha}{C_{pb} A} - \frac{x_1'}{b_0} \right] + 1 \right\}^{1/2} \int_0^{\eta_c'} f d\eta' = \frac{2}{7}, \quad (40)$$

where

$$A = \cos \alpha - \frac{1}{2} - \frac{7}{4} \int_0^{\eta_c'} f^2 d\eta'.$$

Equation (40) gives the value of η_c' for each offset ratio, inclination and initial turbulence intensity. Then θ is determined from Eq.(37), R is determined from Eq.(11), x_c' is determined from Eq.(9) and x_c is determined from Eq.(10). Meanwhile, using the calculated results of a free jet flow, after putting $x' = x_{a'}$, Eq.(8) becomes

$$\left\{ \frac{54912}{20335} \kappa \left(\frac{x_{a'}}{b_0} - \frac{x_1'}{b_0} \right) + 1 \right\}^{1/2} \int_0^{\eta_{a'}} f d\eta' = \frac{2}{7}. \quad (41)$$

Substituting Eq.(12) into Eq.(41), the center angle of reattachment becomes

$$\zeta = \frac{b_0}{R} \left[\frac{2905}{27456} \frac{1}{\kappa} \left\{ \frac{2}{7} \left(\int_0^{\eta_{a'}} f d\eta' \right)^{-2} - \frac{7}{2} \right\} + \frac{x_1'}{b_0} \right] - \alpha. \quad (42)$$

Equation (14) gives

$$\frac{R}{b_0} - \frac{D}{b_0} - 1 = \left\{ \frac{R}{b_0} - \eta_{a'} \left\{ \frac{7}{2} + \frac{27456}{2905} \kappa \times \left(\frac{x_{a'}}{b_0} - \frac{x_1'}{b_0} \right) \right\} \right\} \frac{\cos \zeta}{\cos \alpha}, \quad (43)$$

and substituting Eqs.(41) and (42) into Eq.(43), we obtain

$$\frac{R}{b_0} - \frac{D}{b_0} - 1 = \frac{1}{\cos \alpha} \left\{ \frac{R}{b_0} - \frac{2}{7} \eta_{a'} \left(\int_0^{\eta_{a'}} f d\eta' \right)^{-2} \right\} \cos \left[\frac{b_0}{R} \times \left\{ \frac{2905}{27456} \frac{1}{\kappa} \left\{ \frac{2}{7} \left(\int_0^{\eta_{a'}} f d\eta' \right)^{-2} - \frac{7}{2} \right\} + \frac{x_1'}{b_0} \right\} - \alpha \right]. \quad (44)$$

Using the value of R determined before, $\eta_{a'}$ can be obtained from Eq.(44), ζ can be obtained from Eq.(42). Then $x_{a'}$ is determined from Eq.(12) and the reattachment distance x_a is determined from Eq.(13).

3. Experimental results and discussions

The experimental apparatus is almost the same as the one shown in the previous paper⁽⁸⁾ but that the nozzle of width 10 mm and the reattachment wall of length $L=200b_0$ are used, because the reattachment distances become large as the inclinations increase. The initial turbulence intensity T_0 is 0.004 for a small one and 0.08 for a large one. Reynolds number is defined as

$$Re = 2U_0 b_0 / \nu,$$

where ν is the kinematic viscosity of the fluid and all measurements are made at $Re = 5.0 \times 10^4$.

3.1 Pressure distributions along the reattachment wall

In order to confirm that the value of the coefficient of base pressure which is used in the above mentioned calculations can be regarded as constant, it is necessary to investigate the dependence of the coefficient of base pressure on the inclination of the reattachment wall, therefore to measure the base pressure p_b and the mean bubble pressure p_{mean} . These were obtained from the measurement of the pressure distributions along the reattachment wall.

As an example, the pressure distributions along the reattachment wall in the case of $D/b_0 = 5$ and $\alpha = 30^\circ$ are shown in Fig.2, using the initial turbulence intensity as a parameter. Using p_{mean} obtained by Sawyer's proposal⁽⁹⁾, the coefficient of mean bubble pressure is given as $C_{p_{mean}} = -0.138$ for $T_0 = 0.004$ and $C_{p_{mean}} = -0.161$ for $T_0 = 0.08$. It is found from the point of the maximum pressure that the flow having the larger initial turbulence intensity has a smaller reattachment distance, so a stronger reattachment occurs. The initial turbulence intensity

changes the behaviour of the reattachment flow considerably in spite of the same geometrical parameters.

3.2 Coefficient of base pressure

The value of the coefficient of base pressure C_{pb} contained in Eqs.(31) and (37) has been chosen so that the result deduced from Eq.(5) of the control volume model coincides with that deduced from Eq.(4) of the reattachment point model at $\alpha = 0^\circ$. Then we have obtained $C_{pb} = 2$. It may be considered that there is no dependence of C_{pb} on offset ratio, because the results of the approximate calculation in the case of the flow for $\alpha = 0^\circ$ agree well with the experimental results in a wide range of offset ratios.

From the pressure distributions along the reattachment wall shown in Fig.2, the mean bubble pressure is obtained by using Sawyer's proposal. The base pressure p_b is also obtained by extension of the same distributions to $x/b_0 = 0$. Using these values, C_{pb} may be estimated from Eq.(29), the result being shown in Fig.3. C_{pb} estimated by the above method is smaller than 2 which has been used in the approximate calculation.

Dependence of C_{pb} on the initial turbulence intensity is not so heavy, and dependence of C_{pb} on the inclination is not so heavy either. It may be considered that C_{pb} is constant for any offset and initial turbulence intensity. Therefore, $C_{pb} = 2$ may be used because the results deduced from the equation of reattachment point model coincide with those deduced from the control volume model in the case of the flow for $\alpha = 0^\circ$.

3.3 Length of a reattachment wall

The effects of the length of a reattachment wall on the reattachment flow having no offset were investigated by Bourque and Newman⁽³⁾. But such effects on the flow having various initial turbulence intensities and offsets have not been reported yet. In order to compare the results of experiment with those of the approximate calculation by the authors, which considers the effects of the initial turbulence intensity of the flow, experiments with flows having various lengths of the reattachment wall, prescribing the initial turbulence intensity, have been done by the authors.

The experimental results of the effects of the length of the reattachment wall are shown in Fig.4 for the flow having $D/b_0 = 2$ and $\alpha = 30^\circ$, and $T_0 = 0.004$ and 0.08 . For $T_0 = 0.004$, there exists a region, where the jet is not reattached, for L/b_0 smaller than 19. Once the jet is forced to reattach, reattachment is kept after the force is removed in the region of $19 \leq L/b_0 \leq 25$. For $L/b_0 > 25$ the jet is reattached always. In the last region, as the length of the reattachment wall becomes large, the reattachment distance tends to become smaller until the length of the reattachment wall becomes sufficiently large.

Fig.2 Pressure distributions along the reattachment wall

Fig.3 Coefficient of base pressure

Fig.4 The effects of the length of reattachment wall on a flow

The existence of such phenomena for flows having no offset and various inclination was revealed by Bourque and Newman. Because there are no suitable expressions for each region of the length of the reattachment wall, they are designated as a non-reattachment region, a forced reattachment region and a natural reattachment region, respectively. Kumada et al.⁽¹⁰⁾ suggested from the results by Bourque and Newman that the length of a reattachment wall does not affect the flow when $L/x_a \geq 3.8$.

Fig.5 Reattachment distances ($T_0 = 0.004$)

For the authors' experiment, $L/x_a = 3.8$ corresponds to $L/b_0 \approx 70$. The reattachment distance is independent of the length of the reattachment wall provided it is sufficiently long, as shown in Fig.4. For the flow having $T_0 = 0.08$, the non-reattachment region becomes $L/b_0 \leq 12$, which is narrower than that for $T_0 = 0.004$. The forced reattachment region for $T_0 = 0.08$ can not be found and the natural reattachment region becomes $L/b_0 > 12$. The length corresponding to $L/x_a = 3.8$ is $L/b_0 \approx 40$ in this case. For the flow having large initial turbulence intensity, the length of the reattachment wall does not affect the reattachment flow even when it is relatively small.

3.4 Reattachment distance

The relations between the offset ratio and the reattachment distance are presented in Fig.5 for $T_0 = 0.004$ by using the inclination as a parameter. The results of the approximate calculation agree well with the experimental ones for all inclinations presented in the region $D/b_0 \geq 5$. But the results of the approximate calculation become larger than the experimental ones for all inclinations as the offset ratio decreases. This is due to the fact that the value of the empirical constant κ contained in the eddy kinematic viscosity and the length of the zone of flow establishment x_1 , which are determined from the data in the zone of established flow, are used in the flow for each initial turbulence intensity. Because the jet is reattached at the shorter distance from the nozzle as the offset ratio decreases, the relations between κ 's and T_0 given by Eqs.(25) and (26) become not to hold. The results for $T_0 = 0.08$ are shown in Fig.6. The results of the approximate calculation also agree well with the ex-

Fig.6 Reattachment distances ($T_0 = 0.08$)

Fig.7 Maximum inclinations for natural reattachment and for forced reattachment

perimental ones, even for small offset ratios when inclinations are small. This is due to the fact that for the flow having a large initial turbulence intensity the above mentioned relations between κ 's and T_0 are valid for the upper stream.

3.5 Maximum inclinations for reattachment

Figure 7 shows the maximum inclinations α_n for which a natural reattachment may occur and α_f for which a forced reattachment may occur for each offset ratio. As the case of the flow influenced by the length of reattachment wall, $L/b_0 = 48$ is used, and as the case of the flow not influenced by it any more, $L/b_0 = 200$ is used.

From the results of the approximate calculation, the maximum inclination for

which reattachment may occur is obtained as follows: In Eq.(40) for given values of κ and D/b_0 , a positive value of η_c' can be determined for α smaller than 60° . At $\alpha = 60^\circ$, η_c' is equal to 0, then R becomes infinite and also x_a goes to the infinity. For $\alpha > 60^\circ$, Eq.(40) does not hold. This equation is deduced from Eq.(38), which is meaningless for η_c' smaller than zero. Therefore, the maximum inclination for which reattachment may occur is obtained as $\alpha = 60^\circ$, regardless of the offset ratio and the initial turbulence intensity.

From the experiments for $L/b_0 = 48$, the maximum inclination for natural and forced reattachments tends to decrease rapidly as the offset ratio increases. For $L/b_0 = 200$, the maximum inclinations are slightly dependent on the offset ratio. As well as from the results of calculation, it was found from the experiments that the maximum inclination for reattachment is independent of the initial turbulence intensity.

3.6 Coefficient of mean bubble pressure

The relations between the coefficient of mean bubble pressure and the inclination of the reattachment wall are shown in Fig. 8. According to the results of the approximate calculation, the mean bubble pressure shows the lowest value at a certain inclination for each offset ratio. When the inclination exceeds this value, the mean bubble pressure increases and the reattachment does not occur for $\alpha > 60^\circ$. The experimental values were obtained by Sawyer's proposal from the pressure distributions along the reattachment wall. These experimental results also reveal that there exists such inclination that the mean bubble pressure shows a minimum value for each offset ratio. These results are, for example, available to decide the splitter distance for designing the bistable fluidic elements. The calculated values of inclination α for which the mean bubble pressure is minimum are shown in Fig. 9. The value of α increases for each offset ratio as the initial turbulence intensity decreases.

4. Conclusions

The control volume model has been applied to the approximate calculations of the reattachment flow having the reattachment wall with an arbitrary inclination. The results of the approximate calculation of the two-dimensional free jet flow issuing from a finite width nozzle have been applied to the calculation of the reattachment flow.

It has been experimentally revealed that the base pressure which appears in the control volume model is almost independent of offset ratios, inclinations and initial turbulence intensities. The coefficient of base pressure used in the calculation has been chosen as $C_{pb} = 2$, so that the results from the control volume model and the reattachment point model may have been the same expression at no inclination.

Fig.8 Mean bubble pressures

Fig.9 Calculated values of α_c

$C_{pb} = 2$ is the same result as used by Perry. As a result, for flows having small initial turbulence intensities, the calculated reattachment distances agree well with the experimental ones for offset ratios larger than 5 and inclinations from 0° to 40° . For large initial turbulence intensities, the calculated reattachment distances agree well with the experimental ones for offset ratios larger than 2 when the inclination is smaller than 20° , in addition to the same ranges of offsets and inclinations as for small initial turbulence intensity.

The approximate calculation reveals that the maximum inclination which maintains the reattachment is 60° and independent of both the initial turbulence intensity and the offset ratio. The experimental results show that there exist three distinct regions of the length of the reattachment wall, that is, a non-reattachment region, a forced reattachment region and a natural reattachment region, depending on the inclination of the wall. It was found from the authors' experiments that for a reattachment wall with small

length, the maximum inclination for which the natural reattachment takes place and the maximum inclination for which the forced reattachment takes place are smaller than those for a reattachment wall which has sufficiently large length. It was made clear that the effects of the offset ratio and the initial turbulence intensity on those maximum inclinations decrease, when the length of the reattachment wall becomes sufficiently large.

According to the approximate calculations, there exists for each offset ratio an inclination for which the mean bubble pressure has a minimum value, and therefore, the curvature of the jet center line is maximum. This property is confirmed also by the experiments, though the values do not quantitatively agree. This inclination becomes large as the initial turbulence intensity and the offset ratio become small, and also the dependence of this inclination on the initial turbulence intensity increases as the intensity becomes small.

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