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Theoretical Study on Radiative Properties of Doubly Painted Layer

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The analysis is carried out on the radiative transfer in the real application of a doubly painted layer containing different pigments. The reflective properties for oblique and hemispherical incidences are investigated in the wavelength range from 0.45 to 10 μm . The effects of the scattering properties and volume concentrations of the pigments, optical thicknesses and reflection characteristics of the substrate on the bidirectional and hemispherical reflectances are examined.

1. Introduction

The present study extends the analyses carried out on the radiative properties of a single painted layer⁽¹⁻⁴⁾ to the more practical system of a doubly painted layer. The double-layer is considered to consist of two different coats of absorbing and anisotropically scattering media. The analytical method is developed treating the radiative transfer by Chandrasekhar's theory, and applying the Gaussian quadrature formula for the cases of oblique, axisymmetric and hemispherical incidences. Various combinations of the top and under coats are selected to meet the requirements of industrial applications. The effects of scattering properties and volume concentrations of the pigments, optical thicknesses, and the reflection characteristics of the substrate on the bidirectional and hemispherical reflectances are examined in the wavelength range from 0.45 to 10 μm .

Nomenclature

F, I: monochromatic net flux and intensity, respectively, of the diffuse radiation
 K_{av} : absorption coefficient of the vehicle, μm^{-1}
 K_{sp} , K_{ep} : scattering and extinction coefficients of the pigment, μm^{-1}
 L_k : thickness of the layer, μm
 n_v : real refractive index of the vehicle
 $p_k(\cos\theta)$: scattering phase function
 R : directional-hemispherical or hemispherical reflectance
 r_{OD} : bidirectional reflectance
 v_k : pigment volume concentration
 w_i : Gaussian weights
 θ : angle between directions (μ, ϕ) and (μ', ϕ')
 λ : wavelength in the vacuum, μm
 μ_i : Gaussian divisions
 ρ_D : hemispherical reflectance of a perfect diffuse substrate
 ρ_1, ρ_2 : specular reflectivities at the upper and lower boundaries of double-layer, respectively

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τ : normal optical depth
 τ_k : optical thickness
 ω_k : scattering albedo of the layer for single scattering

Suffixes

k: top coat (=1), under coat (=2)
 m: single layer of mixed pigments

2. Analysis

Figure 1 shows the model of a doubly painted layer with plane-parallel geometry. Two pigments of different shapes and sizes are dispersed in the same vehicle in the top and under coats. The layer is irradiated by unpolarised light of unit flux (per unit area of the layer) with oblique, axisymmetric or hemispherical incidence. The substrate may have a specular or perfect diffuse surface. Both the top and under coats are considered homogeneous, isothermal, absorbing and anisotropically scattering but nonemitting media. The air-vehicle interface is assumed to be specular.

2.1 Oblique or axisymmetric incidence

The transfer equation for the diffuse radiation field within a double-layer on specular substrate may be expressed by

$$\mu \frac{dI_k(\tau, \mu, \phi)}{d\tau} = I_k(\tau, \mu, \phi) - \frac{\tilde{\omega}_k}{4\pi} \int p_k(\cos\theta) \times I_k(\tau, \mu', \phi') d\Omega' - H_k(\tau, \mu, \phi) - G_k(\tau, \mu, \phi), \quad (k = 1, 2), \quad (1)$$

where H_k and G_k are the source functions due to the inward and outward components of the attenuated incident flux, and they are given by

Fig.1 Radiative model of the doubly painted layer

Fig.2 Effect of the optical thickness τ_1 on the bidirectional reflectance r_{0D}

Fig.3 Effect of scattering and absorption properties of the pigment in the top coat on the τ_1 dependence of r_{0D}

Fig.4 τ_1 dependence of the directional-hemispherical reflectance R_{0h}

$$K_{em} = \sum_{k=1}^2 v'_k K_{epk} + (1 - \sum_{k=1}^2 v'_k) K_{av}. \quad (16)$$

The equivalent phase function of scattering

Fig.5 Effect of the reflection characteristics of the substrate on R_{0h}

Fig.6 Effect of scattering and absorption properties and volume concentration of the pigment in the top coat of R_{0h} (optically thick under coat)

is expressed as a series in Legendre polynomials by

$$p_m(\cos\theta) = \sum_{l=0}^N c_{lm} P_l(\cos\theta), \quad (17)$$

where the equivalent angular distribution coefficient c_{lm} is given in terms of those for each pigment c_{lk} by

$$c_{lm} = \frac{1}{K_{sm}} \sum_{k=1}^2 v'_k K_{spk} c_{lk}. \quad (18)$$

5. Results and Discussions

The pigments selected for the present study are TiO_2 , Fe_2O_3 , ZnO and metallic Al pigment. The vehicle of alkyd resin and the specular surface of Al substrate are considered. The data on the shape and size of each pigment and the optical constants, together with the calculation procedure for the scattering properties can be found in

$$H_k = \frac{\tilde{\omega}_k (1-\rho_1) \exp[-(\tau+\delta_k \tau_1)/\mu_0]}{4\pi\mu_0 [1-\rho_1 \rho_2 \exp(-2\tau^*/\mu_0)]} p_k(\cos\theta_0), \quad (2)$$

$$G_k = \frac{\tilde{\omega}_k \rho_2 (1-\rho_1) \exp[-(2\tau^*-\delta_k \tau_1-\tau)/\mu_0]}{4\pi\mu_0 [1-\rho_1 \rho_2 \exp(-2\tau^*/\mu_0)]} \times p_k(\cos\theta'_0), \quad (3)$$

$$\delta_k=0, \tau=\tau' \text{ for } k=1; \quad \delta_k=1, \tau=\tau' \text{ for } k=2 \quad (4)$$

and $\tau^*=\tau_1+\tau_2$. The expressions for the phase function $p_k(\cos\theta)$, the scattering albedo $\tilde{\omega}_k$ and the optical thickness τ_k can be found in Ref.(1). The solution of Eq.(1) is(3)

$$I_k(\tau, \mu, \phi) = \sum_{m=0}^N I_k^{(m)}(\tau, \mu) \cos[m(\phi_0 - \phi)], \quad (5)$$

where the order N is taken to be the larger one of N_1 and N_2 orders for the finite Fourier series of $p_{1,2}(\cos\theta)$. The intensity coefficients $I_k^{(m)}(\tau, \mu)$ can be calculated numerically for every value of the order m using the Gaussian quadrature formula in the n th approximation. They are given (without m notation) by the following equation for the azimuth-independent intensity $I_k(\tau, \mu_1)$ in the case of axisymmetric incidence.

$$I_k(\tau, \mu_1) = \sum_{\alpha=1}^{2n} g_{k\alpha} b_{k\alpha} \exp(\gamma_{k\alpha} \tau) + s_{ki} \exp(\tau/\mu_0) + t_{ki} \exp(-\tau/\mu_0), \quad (i = 1, \dots, 2n). \quad (6)$$

The constants $\gamma_{k\alpha}$, $b_{k\alpha}$, s_{ki} and t_{ki} are determined for the top and under coats, separately. The constants of integration $g_{k\alpha}$'s are determined simultaneously for the two coats by the application of the following boundary and continuity conditions.

$$I_1(0, -\mu_1) = \rho_{\mu_1} I_1(0, \mu_1), \quad (i = 1, \dots, n), \quad (7a)$$

$$I_2(\tau_2, \mu_1) = \rho_{-\mu_1} I_2(\tau_2, -\mu_1), \quad (i = 1, \dots, n), \quad (7b)$$

$$I_1(\tau_1, \pm\mu_1) = I_2(0, \pm\mu_1), \quad (i = 1, \dots, n). \quad (7c)$$

Substituting Eq.(6) in Eqs.(7a)~(7c) we obtain $4n$ linear independent equations of $4n$ unknowns $g_{k\alpha}$, which can be solved by the Gaussian elimination procedure.

The case of a double-layer on a diffuse substrate is treated similarly to a single layer, with the lower boundary condition

$$I_2(\tau_2, \mu_1) = 2\rho_D \sum_{j=n+1}^{2n} w_j |\mu_j| I_2(\tau_2, \mu_j) + \rho_D \frac{(1-\rho_1)}{\pi} \exp(-\tau^*/\mu_0). \quad (8)$$

When τ_1 increases by a large extent the problem reduces to one of an optically thick single layer ($\tau_1 \rightarrow \infty$)(2). Keeping τ_1 finite and increasing τ_2 to infinity ($\tau_2 \rightarrow \infty$) the transfer equation and the corresponding numerical solution are expressed by Eqs.(1) and (6) for $\tau^*=\tau_2 \rightarrow \infty$ except that the summation in Eq.(6) for $I_2(\tau'', \mu_1)$ is carried out only on n orders of α corresponding to the negative values of $\gamma_{2\alpha}$. The $3n$ constants $g_{k\alpha}$ are determined by applying the upper boundary condition Eq.(7a) and the continuity condition Eq.(7c).

2.2 Hemispherical incidence

Considering uniform irradiation ($\pi I_0=1$) to a double-layer on specular substrate the transfer equation may be expressed by

$$\mu \frac{dI_k(\tau, \mu)}{d\tau} = I_k(\tau, \mu) - \frac{\tilde{\omega}_k}{4\pi} \int_{4\pi} p_k(\cos\theta) \times I_k(\tau, \mu') d\Omega' - \int_{2\pi} [\tilde{H}_k(\tau, \mu) + \tilde{G}_k(\tau, \mu)] I_0 \mu'_0 d\Omega'_0, \quad (9)$$

where $\tilde{H}_k$ and $\tilde{G}_k$ are the azimuthal averages of the functions in Eqs.(2) and (3). Equation (10) is solved numerically in a manner similar to the case of a single layer. For all cases of incidence and for all conditions of the double-layer the continuous functions $I_k(\tau, \mu)$ can be obtained in terms of the numerical solution(2).

3. Analysis of calculated results

The bidirectional relative diffuse reflectance r_{OD} in a reflection direction (μ', ϕ) for oblique incidence in a direction (μ_0, ϕ_0) is defined by

$$r_{OD}(\mu', \phi) = \pi(1-\rho_{\mu'}) I_1(0, \mu, \phi) / n_v^2, \quad (10)$$

where μ and μ' satisfy Snell's law.

The net flux $F_k(\tau)$ of the diffuse radiation inside the layer due to axisymmetric or hemispherical incidence (with uniform irradiation) can be calculated by

$$F_k(\tau) = 2\pi \sum_{j=1}^{2n} w_j \mu_j I_k(\tau, \mu_j). \quad (11)$$

For oblique incidence, $I_k(\tau, \mu_j)$ is replaced by $I_k^{(0)}(\tau, \mu_j)$ to calculate $F_k(\tau)$. The diffuse component R_{OD} of the directional hemispherical reflectance R_{Oh} ($=R_{OD}+R_{OS}$) is given by

$$R_{OD} = F_1(0) \quad (12)$$

and the specular component R_{OS} is given by the same equations in Ref.(3). In the case of hemispherical incidence with uniform irradiation the diffuse component R_D of the hemispherical reflectance R_h ($=R_D+R_S$) is also given by Eq.(12), while the specular component R_S is given by

$$R_S = 2 \sum_{j=1}^n w_j \mu'_j R_{jS}, \quad (13)$$

where R_{jS} is the specular reflectance for an incidence direction of μ'_j .

4. Equivalent single layer of mixed pigments

We compare the results of the double-layer with those of an equivalent single layer of mixed pigments having the same total optical thickness τ^* . The volume concentrations of the pigments and the scattering and extinction coefficients of the equivalent single layer are given, respectively, by

$$v'_k = \frac{L_k}{L_m} v_k, \text{ where } L_m = L_1 + L_2, \quad (14)$$

$$K_{sm} = \frac{2}{\sum_{k=1}^2 v'_k} K_{spk}, \quad (15)$$

Fig.7 Spatial distributions of diffuse intensities and flux (comparison)

Ref.(1) and (4). Figure 2 shows the effect of the optical thickness of the top coat τ_1 on the angular distribution of the bidirectional reflectance r_{OD} for oblique incidence at a polar angle $\theta_0' = 60^\circ$. The curve for $\tau_1 = 0$ (only under coat with $\tau_2 = 0.5$) shows more forward reflection due to the combined effects of the absorption ($\tilde{\omega}_2 = 0.9608$), the predominant forward scattering by Fe_2O_3 and the multiple reflections on the specular substrate. The addition of an equal optical thickness $\tau_1 = 0.5$ of the top coat ($\tilde{\omega}_2 = 0.9999$) increases the average value of r_{OD} without a noticeable change in the angular distribution. As τ_1 increases, the angular distribution changes gradually to a uniform one. Figure 3 shows the influence of the scattering and absorption properties of the pigment in the top coat on the τ_1 dependence of r_{OD} . Both TiO_2 and ZnO show the increase of r_{OD} with the increase of τ_1 due to a considerable scattering and no absorption by them (the absorption is only by vehicle). On the other hand a considerable absorption by the metallic Al pigment leads to a decrease of r_{OD} with an increase of τ_1 . In all cases the angular distribution does not change much with the increase of τ_1 due to the large value of $\tau_2 = 2.0$ compared with the case of $\tau_2 = 0.5$ in Fig.2. Shown in Fig.4 is the τ_1 dependence of the directional-hemispherical reflectance R_{Oh} of a painted layer on a perfect diffuse substrate. Compared with its equivalent single layer of mixed pigments the double layer shows different results depending on values of ρ_D and τ_2 . The results for both cases approach each other and coincide at $\tau_1 = \infty$. Figures 5 and 6 show the wavelength dependence of R_{Oh} . Figure 5 shows the enhanced effect of the substrate. A specular Al substrate and diffuse gray substrates with $\rho_D = 0, 1$ are considered. The spectra for the equivalent single layer and the two cases of double-layers show remarkable differences in the visible and near infrared regions, while they nearly coincide in the infrared region due to the similarity of the scattering and absorption properties of the two pigments (TiO_2, Fe_2O_3). Figure 6 shows the results for an optically thick under coat ($\tau_2 = \infty$).

Clear effect can be seen for the pigment volume concentration v_1 depending on the scattering and absorption properties of the pigments. Figure 7 shows a comparison of the results for hemispherical incidence of uniform irradiation with those for axisymmetric incidence at $\theta_0' = 60^\circ$ (3). The spatial distributions of the outward and inward normal diffuse intensities $I(\tau, \pm 1)$ and the net diffuse flux $F(\tau)$ are plotted together with the values of R_D (or R_{OD}) and R_S (or R_{OS}) for completeness of comparison. The results for both cases of incidence nearly coincide with each other; except small differences of $I(\tau, \pm 1)$ due to the effect of the scattering phase function. Therefore, the results of axisymmetric incidence at $\theta_0' = 60^\circ$ can be used as a substitute for the hemispherical incidence of uniform irradiation.

6. Conclusions

An analytical method was proposed to study the radiative properties of a doubly painted layer. The bidirectional reflectance is mainly affected by the scattering and absorption properties in the top coat in the case of the under coat optical thickness $\tau_2 > 1$. The treatment of the double layer as an equivalent single layer of mixed pigments leads to a considerable error depending on the wavelength and the scattering and absorption properties of the pigments relative to each other. The results of axisymmetric incidence at a polar angle of 60° can be used as a substitute for the hemispherical incidence of uniform irradiation.

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