543.11::621-185.7

Paper No. 200-17

Q-Factor Analysis of Unbalance Vibration in Rotor-Bearing Systems *

By Osami MATSUSHITA **
 Michiaki IDA **
 Katsuaki KIKUCHI ***

Q-factor is usually a practical means for predicting maximum amplitude at critical speeds. However, it is not accurate in the case of systems with strong gyroscopic effect, large damping coefficients and asymmetric properties in bearing dynamics.

In this study these problems are solved through a new definition of the Q-factor.

The unbalance vibration is formulated at each damped critical speed, based upon orthogonality of the undamped critical speed mode, and rearranged by the equation derived from the corresponding damped eigenvalues. The result proves that Q-factor and damped critical speed Ω_c are not functions of only modal damping ratio ζ , but also vary with gyroscopic factor γ which is the ratio of modal gyro per modal mass, as follows.

$$\frac{\Omega_c}{\omega_0} = \frac{1}{1 - \gamma^2} \quad Q = \frac{1}{1 - \gamma^2} \frac{1}{2\zeta\sqrt{1 - \gamma^2}} \quad \text{where } \gamma = \frac{1 - \gamma^2/2}{1 - \gamma^2} \zeta$$

The new definition is characterized by value γ reflecting the gyroscopic effect in the system. The value $\gamma=0$ is an old and incorrect definition as it does not consider gyroscopic effect.

To demonstrate the validity of the method, numerical examples are shown for a 3-disk rotor supported with symmetric and tilting pad type bearings.

1. Introduction

Because of the trends toward weight reduction and increased operational speed, a turbomachine is required to be so super critical that the increment of the number of critical speeds and the growth of unbalance response sensitivity can not be avoided. In the design of a super critical rotor, unbalance vibration is theoretically controlled by the location of critical speeds and the change of modal damping ratios (1, 2, 3), which are given by complex eigenvalue analysis. In addition, experimental research (4) has been done about highly damped bearings employing external damper support.

It is the purpose in such research to reduce amplitude severity at an unbalance resonance, called the Q-factor.

The Q-factor is defined by the ratio of the resulting maximum amplitude to the mass unbalance eccentricity. The modal damping ratio is so convenient for estimating unbalance response severity that it is widely applied to rotor design.

However, the authors doubt the validity of the relationship between the Q-factor and the modal damping ratio ζ in fields such as rotor dynamics, even though it guarantees good agreement in structure dynamics. The doubts are summarized in the following cases.

- (1) Q-factor can be calculated by a well known formula, i.e. $Q=1/2\zeta$, after deriving the modal damping ratio in complex eigenvalue analysis. But the predicted maximum amplitude which corresponds to the calculated Q-factor does not always show good agreement with the peak amplitude on an actual response curve.
- (2) By examining the sharpness of the response curve, the Q-factor is found by a well known formula, i.e. $Q=(\text{natural frequency})/(\text{frequency width of 70\% amplitude}) = f_n/\Delta f$. But the Q-factor which is thus derived does not always show good correspondence with the actual severity which is the division of a peak amplitude by an unbalance eccentricity.

Appendix (D) gives a good example of these doubts, employing numerical values for a typical rotor. There is also contained the resolution of doubts based upon the result of the present study on a new Q-factor definition.

The authors believe that the reason for the disagreement stated above seems to be a lack of consideration on gyroscopic force and damping force in bearings. It is the purpose of the present analysis to establish a method which can accurately predict the Q-factor so as to solve the disagreement.

In this paper, an unbalance response is separated into its eigen mode components by modal techniques based upon the orthogonality (5) of the eigensolution, including its gyroscopic effect. Also, the modal unbalance response obtained by the separation is rearranged according to the result of the complex eigenvalue analysis. Finally,

* Received 10th October, 1980
 ** Researcher, Mechanical Engineering Research Laboratory of Hitachi Ltd., 502, Kandatsu-MACHI, Tsuchiura-SHI, IBARAKI
 *** Senior Researcher, ditto.

the relationship between the modal damping ratio, Q-factor and peak amplitude at a certain resonance is made clear, and a new Q-factor definition is proposed.

The result of the new Q-factor analysis is applied to the problem of how to find the exact Q-factor on an unbalance response curve.

In addition, to demonstrate the appropriateness of the method, examples are shown for a 3-disk rotor supported with symmetric and asymmetric tilting pad type bearings.

Nomenclature

B, A ; coefficient matrices in an eigenvalue problem

$C_f = C_{fd} + iC_{fo}$; damping matrix
 $= [(c_{xx} + c_{yy})/2] + i[(c_{yx} - c_{xy})/2]$

$C_b = C_{bd} + iC_{bo}$; damping matrix
 $= [(c_{xx} - c_{yy})/2] + i[(c_{yx} + c_{xy})/2]$

C_g ; gyroscopic matrix

C_p^* ; modal gyro

$K_f = K_{fd} + iK_{fo}$; stiffness matrix
 $= [(k_{xx} + k_{yy})/2] + i[(k_{yx} - k_{xy})/2]$

$K_b = K_{bd} + iK_{bo}$; stiffness matrix
 $= [(k_{xx} - k_{yy})/2] + i[(k_{yx} + k_{xy})/2]$

M ; mass matrix

M^* ; modal mass

Q ; Q-factors $Q_{70}, Q_{A0}, Q_{D0}, Q_D, Q_A$

s, s_i ; normal coordinate

U ; unbalance distribution

z ; complex displacement $= x + iy$

β ; nondimensional parameter $= (1 - \gamma/2)/(1 - \gamma)$

γ ; gyro factor $= -C_g^*/M^*$

e^* ; modal mass unbalance eccentricity

ζ ; modal damping ratio $= -\alpha/\omega_0$

η ; modified modal damping ratio $= \beta\zeta$

λ ; complex eigenvalue $= \alpha + i\eta$

ϕ, ϕ_f ; forward eigen mode vector

ϕ_b ; backward eigen mode vector

Ω ; rotational speed

Ω_0 ; undamped critical speed

ω_0 ; tuning frequency $= \sqrt{\alpha^2 + \eta^2}$

$\approx$; nearly equal

$=$; definitely equal

[]^t ; transpose operator of matrix

i ; imaginary unit $= \sqrt{-1}$

$\bar{}$; conjugate operator

|| ; absolute value

2. Complex Eigenvalue and Modal Unbalance Vibration

This paper deals with the dynamics of a rotor with many disks and bearings such as the one shown in Fig.1. Assuming that a complex eigenvalue is known, the corresponding unbalance response can be approximated at each eigensolution in order to develop a practical rotor design. First of all, the unbalance vibration of a rotor supported by symmetric bearings is discussed in order to make the mathematical description simpler and to facilitate the comprehension of the physical thinking behind this formula. This discussion is followed by one about rotor vibration concerning only asymmetric bearings where a whirl orbit is generally elliptical. Finally, the unbalance vibration is formulated by separation at each eigenmode throughout both bearing conditions, and a new definition of the Q-factor and a prediction method for the maximum resonance amplitude are established corresponding to each critical speed.

2.1 Undamped Critical Speed and Orthogonality

The complex form of the displacement of a rotor is here denoted by $z = x + iy$ where the displacements in the X-direction and Y-direction are x and y, respectively as shown in Fig.1.

Fig.1 Rotor-Bearing System

Tab.1 Undamped Critical Speed Analysis

No	ITEM	I	II
1	SYSTEM	symmetric support circular whirl orbit Vertically Suspended Rotor	asymmetric support elliptical whirl orbit Horizontally Suspended Rotor
2	DYNAMIC PROPERTIES OF BEARING	$k_{xx} = k_{yy} \quad k_{xy} = -k_{yx} = 0$ $c_{xx} = c_{yy} = c_{xy} = -c_{yx} = 0$	$k_{xx} \neq k_{yy} \quad k_{xy} = k_{yx}$ $c_{xx} = c_{yy} = c_{xy} = c_{yx} = 0$
3	EQUATION OF MOTION OF FREE VIBRATION	$M \ddot{z} + i\Omega C_g \dot{z} + K z = 0$ where $K = [k_{xx}]$	$M \ddot{z} + i\Omega C_g \dot{z} + K z + K_b \bar{z} = 0$ where $K = \left[\frac{k_{xx} + k_{yy}}{2} \right] \quad K_b = \left[\frac{k_{xx} - k_{yy}}{2} \right] + i \left[\frac{k_{xy} + k_{yx}}{2} \right]$
4	SOLUTION	$Z = \phi e^{i\Omega t}$ ϕ : real number	$z = \phi_f e^{i\Omega t} + \bar{\phi}_b e^{-i\Omega t}$ ϕ_f, ϕ_b : complex number
5	EIGENVALUE PROBLEM $\Omega^2 B \phi = A \phi$	$B = M + C_g \quad A = K$	$B = \begin{bmatrix} M + C_g & 0 \\ 0 & M - C_g \end{bmatrix} \quad A = \begin{bmatrix} K & K_b \\ \bar{K}_b & K \end{bmatrix} \quad \phi = \begin{bmatrix} \phi_f \\ \phi_b \end{bmatrix}$
6	MATRIX B MATRIX A	real symmetric and indefinite real symmetric and positive definite	real symmetric and indefinite hermite and positive definite
7	UNDAMPED CRITICAL SPEED Ω_0	$\Omega_0 = \sqrt{\Omega^2}$ for positive eigenvalue Ω^2	
8	ORTHOGONALITY CONDITION	$\bar{\phi}_i B \phi_j = \begin{cases} = 0 & i \neq j \\ \neq 0 & i = j \end{cases} \quad \bar{\phi}_i A \phi_j = \begin{cases} = 0 & i \neq j \\ \neq 0 & i = j \end{cases}$	

Eigensolutions with undamped critical speeds and mode shapes are summarized in Table 1. The table is made with the column I (6) for symmetrical bearing systems and the column II for asymmetric ones.

In the conservative systems concerned with bearing dynamic properties as described in the line No.2, the equation of motion is formulated in the line No.3. The eigensolution shall be assumed to be as shown in the line No.4 and synchronized with the rotational speed Ω . The corresponding eigenproblem leads to the line No.5 to determine the undamped critical speed and its eigen mode shapes. The coefficient matrix has such a particular form, as shown in the line No.6, that eigenvalue Ω^2 is a real numbers, but not always positive. If and only if eigenvalue Ω^2 is positive, the undamped critical speed Ω_0 will exist. Eigen mode shapes whose gyroscopic effects are too strong to produce negative eigenvalues, and as a result these do not give critical speeds. The orthogonality of eigensolutions is indicated in the line No.8, in bearings with both symmetry and asymmetry.

2.2 Bearing with Symmetric Property

2.2.1 Unbalance Vibration at Each Mode

The equation of motion for unbalance vibration in a damped system is written as follows :

$$M\ddot{z} + i\Omega C_g \dot{z} + K_f z + C_f \dot{z} = U Q_0 e^{i\Omega t} \quad \dots\dots\dots (1)$$

$$\text{where } K_{xx} = K_{yy}, K_{xy} = -K_{yx}, C_{xx} = C_{yy}, C_{xy} = -C_{yx}$$

The modal analysis technique is most convenient to solve the unbalance vibration in a system with a multi-degree of freedom of motion. Hence the modal matrix is assumed to consist of eigen modes at undamped critical speeds. Thus transformation from the physical coordinate z to the normal one s is done through the matrix relationship.

$$z = [\phi_1 \dots \phi_k] s \equiv [\phi] s \equiv [\phi] s e^{i\Omega t} \quad \dots\dots\dots (2)$$

Substituting the above Eq.(2) into Eq.(1), and premultiplying the resulting equation by the transposed modal matrix $[\phi]^t$, the orthogonality concerned with the conservative components M , C_g and K_f gives the diagonality of matrix. Other components K_{fc} and C_f are non-conservative, and they can not be mathematically orthogonal. Here from an practical consideration, it may be assumed that non-conservative components K_{fc} and C_f can be also diagonalized by orthogonality as well as conservative ones. This assumption guarantees division of unbalance vibration into separate modes and makes the response easy to solve.

Under this assumption, the unbalance vibration on a normal coordinate system corresponding to the k -th eigensolution implies

$$s_k = \frac{\phi_k^t U Q_0 e^{i\Omega t}}{-\Omega^2 (M^* + C_g^*) + i\Omega (C_r^* + iC_f^*) + K_r^* + iK_f^*} \quad \dots\dots\dots (3)$$

where superscript * means the quadratic form of a matrix using the undamped eigen mode, as is shown in the following.

$$\left. \begin{aligned} M^* &= \phi_k^t M \phi_k, & C_g^* &= \phi_k^t C_g \phi_k \\ C_r^* + iC_f^* &= \phi_k^t C_r \phi_k, & K_r^* + iK_f^* &= \phi_k^t K_f \phi_k \end{aligned} \right\} \quad \dots\dots\dots (4)$$

The unbalance response is described by Eq.(3) for each eigen mode and is termed the modal unbalance vibration.

Next, we replace the quadratic form in Eq.(3) with the corresponding complex eigenvalue. The relationship between the quadratic form and the complex eigenvalue is derived in detail in Appendix(A), and it may approximately satisfy the following formulas.

$$\left. \begin{aligned} K_r^* - \Omega C_g^* &\approx (M^* + C_g^*) \omega_0^2 \\ C_r^* + K_f^* / \Omega &\approx (M^* + C_g^*) 2\zeta \beta \omega_0 \end{aligned} \right\} \quad \dots\dots\dots (5)$$

The approximation demands small values for C_g^* and K_f^* , but there is no restriction for common damping C_r^* . Therefore, the relationship can be considered applicable to many cases involving damped systems.

Substitution of Eq.(5) into Eq.(3) provides the following formula, called the modal unbalance response.

$$s_k = \frac{\phi_k^t U}{M^*} \frac{1}{1-\gamma} A(p) e^{i\Omega t} = \frac{\phi_k^t U}{M^*} \hat{s}_k e^{i\Omega t} \quad \dots\dots\dots (6)$$

where $p = \Omega / \omega_0$, $A(p) = p^2 / (1 - p^2 + 2i\beta\zeta p)$

$$\gamma = -C_g^* / M^*, \quad \beta = \left(1 - \frac{\gamma}{2}\right) / (1 - \gamma)$$

Gyroscopic factor γ is a nondimensional value representing the gyroscopics effect. It is limited in the range $0 < \gamma < 1$ because of its definition as shown in Table 1-I. Since the configuration of an unbalance response curve is described by the function $\hat{s}_k$, it is changed according to the gyroscopic factor γ as shown in Fig.2.

In Fig.2, attention should be paid to the influence of the gyroscopic factor γ which makes the value and location of the peak different even for the same modal damping ratio ζ . The curve indicating $\gamma = 0$ is a well-known conventional form (8) appearing in every textbook on mechanical vibration and it is available for harmonic excitation problems only in non-rotating structure dynamics.

Now, it has been found in the above discussion that the rotor unbalance vibration is characterized by the gyroscopic factor γ because $\gamma \neq 0$ in rotor dynamics. The influences of the gyroscopic factor γ on the response curves shown in Fig.2 are summarized in the following two results.

Fig.2 Unbalance Response Curve

- (1) Peak of Amplitude; In the larger gyroscopic effect, when $\gamma \rightarrow 1$, the peak value of the amplitude becomes larger even if the modal damping ratio remains constant.
- (2) Form of Response Curve; The sharpness of the response curve depends upon the value η , called the modified modal damping ratio, but not upon the modal damping ratio ξ alone such as in structure dynamics. The former η is larger than the latter ξ because $\beta > 1$. The response curve with stronger gyroscopic effect, i.e., $\gamma \rightarrow 1$, has lower sharpness and larger expansion of the curve around the peaks. To the ordinary observer, it actually looks like an increment of the modal damping ratio ξ .

2.2.2 Damped Critical Speed and Q-factor

The unbalance response has been described by Eq.(6) at each eigen mode. Therefore, the damped critical speed Ω_c , at which an amplitude reaches the actual maximum value on a peak, and the Q-factor which is the sensitivity to unbalance at actual resonance, are defined as follows.

$$\Omega_c = \omega_0 / \sqrt{1 - 2\eta^2} \quad (7)$$

$$Q = \frac{\varepsilon_{K\text{peak}}}{\varepsilon^*} = \frac{1}{1 - \gamma} \frac{1}{2\eta\sqrt{1 - \eta^2}} \quad (8)$$

where

$$\varepsilon^* = \phi_k^* U / M^*, \quad \eta = \beta \xi$$

Since the modal damping ratio ξ and the gyroscopic factor γ are derived by solving a complex eigenvalue problem, the accurate damped critical speed and Q-factor can be predicted from the calculations for a rotor design.

If $\gamma = 0$ in structure dynamics, the relationship as has been stated above reduces to a well-known formulas, where η becomes ξ .

$$\Omega_c = \omega_0 / \sqrt{1 - 2\xi^2}, \quad Q = \frac{1}{2\xi\sqrt{1 - \xi^2}} \quad (8')$$

We can find this formula in almost every textbook about harmonic forced vibration, and it has been used for the prediction of resonance vibration.

However, this study reveals that the well-known formula in Eq.(8') is valid only for structure dynamics ($\gamma = 0$), but not for rotor dynamics ($\gamma \neq 0$). Consequently, instead of Eq.(8'), the formula in Eq.(8) appears to be appropriate for the process of estimating unbalance resonances in rotor dynamics.

2.3 Bearing with Asymmetric Property

2.3.1 Unbalance Vibration at Each Mode

In systems that have bearings with asymmetric properties the rotor whirls elliptically. For such rotor-bearing systems the equation of motion can be written in the following form.

$$M\ddot{z} + iQC_g\dot{z} + K_f z + K_b \bar{z} + C_f \dot{z} + C_b \dot{\bar{z}} = UQ^2 e^{i\Omega t} \quad (9)$$

Since an elliptical orbit is the sum of forward and backward orbits, the unbalance vibration z contains the forward component z_f and the backward component z_b which are synchronized with the rotational speed.

From this, we can suppose the relationship

$$z = z_f + \bar{z}_b = A_f e^{i\Omega t} + \bar{A}_b e^{-i\Omega t} \quad (10)$$

By substituting Eq.(10) into Eq.(9) and separating z_f and z_b from each other, the following equations can be derived.

$$\left. \begin{aligned} M\ddot{z}_f + iQC_g\dot{z}_f + K_f z_f \\ + K_b z_b + C_f \dot{z}_f + C_b \dot{z}_b = UQ^2 e^{i\Omega t} \\ M\ddot{z}_b - iQC_g\dot{z}_b + \bar{K}_b z_f + \bar{K}_f z_b + \bar{C}_b \dot{z}_f + \bar{C}_f \dot{z}_b = 0 \end{aligned} \right\} \quad (11)$$

Hence all of the vibration components z_f and z_b are changed into forward form so that the results for the symmetric system discussed in the previous chapter may be applied to Eq.(11).

A new additional vector $[z_f^t, z_b^t]$ is then introduced and a transformation from the physical coordinates to normal ones is performed, in the same manner as with Eq.(2).

$$\begin{bmatrix} z_f \\ z_b \end{bmatrix} = \begin{bmatrix} \phi_f \\ \phi_b \end{bmatrix} s \equiv \begin{bmatrix} \phi_{f1} \cdots \phi_{fk} \\ \phi_{b1} \cdots \phi_{bk} \end{bmatrix} s e^{i\Omega t} \quad (12)$$

where the eigen modes are obtained by analysis of the undamped critical speed shown in Table 1-II. Normal transformation is executed and orthogonality of the conservative components M , C_g , K_{fd} , and K_b insures the diagonalization of the post normal transformation matrix. By assuming the orthogonality of the remaining non-conservative components C_f , C_b and K_{fc} , the unbalance vibration response is separated into forms corresponding to each eigen mode. The results are described by using an expression similar to Eq.(3).

$$\left. \begin{aligned} M^* &= \bar{\phi}_{fk}^t M \bar{\phi}_{fk} + \bar{\phi}_{bk}^t M \bar{\phi}_{bk} \\ C_g^* &= \bar{\phi}_{fk}^t C_g \bar{\phi}_{fk} - \bar{\phi}_{bk}^t C_g \bar{\phi}_{bk} \\ C_f^* + iC_b^* &= [\bar{\phi}_{fk}^t, \bar{\phi}_{bk}^t] \begin{bmatrix} C_f C_b \\ \bar{C}_b \bar{C}_f \end{bmatrix} \begin{bmatrix} \bar{\phi}_{fk} \\ \bar{\phi}_{bk} \end{bmatrix} \\ K_f^* + iK_b^* &= [\bar{\phi}_{fk}^t, \bar{\phi}_{bk}^t] \begin{bmatrix} K_f K_b \\ \bar{K}_b \bar{K}_f \end{bmatrix} \begin{bmatrix} \bar{\phi}_{fk} \\ \bar{\phi}_{bk} \end{bmatrix} \\ U^* &= [\bar{\phi}_{fk}^t, \bar{\phi}_{bk}^t] \begin{bmatrix} U \\ 0 \end{bmatrix} = \bar{\phi}_{fk}^t U \end{aligned} \right\} \quad (13)$$

However, in this case of asymmetric support, the quadratic form implies the sum of the values of the forward eigen mode ϕ_f and backward eigen mode ϕ_b . The form of Eq.(13) is generally applicable to even symmetric bearing systems since Equation (13) in which $\phi_f = \phi$ and $\phi_b = 0$ reduces to the former form of Eq.(4). Hence, the modal unbalance response is also defined by Eq.(6) even in systems with asymmetric bearing properties.

Now by referring to the approximate relationship between the modal quadratic values and the complex eigenvalue derived in Appendix (B), the damped critical speed and the corresponding Q-factor can also be predicted by Eq.(7) and (8), as in case of symmetry previously described.

It is thus concluded that in asymmetric bearing systems the unbalance response at each eigen mode results in the same formula as for symmetric bearing systems, the only difference being in the variance between the quadratic forms Eq.(4) and Eq.(13). Also in these results, it is worth noting that the unbalance vibration is generated by only the forward eigen mode, but not the backward eigen mode. This is because the modal unbalance force $\bar{\phi}_f^t U$ means an inner product of the forward eigen vectors $\bar{\phi}_f$ and the unbalance vector U for the asymmetry of the bearing property.

2.3.2 Eigen Mode and Modal Unbalance Response

The influence of the forward eigen mode ϕ_f and the backward one ϕ_b on the unbalance response will now be discussed.

- (1) $|\phi_f| > |\phi_b|$ Knowing now that the eigen mode describes the forwardly elliptical orbit and $0 \leq \gamma < 1$, the corresponding unbalance response implies a forward precession similar to the case of a symmetric bearings characterized by a circular orbit.
- (2) $|\phi_f| < |\phi_b|$ Knowing that the eigen mode describes a backwardly elliptical orbit as well as $\gamma < 0$, the corresponding unbalance response becomes a backward precession.

$$z_k = \phi_{fk} s_k + \bar{\phi}_{bk} \bar{s}_k \quad (14)$$

For example, the curve of an unbalance response where $\gamma = -0.5$ is shown in Fig.2. As this example shows, the negative value γ makes a curve around a peak sharper and the maximum amplitude smaller than values when $\gamma = 0$. This is because the modified modal damping ratio η with negative value $\gamma < 0$ becomes smaller than the ratio ζ when $\gamma = 0$.

- (3) $|\phi_f| \approx |\phi_b|$ In this case, the eigen mode describes an especially sharp elliptical orbit, and the direction of the whirling depends on which of the magnitudes of ϕ_f and ϕ_b is greater. As the gyroscopic factor γ gets closer to zero, i.e. $\gamma \rightarrow 0$, the gyroscopic effect decreases, and finally vanishes as if in structure dynamics.

2.4 Prediction of Q-Factor

Knowing the complex eigenvalue λ , the procedure can be described by the following sequence of steps to determine the Q-factor and the peak amplitude at each modal unbalance vibration.

To summarize the results of previous paragraphs ;

- (1) Eigenvalue $\lambda = \alpha + i\beta$ (where $\beta > 0$)
Eigen mode $Z = \phi_f e^{\lambda t} + \bar{\phi}_b e^{\bar{\lambda} t}$
- (2) Tuning frequency $\omega_0 = |\lambda| = \sqrt{\alpha^2 + \beta^2}$
Modal damping ratio $\zeta = -\alpha/\omega_0$
- (3) Gyroscopic factor $\gamma = -C_g^*/M^*$
- (4) After obtaining these values by complex eigenvalue analysis, the prediction values of the damped critical speed and the Q-factor can be calculated.

$$\left. \begin{aligned} Q_0 &= \omega_0 / \sqrt{1 - 2(\beta\zeta)^2} \\ Q &= \frac{1}{1 - \gamma} \frac{1}{2\beta\zeta\sqrt{1 - (\beta\zeta)^2}} \end{aligned} \right\} \quad (15)$$

Furthermore, the determination of a maximum amplitude at the peak of the response curve requires the following sequences.

- (5) Modal unbalance eccentricity ε^* rearranged by unbalance distribution U, in such a way that $\varepsilon^* = \bar{\phi}_f^t U / M^*$
- (6) Amplitude at a resonance on a normal coordinate system S peak $= Q \varepsilon^*$
- (7) Elliptical orbit at a resonance with a major axis; magnitude $(\bar{a}_f + \bar{a}_b) Q \varepsilon^*$
phase $(\theta_f - \theta_b)/2$
minor axis; magnitude $|\bar{a}_f - \bar{a}_b| Q \varepsilon^*$
where $\phi_f = a_f e^{i\theta_f}$, $\phi_b = a_b e^{i\theta_b}$

- (8) Amplitude at a resonance in the X and Y directions on a physical coordinate system.

$$\left. \begin{aligned} A_{x\max} &= |\phi_f + \phi_b| Q \varepsilon^* \\ A_{y\max} &= |\phi_f - \phi_b| Q \varepsilon^* \end{aligned} \right\} \quad (16)$$

3. Unbalance Response Curve and Q-Factor

The subject discussed here is how to examine a curve of the unbalance response in order to find the Q-factor, taking the gyroscopic effect into account.

3.1 Q-factor at 70% Amplitude

The definition of the Q-factor, which is widely known, is shown in Fig.3. It is here denoted as Q_{70} ,

$$Q_{70} = \frac{Q_0}{\Delta\Omega} \quad (17)$$

The Q_{70} formula is reasonable for systems with a small damping ratio and no gyroscopic effect.

However the present analysis deals with systems characterized by large damping and spinning effect. Therefore, it is necessary to review the exact relationship between the modal unbalance response defined by Eq.(6) and Q_{70} . At a 70% amplitude on a response curve, $\Delta\Omega/\omega_0$ and $\eta = \beta\zeta$ are combined by the equation.

$$\frac{1}{\sqrt{2}} \frac{1}{2\beta\zeta\sqrt{1 - (\beta\zeta)^2}} = A \left(\frac{Q}{\omega_0} \right) \quad (18)$$

The combination of Eq.(17) and (18) gives the following formula of Q_{70} by which we may exactly define Q_{70} .

$$Q_{70} = \frac{\sqrt{1 - 8\eta^2 + 8\eta^4} (\sqrt{1 - 2\eta^2} + 2\eta\sqrt{1 - \eta^2} + \sqrt{1 - 2\eta^2 - 2\eta\sqrt{1 - \eta^2}})}{4\eta\sqrt{1 - \eta^2}\sqrt{1 - 2\eta^2}} \quad (19)$$

Especially when $\eta = \beta\zeta \ll 1$, it becomes

$$Q_{70} \approx 1/2\eta = 1/2\beta\zeta \quad (20)$$

This formula is very similar to the conventional one in which $Q_{70} \approx 1/2\zeta$ in structure dynamics. The relationship of Q_{70} to η is shown in Fig.4 according to Eq.(19).

3.2 Finding of Actual Q-Factor

After finding Q_{70} as shown in Fig.3, the modified modal damping ratio η is algebraically given by the inverse function of Eq.(19) or graphically by the Q_{70} chart in Fig.4. On the other hand, the actual Q-factor Q_A is a function of η and γ as shown in Eq.(8), i.e. $Q_A = Q_A(\eta, \gamma)$, where the actual Q-factor gives the curves in correspondence to the gyroscopic factor γ , denoted by Q_A in Fig.4.

Fig.3 Response Curve and Q-Factor

Therefore, the nondimensional chart in Fig.4 is a very practical means for determining the actual Q-factor Q_A . In fact, Q_{70} is identified by a sharpness of the response curve around its peak as shown in Fig.3, and it is located on a perpendicular axis in Fig.4.

After finding Q_{70} on the chart in Fig.4, we go rightward along the path shown by the arrow, come to the point of intersection with the Q_{70} curve, and then go downward. Finally, the modified damping ratio γ is obtained.

Again we go upward from the intersection on the Q_{70} curve, come to the Q_A curve associated with the corresponding gyroscopic factor γ , and turn left at the point of intersection at the $Q_A(\gamma, \gamma)$ curve. This course gives the actual Q-factor Q_A on the perpendicular axis, as shown by the leftward-pointing arrow in Fig.4. This type of graphical procedure is an effective method for finding the actual Q-factor while examining the response curve.

In special cases where there is a small damping ratio with $\gamma \ll 1$, the actual Q-factor Q is given by the following approximate formula.

$$Q \approx Q_{70}/(1-\gamma) \quad (21)$$

4. Numerical Simulation

The rotor-bearing system given in Fig. 5 is employed to demonstrate the effectiveness of the new Q-factor definition. The configuration of the rotor is fully presented in ref.(8). Calculation of unbalance vibration is performed by the transfer matrix method and the complex eigenvalue problem is solved by the finite element method and QR procedure.

Two types of bearings are selected for the simulation.

- (1) Support of Spring K - Damping C; The coefficients of the spring and the damping are constant at all rotational

Fig. 4 Q-Factor Chart

speeds. This is an example of a symmetric rotor-bearing system

- (2) Tilting Pad Type; This is a sliding type of bearing, its dynamic properties vary with the rotational speed. It is selected as an example of an asymmetric rotor-bearing system.

Notations of the Q-factor should be defined at this point.

- (a) Q ; the exact Q-factor which is precisely defined by the ratio of a maximum amplitude to the modal mass unbalance eccentricity.

By complex eigenvalue analysis, the following two types of Q-factor can be derived.

- (b) Q_{D0} ; value especially when $\gamma = 0$ in Eq.(8)

- (c) Q_D ; value calculated in Eq.(8) using γ and γ .

On the other hand, by examining the sharpness of the unbalance response curve, the following three types of Q-factors can be determined.

- (d) Q_{70} ; expressed in Fig.3

- (e) Q_{A0} ; the value calculated by the insertion of the modified damping ratio γ , which is graphically obtained by the given Q_{70} in Fig.3, into Eq.(8) where $\gamma = 0$.

- (f) Q_A ; defined by $Q_A = Q_{A0}/(1-\gamma)$ (22)

Table 2 concludes with a short description of each type of notation for the Q-factor. Among these, Q_{D0} , Q_{70} and Q_{A0} are based upon the conventional definition with $\gamma = 0$, and do not consider the gyroscopic effect. The others Q_D and Q_A fall under new definition with $\gamma \neq 0$. In this chapter, errors of each Q_i ($i = D0, D, 70, A0, A$) in relation to the exact value Q are discussed.

Results of the undamped critical speed analysis are presented in Fig.6 with regard to each order of critical speed and the modal quanta with modal mass M^* and modal gyro C_g^* in relation to the varying support stiffness k in the bearings. Modal masses in the first and second modes account for about 65 percent of the physical rotor weight. Modal gyro in the second mode is greater than that in the first mode because the inclination of disks steeper in the second mode than in the first.

Tab.2 Notation of the Q-Factor

Q	Amp. peak / ε^*
Q_{70}	$\Omega_c / \Delta \Omega$
Q_{A0}	$1/2 \eta \sqrt{1-\eta^2}$ ($\eta = \eta_{70}$)
Q_A	$Q_{A0}/(1-\gamma)$
Q_{D0}	$1/2 \zeta \sqrt{1-\zeta^2}$
Q_D	$1/(1-\gamma) (2 \eta \sqrt{1-\eta^2}) (\eta = \beta \zeta)$

Fig. 5 Rotor

The gyroscopic effect is evaluated by the gyroscopic factor γ which can be calculated by a division of the modal gyro per modal mass. For example, where $k=1600$, the values of γ in the first mode and the second mode become respectively.

$$\gamma = 1.4/34 \approx 0.04 = 4\% \quad \text{for 1st mode}$$

$$\gamma = 5.8/34 = 0.17 = 17\% \quad \text{for 2nd mode}$$

4.1 Symmetric Properties in Bearings

Putting unbalance weights (1 grf, 80mm radius) on both disks L and R where the phase difference between them is 90 degrees, results in the unbalance vibration as described on the response curves shown in Fig.7. The corresponding complex eigenvalues are given in the damped natural frequency map and the modal damping ratio map as shown in Fig.8. It is obvious in Fig.8 that critical speeds are the points of intersection between rotational speed and forward natural frequency on solid lines, in other words, 46 rps and 192 rps. Also, peaks of the response curves in Fig.7 are located at the speeds predicted in Fig.8.

The Q-factor analysis developed in the previous chapters will now be explained in detail by using a numerical example. For this example, the second eigen mode is chosen because it has a larger gyroscopic effect than the first one. The modal mass unbalance eccentricity ϵ^* is determined by the relationship of the modal mass and the unbalance distribution.

$$\epsilon^* = (1.0e^{i0^\circ} \times 1.0 \times 80e^{i0^\circ} + 1.0e^{i90^\circ} \times 1.0 \times 80e^{i90^\circ}) / M^* = 4.95 \mu\text{m}$$

The maximum amplitude A peak at the second critical speed is seen in Fig.7, that is

$$A_{\text{peak}} = 37.3 \mu\text{m}$$

Hence the exact Q-factor implies that

$$Q = A_{\text{peak}} / \epsilon^* = 37.3 / 4.95 = 7.54$$

On the other hand, Q-factors predicted by the results of complex eigenvalue analysis

are obtained using Eq.(8) with $\xi = 0.07$ and $\gamma = 0.17$.

$$Q_{D0} = 6.98, \quad Q_D = 7.63$$

The configuration of a response curve as shown in Fig.7 gives a Q-factor with the sharpness at the 70% amplitude around the second peak, that is

$$Q_{70} = 6.22$$

Other Q-factor are then determined by using the nondimensional chart in Fig.4.

$$Q_{A0} = 6.43 \quad Q_A = 7.75$$

These values are concerned with the second mode. In the same manner, the values of Q-factor are obtained for the first mode. All of these values are finally summarized in Table 3-I. It is thus revealed that the predicted values Q_D and Q_A are closer to exact one than the others, Q_{D0} , Q_{70} and Q_{A0} .

A survey about the error in each Q-factor prediction is graphically summarized in Fig.9, and includes a variety of the values for stiffness K and damping C . The perpendicular axis indicate the ratio of each Q_i ($i=D0, D, 70, A0, A$) to the exact Q , where the ratio comes so close to unity that it is a better prediction.

Fig.7 Unbalance Response
in Symmetrical System

Fig.6 Modal Quanta for Mode
at Undamped Critical Speed

Fig.8 Complex Eigenvalue

Looking at Fig.9, it can be said that with the new Q-factor prediction, Q_D and Q_A , identified by solid marks $\blacklozenge$ and $\bullet$ respectively, are located more closely to unity than with the other conventional predictions of Q-factor as shown throughout the six charts.

With respect to the first mode, the error for Q_D stays within 5%, and for Q_A remains within 9%. The latter becomes a little higher than an former because of the unavoidable error involved when measuring the sharpness of a peak. The other Q-factors, Q_{D0} , Q_{70} and Q_{A0} , contain a maximum error as large as 10%. It is thus acceptable that the introduction of the gyroscopic factor is very important for a more accurate method to predict the Q-factor even in the case with a small γ^* , such as in the first eigen mode.

With respect to the second mode, the necessity for consideration of the gyroscopic factor γ^* is even more obvious. Error in the second mode is kept within a maximum of 4% for Q_D and Q_A in Fig.9. But it reaches 10% for Q_{D0} and almost 20% for Q_{A0} and Q_{70} . The conventional prediction method induces sizable errors for eigen mode with large gyroscopic effects, such as the second mode. The new prediction method based upon a consideration of gyroscopic factors is thus a very desirable one.

4.2 Asymmetric Properties in Bearings

The unbalance response of rotors supported by tilting pad type bearings is shown in Fig.10. Two peaks appear at about the first critical speed, showing superior vibration in the vertical and horizontal directions.

The corresponding complex eigenvalue map is shown in Fig.11. The curve for the first damped natural frequency has two points of intersection with the curve for the rotational speed; one is on the forward precession (solid line, horizontally remarkable mode) and the other on the backward precession (dotted line, vertically remarkable mode). The critical speed corresponding to the backward natural frequency is a little higher than the forward one, but the modal damping ratio at the former critical speed is lower than the one at the latter critical speed. This is why the resonance of the first eigen mode is superior in the vertical direction.

Fig.10 Unbalance Response in Asymmetrical System

Fig.9 Comparison of Accuracy between Q-Factors

The results of Q-factor analysis are listed in Table 3-II in regard to the two peaks around the first and second critical modes. In comparison with the exact Q-factor, Q_D and Q_A represent the most acceptable accuracy, while the others Q_{D0} , Q_{A0} and Q_{70} are tolerable.

For both the first and second eigen modes, the whirl orbits in the unbalance response and eigen mode are so sharply elliptical that $|\phi_f| \approx |\phi_b|$ implies a very small gyroscopic factor as is shown where $\gamma = 0.0019$, -0.0002 and 0.0074 . It is why every Q_i ($i=D0, D, 70, A0, A$) becomes almost the same and there is little difference caused by the gyroscopic effect. One of the first eigen modes describes an elliptical and backward whirl orbit, and its gyroscopic factor γ is negative.

It is obvious that the gyroscopic factor γ becomes zero and its effect vanishes in the case of a resonance with the specialization of such an elliptical orbit created by strong asymmetrical bearing properties. For this reason, gyroscopic factor γ plays a very important role in estimating the Q-factor in systems with even asymmetric supports.

5. Conclusions

From the results of the present study, the following conclusions have been drawn.

- (1) For a general rotating system with many disks and bearings, the unbalance vibration is analyzed corresponding to each eigensolution, based upon the orthogonality of the undamped eigen modes.
- (2) Modal unbalance responses are formulated taking gyroscopic effect into con-

sideration, and a new Q-factor definition is presented in order to predict the Q-factor by complex eigenvalues or unbalance response curves.

- (3) Using the new Q-factor definition, it has become possible to accurately estimate resonance sensitivity and actual peak amplitude even in systems with large gyroscopic forces, large damping forces and strong asymmetrical properties in bearings.

6. Acknowledgements

The authors would like to thank Dr. S. Yanabe, assistant professor of the Technological University of Nagaoka, and Dr. S. Saito, research engineer of the Research Institute of Ishikawajima-Harima Heavy Industries Co. Ltd., for their valuable comments on the results of the new Q-factor prediction method.

7. References.

- (1) Siraki, K. et.al; Technical Review of Mitsubishi Juko (in Japanese), 16-12 (1979-3), 135
- (2) Saito, S.; Trans. JSME (in Japanese), 46-401 (1980-1), 8
- (3) Lund, J.W.; Trans. ASME Ser. K, 102-1 (1980-1), 115
- (4) Cunningham, R.E.; Trans. ASME Ser. B, 97-4 (1975-11), 1383
- (5) Matsushita, O. et.al; Trans. JSME (in Japanese), 72-610 (1980-3), 245
- (7) Kikuchi, K.; Bulletin of JSME, 13-61 (1970), 864
- (8) Tajima, K.; Vibration in Engineering (in Japanese) (1974), 126, Sangyo-Tosyo

8 Appendix

(A) Complex Eigenvalue with Symmetric Support

Assuming that an eigen mode ϕ at an undamped critical speed is available around a damped critical speed, a free vibration can be expressed by

$$z = \phi e^{\lambda t} \quad (23)$$

where the complex eigenvalue λ is unknown.

Fig.11 Complex Eigenvalue

Tab.3 Q-Factors in Rotor-Bearing System

	I		II			
3DISK -ROTOR	Symmetric Support		Asymmetric Support			
	First Mode	Second Mode	First Mode		Second Mode	
M *	33.52	34.30	64.67	68.06	65.60	
ϵ *	3.67	4.95	3.95	3.67	4.96	
γ	0.0404	0.17	0.0019	-0.0002	0.0074	
A _{peak}	84.3	37.3	7.0	77.8	26.2	
Q	22.98	7.54	1.77	21.20	5.28	
Eigenvalue	ζ	0.0228	0.0718	0.2680	0.0249	0.0938
	Ω_c	45.98	192.32	43.15	46.09	166.90
	Q _{D0}	21.90	6.98	1.94	20.09	5.35
	Q _D	22.35	7.63	1.94	20.08	5.37
Response	Q ₇₀	20.79	6.22	-	19.88	5.54
	Q _{A0}	21.28	6.43	-	20.01	5.84
	Q _A	22.18	7.75	-	20.01	5.86

substituting Eq.(23) into the equation of motion Eq.(1) and premultiplying the transposed eigen mode ϕ^t , the following equation is obtained on a normal coordinate system

$$M^*\lambda^2 + i\Omega C_g^*\lambda + (K_r^* + iK_j^*) + (C_r^* + iC_j^*)\lambda = 0 \quad (24)$$

This is an eigen equation used to determine an complex eigenvalue corresponding to the eigen mode ϕ .

After putting $\lambda = \alpha + i\eta$ into Eq.(24), the separation of the real part from the imaginary part is performed and operations such as (real part) $\times \alpha +$ (imaginary part) $\times \eta$ and -(real part) $\times \eta +$ (imaginary part) $\times \alpha$ give the equations.

$$\alpha\omega_0^2 M^* + \alpha K_r^* + \omega_0^2 C_r^* + \eta K_j^* = 0 \quad (25)$$

$$\eta\omega_0^2 M^* + \Omega\omega_0^2 C_g^* - \eta K_r^* + \omega_0^2 C_j^* + \alpha K_j^* = 0 \quad (26)$$

Therefore, the relationships between the matrices of the quadratic form and the complex eigenvalue are approximated.

- (1) In a case where $Cr^* \neq 0$ and $Kj^* = Cj^* = 0$,

$$\text{i.e. } C_{xx} = C_{yy} \neq 0, \quad C_{xy} = -C_{yx} = K_{xy} = -K_{yx} = 0$$

If the eigenvalue λ is almost constant at a rotational speed around the critical speed, i.e. $\Omega \approx q$, from Eq.(26), it follows that

$$\omega_0^2 = \frac{K_r^*}{M^* + C_g^*} \approx \Omega_0^2 \quad (27)$$

Therefore, the combination of Eq.(25) and Eq.(27) gives.

$$\left. \begin{aligned} \alpha &= -C_r^* \omega_0^2 / (\omega_0^2 M^* + K_r^*) \\ &\approx -C_r^* / (2M^* + C_g^*) \\ \text{i.e. } \frac{C_r^*}{M^* + C_g^*} &\approx -2\alpha \frac{1-\gamma/2}{1-\gamma} \approx 2\zeta\beta\omega_0 \end{aligned} \right\} \quad (28)$$

- (2) In a case where $Cr^* \neq 0$, $Kj^* \approx 0$, $Cj^* \approx 0$

$$\text{i.e. } C_{xx} = C_{yy} \neq 0, \quad K_{xy} = -K_{yx} \approx 0, \quad C_{xy} = -C_{yx} \approx 0$$

Supposed here that $\Omega \approx \omega_0 \approx q$ and $\alpha \approx 0$ near a critical rotational speed, it follows from Eq.(26) that,

$$\left. \begin{aligned} q\omega_0^2 (M^* + C_g^*) &\approx qK_r^* - \omega_0^2 C_j^* \\ -\alpha K_j^* &\approx qK_r^* - q\Omega C_j^* \\ \therefore \frac{K_r^* - \Omega C_j^*}{M^* + C_g^*} &\approx \omega_0^2 \approx \Omega_0^2 \end{aligned} \right\} \quad (29)$$

and from Eq.(25) that

$$\left. \begin{aligned} \alpha(\omega_0^2 M^* + K_r^*) &= -\omega_0^2 C_r^* - qK_j^* \\ &\approx -\omega_0^2 (C_r^* + K_j^* / \Omega) \\ \therefore \frac{C_r^* + K_j^* / \Omega}{M^* + C_g^*} &\approx 2\zeta\beta\omega_0 \end{aligned} \right\} \quad (30)$$

Therefore, since the relationship of Eq.(5) is approximately acceptable in both cases (1) and (2) as is stated above, Equation (3) can be reduced to Eq.(6).

(B) Complex Eigenvalue with Asymmetric Support

If ϕ_f and ϕ_b respectively indicate the forward and the backward eigen modes at an undamped critical speed, an eigensolution may be assumed by the following form with

as the complex eigenvalue

$$z = \phi_f e^{i\Omega t} + \phi_b e^{-i\Omega t} \quad (31)$$

The substitution of Eq.(31) into Eq.(9) and premultiplication of the resulting equation by the transposed eigen mode vector $[\bar{\phi}_f^t, \bar{\phi}_b^t]$ finally give the eigen equation of Eq.(24) as shown in the previous Appendix (A). In this case it should be noted that the coefficients of the modal quanta must be calculated by Eq.(13) instead of Eq.(4).

In either case of (1) or (2)

$$(1) \quad Cr^* \neq 0, \quad Kj^* = Cj^* = 0$$

$$\text{i.e. } K_{xy} = K_{yx}, \quad C_{xy} = C_{yx}$$

$$(2) \quad Cr^* \neq 0, \quad Kj^* \neq Cj^* \neq 0$$

$$\text{i.e. } K_{xy} \approx K_{yx}, \quad C_{xy} \approx C_{yx}$$

the approximate expression of Eq.(5) can be derived in the same manner as shown in Appendix (A).

(C) Elliptical Whirl Orbit

If the vibration is described by the complex displacement form

$$z = \phi_f e^{i\Omega t} + \phi_b e^{-i\Omega t} \quad (32)$$

where ϕ_f and ϕ_b represent the forward and backward, respectively, this equation will be written by the following significant expression of the elliptical whirl orbit.

$$z = \exp\{i(\theta_f - \theta_b)/2\} \{a_f e^{i(\Omega t + \theta_f)} + a_b e^{-i(\Omega t + \theta_b)}\} \quad (33)$$

$$\text{where } \theta_1 = (\theta_f + \theta_b)/2$$

The corresponding whirl orbit is specified in Fig.12.

Next, let us decompose the complex displacement z into the real displacements x and y in X and Y directions. Since it is clear that the relationship between both forms in regard to the displacement is defined by the equation,

$$z = x + iy = \text{Re}[\phi_f e^{i\Omega t}] + i \text{Re}[\phi_b e^{i\Omega t}] \quad (34)$$

the results of the decomposition becomes as follows

$$\left. \begin{aligned} \phi_f + \phi_b &= \phi_x \\ \phi_f - \phi_b &= i\phi_y \end{aligned} \right\} \quad (35)$$

$$\therefore \left. \begin{aligned} x &= \text{Re}[(\phi_f + \phi_b)e^{i\Omega t}] \\ y &= \text{Im}[(\phi_f - \phi_b)e^{i\Omega t}] \end{aligned} \right\} \quad (36)$$

Fig.12 Elliptical Whirl Orbit

(D) Q-factor Analysis of a Rotor with an Overhanging Disk

In order to explain the insufficiency of the conventional Q-factor formulas, a typical rotor is shown in Fig.14. This rotor has an overhanging disk and is supported by pin type bearings and a damper.

This is a discussion on unbalance vibrations with respect to fat and thin disks, where either of these disks has eccentricity e and constant weight. A greater gyroscopic force acts on the latter disk than on the former one.

The corresponding unbalance response curves are shown in Fig.13. Response curves with non-dimensional rotating speeds so as to coincide a critical speeds with unity are drawn in more detail around the peak in Fig.14. Values related to Q-factor analysis are summarized in Table 4.

Now, let us numerically explain the Q-factor along with Table 4. The modal damping ratio ζ is given by the complex eigenvalue analysis of the systems. The Q-factor Q_{D0} is then calculated according to the conventional definition, and the maximum amplitude A_{peak} can be predicted by the calculated value of Q_{D0} , as shown in Table 4-I.

Comparing the predicted values A_{peak} as is stated above, the value of predicted $A_{peak} = 49$ in the system with the disk $L/D=4$ agrees well with the peak amplitude on the corresponding response curve. However, in the case of the disk $L/D=0.02$ the predicted value of $A_{peak} = 34$ is very different from the actual peak amplitude. The gyroscopic effect surely generates this discrepancy because it is larger in the latter system than in the former one. This unsatisfactory fact is pointed out in the doubt (1) mentioned in the Introduction.

The next subject is the sharpness of the response curve around a peak. Since the maximum amplitude is about $50e$, and the modal unbalance eccentricity e^* is about $0.6e$ in either of the two systems, it is obvious that the exact Q-factor becomes $Q=72.5 \sim 76.9$.

On the other hand, the examination of the sharpness on response curves gives the Q-factor, denoted by Q_{70} , as shown in Table 4-I. Comparing the examined values Q_{70} with the exact value Q , the error in the case of the fat disk is very small, but for the thin disk, it is in marked disagreement with the

exact value. It is clear that the gyroscopic effect causes this disagreement. This discrepancy is the subject of the doubt (2) in the Introduction.

The authors would like to introduce the gyroscopic factor γ ranging in nondimensional value from 0 to 100%, which measures the gyroscopic effect in quantum as is stated in detail in chapter 2, as are shown in Tab. 4-II.

For the two systems, the gyroscopic factor becomes $\gamma=3.2\%$ and $\gamma=62.6\%$ for the fat disk and the thin disk, respectively. Based upon the new Q-factor definition developed here which takes the gyroscopic effect into consideration, the Q-factor Q_D is obtained by the complex eigenvalue and the other Q-factor Q_A is given by examining the sharpness of the response curve. These

Fig.13 Unbalance Response Curve

Tab.4 Conventional Q-Factor and New Q-Factor

SYSTEM		GYRO EFFECT	ACTUAL VALUE			I CONVENTIONAL			II	III NEW METHOD		
L/D	ζ		A.peak	e^*	Q-FACTOR	Q_{D0}	A.peak	Q_{70}	Gyro Factor	Q_D	A.peak	Q_A
0.02	0.010	Large	50	0.69	72.5	50	34	26	0.626	72.9	50.3	70.4
4	0.0066	Small	50	0.65	76.9	76	49	71	0.032	77.3	50.2	74.8

values are $Q_D = 77.3$ and $Q_A = 74.8$ for the fat disk, and $Q_D = 72.9$ and $Q_A = 70.4$ for the thin disk. All of these values obtained by the new concept are in good agreement with the exact Q-factor. It can thus be said that the new method gives accurate prediction values for a peak amplitude, as shown in Tab. 4-III.

According to the result of the Q-factor study in the example of a rotor with an overhanging disk, it has been clarified that the new Q-factor definition encompassing the effect of the gyroscopic force can very accurately predict the Q-factor. Accordingly doubts involved in the conventional Q-factor prediction method are eliminated.