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Finite Element Analysis of the Threaded Connections subjected to External Loads*

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Although there are many reports referred to the threaded connections subjected to an external load, no reports have fully investigated the contact surfaces. This paper introduces a numerical method to analyse such cases as the threaded connections subjected to a transverse displacement and the flange coupling under an arbitrary type of load for the typical examples of the threaded connections subjected to external load. And the self-loosening mechanism of the threaded connections and the relation between the axial tension and the external load are discussed by the analytical results.

1. Introduction

So far, there have been many studies on threaded connections subjected to an external load.⁽¹⁾⁻⁽³⁾ In these studies, solutions have been obtained by using the elementary methods of materials, so it is difficult in them to take the effect of contact surfaces into account. Also each of them is based on the most idealized model, and the assumption used in the analysis generally has no physical meanings. For example, in the case of the self-loosening of threaded connections under a transverse displacement, it is not so easy to understand the self-loosening mechanism on a simple assumption without analytically checking the distribution of deformation and loads in each part of the members.

Especially it is not so preferable to obtain the relation between a transverse displacement and the loads acting on the threads which are statically indeterminate, by using methods of statics or elementary theories. And this is far from being a physical interpretation and an investigation on the causality. In the case of the flange coupling under an external load, it is very difficult to find out the load distribution and the correlation of a threaded fastener to other members by idealizing a bolt-nut as a rod or cylinder and the flange coupling as a hollow cylinder.

On the other hand, during recent years a numerical technique called the finite element method has been

much used in the investigation of the threaded connections, and is receiving attention in the design of the threaded connections and the stress analysis. The authors have been studying the applications of the finite element method to the threaded connections.⁽⁴⁾ Applying these methods, it is very easy to analyse the threaded connections subjected to an external load.

In this paper, considering the cases of the threaded connections subjected to a transverse displacement and the flange couplings under an arbitrary acting load as typical examples of the threaded connections subjected to an external load, the numerical method and the obtained results will be introduced.

2. Threaded Connections subjected to a transverse displacement

Several investigators have stated about the phenomena of the self-loosening of the threaded connections. But there seem to be many unsolved problems and a clear explanation of the self-loosening mechanism has not been given yet. The self-loosening of the threaded connections is primarily responsible for the rotational slip caused by a slip on the nut surface. Strictly speaking, it is difficult to say that the mechanism of the self-loosening process used by them is complete and a precise solution has been obtained.

In this section analysing the threaded connections subjected to a transverse displacement as an application of the finite element method, the authors will discuss, though qualitatively, the self-loosening mechanism of the threaded connections from the obtained results.

2-1. Analysis of the variation in the flank load distribution

One of the simplest ways to investigate the threaded connections

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subjected to a transverse displacement or a bending moment is to consider the system as an axi-symmetric solid with an asymmetric load. The nodal displacements and forces can be divided into symmetric and anti-symmetric components, and it is enough to take the symmetric components into account for discussion on the threaded connections subjected to a transverse displacement. Superposing the partial effect of axi-symmetric load only due to the axial tension on the obtained results, unlike under the usual assumption which has been made, it becomes clear that both the contact areas and the non-contact areas exist in the pressure flank surfaces of threads.

In spite of the advantages of this approach that the threaded conn-

Fig.1 Two dimensional modeling of threaded connection

Fig.2 Variation of the ratio of flank load to axial tension

ections with fastened plates are quite easily taken into account, because of presence of non-contact areas the symmetry is lost and solutions for the variation in the flank load can not be obtained. For this reason, after Yamamoto's model⁽²⁾ the bolt is analysed with a two-dimensional model shown in Fig.1. It is assumed that the number of threads in contact is 6, and that the nut and the fastened plates are regarded to be rigid.

If the nut slides a distance of 0.1 mm to the right after the bolt has been tightened to an initial force of 63 Kgf, the variations of the ratio of the flank loads to the axial tension in both the right and the left threads are shown in Fig.2. Although a reduced matrix for nodal components on the contact surfaces is used in the analysis from an economic stand point of the short computational time involved, the detailed procedure to introduce the reduced matrix will be described in the following sections.

In Fig.2 for physical reason of releasing the nodal points where the negative nodal forces are acting from the contact surfaces, the axial tension increases and the flank loads are the greatest at the first thread on the left. Either the nut bearing surface does not contact as completely as the threads. The axial tension finally increases to 90 Kgf and the contact points at the last stage are marked by dark spots in Fig.1.

Furthermore for the purpose of investigation on the behaviour of the model which simulates the actual threaded connection, the system as shown in Fig.3 is used too in this study.

Fig.3 Three dimensional modeling of threaded connection

the computational time and the core storage of the available computer, it is not so easy to investigate the self-loosening process in consideration of static-kinetic friction, the geometry of the lead and deformation of the nut and the fastened plates. However some deficiencies in those studies which have been performed by the conventional elementary theory of beam can be pointed out by the analytical results.

Namely, in the threaded connections under a transverse displacement, considerably eccentric loads are present, and the pressure flank does not entirely contact with the whole contact surfaces. It is not simple equilibriums⁽²⁾ for loads on the pressure flank but the geometric shape and the flexibility of each part in assembly that relate to the magnitude and the location of these eccentric loads. Also it is not preferable to obtain the bending moment and the transverse displacement without regard for the load which is applied perpendicular to the axis of the bolt. And the axial tension and the torque moment increase. If the torque caused in the bolt increases while the axial tension decreases,⁽²⁾ the self-support condition fails to hold, and the loosening of the threaded connections will occur without a slip on the nut bearing surface.

Although it is not so easy to simulate the phenomenon of the self-loosening process exactly even with the help of the finite element method a qualitative explanation for the self-loosening mechanism of the threaded connections can be made below. But the hysteresis caused by the friction or the like is ignored in order to simplify the process, and a two-threaded model shown in Fig 6 is used for convenience. When the bolt and the nut are in a state of neutral equilibrium, no loads but axial tension are applied to the threaded connection and accordingly the loads acting on both the right and left threads are equal. This relation may be written as $F_1^L = F_1^R$ and $F_2^L = F_2^R$ (Fig.6-a). If the nut moves to the right in a direction perpendicular to the bolt axis, the bolt is

pushed and bent to the right by the load acting on the first thread in the left. And this time, the contact force decreases in the first right-side thread. Owing to the inclination of the bolt, the contact forces on the second thread increase in the right and decrease in the left. Namely, both F_1^L and F_2^R increase and both F_1^R and F_2^L decrease. If either F_1^R or F_2^L decreases to zero, the axial tension begins to increase. Simultaneously because of the relative slip between the threads of the bolt and the nut, the bolt is twisted in the direction of loosening rotation.

The nut fastens the bolt more tightly with the nut bearing surface fixed to the fastened plates and moves upward along thread helix, this the system keeping its equilibrium. Naturally, F_1^L and F_2^R cause a counter-clockwise bending moment. If the ratio of the lateral load (perpendicular to the axis) which the bolt is subjected to from the nut, namely the static friction force R_1-R_2 acting on the nut bearing surface, to the normal force N_1+N_2 exceeds the maximum coefficient of static friction, there appears a relative rectilinear slip between the bolt-nut assembly and the fastened plates. Then, the torque of the bolt reduces to zero, because the nut is forced to slip in the direction of loosening rotation.

Fig.6 Contact force on threaded in the self-loosening process

Table 1. Rearrangement of the stiffness matrix

CLASS 1	components unconnected with any other units
CLASS 2	components connected with the preceding unit
CLASS 3	components connected with the following unit
CLASS 4	
CLASS 5	components which are necessary for calculation

For convenience, the number of threads is set at 4 and the bottom of the shank is axially fixed. Although the geometry of the bolt is axi-symmetric, the effects of the helix angle on the loads and the deformations at the contact points are taken into account. Also the nut and the fastened plates are assumed to be rigid and all friction at the thread surface is neglected for convenience. Although the application of the finite element method makes it possible to increase the number of nodal points on the contact surfaces, the criterion for the forces or the pressure on the contact surfaces is not so brief due to the use of high order elements. So the contact points used in this paper are represented by 4 points which are located in the positions of 0, 90, 180 and 270 deg. on the pitch circle of each thread. From a point of view taken in the usual assumption which has been considered, it seems to be quite all right that two points of 90 and 270 deg. were chosen as the subject of a study by T. Sakai and two points of 0 and 180 deg. were chosen by A. Yamamoto et al.

The solutions are obtained by the process of iteration. And the convergence after the procedure of deriving the reduced matrix for nodal points on the contact surfaces is the same as in the case of two-dimensional analysis. The variations of the flank load distribution at the points of 0 and 180 deg. are shown in Fig.4, when the nut slides a distance of 0.1 mm in parallel to the 0-degree orientation after the bolt has been tightened to an initial tension of 686 Kgf.

Fig.4 Variation of flank load distribution at the points of 0 and 180 deg.

The variations of the flank load distribution show a certain pattern, whether or not a helix angle is provided. Also the effects of the presence and the absence of the helix angle on the points of 90 and 270 deg. are shown in Fig.5. In the case of the presence of the helix angle, the contact forces at the points of 90 deg. in the first and second threads are greater than those at the points of 270 deg, but the tendency of the contact forces in the third and fourth threads are contrary to each other.

Generally whether the contact forces increase or decrease depends upon the position of threads on the nut which thrust the bolt or depart from it. But the position subjected to the increasing contact forces in the upper threads near the top of the nut is on the opposite side of the increasing force acting position in the first thread under the effect of the bending caused in the bolt. The axial tension increases in a similar manner to two-dimensional results, and because of the small-deformation theory, the tangential force analysed by providing a helix angle increases at the same rate as the axial tension.

2.2 Consideration on the Self-loosening Mechanism of the Threaded Connections

Although the most idealized model has been considered in this paper, an attempt was made to analyse the self-loosening problem of the threaded connections by using the finite element method for the first time. Because of a considerable amount of

Fig.5 Variation of flank load distribution at the points of 90 and 270 deg.

by the restoring torque moment caused in the bolt. The rotations of the bolt and the nut which are in a state where the nut fastens the bolt in higher position than the neutral release the restoring torque of the bolt, and the system changes the position of equilibrium and loosens.

A consideration on the fundamental mechanism of the threaded connection is made above qualitatively by the analytical results using the finite element method. As mentioned before there exist the complicated phenomena of friction acting on the contact surface in actual threaded connection. And the actual self-loosening phenomena will be more complicated by these effects, and furthermore by the deformation of each part of the assembly. But the fundamental mechanism of the self-loosening phenomena may be as stated above.

Fig. 7 Force-deformation relation of the threaded connection

Fig. 8 Division of the threaded connection into finite element meshes

3. Threaded Connections carrying Axial Loads

There are not a few cases in the design of threaded connections where the value of the loading factor is controversial. Although this loading factor is usually obtained from the diagram shown in Fig. 7 that presents the relation between the load and the deformation of threaded connection, it seems to be doubtful that such a simple relation holds in the actual threaded connection. Because the actual threaded connection consists of very complicated elements and makes a delicate contact problems.

Generally, if the threaded connection is subjected to an external load, the contact area and the contact force vary its range and magnitude respectively. So it is very difficult to analyse the threaded connection subjected to an external load in a traditional way which has been customary used. In this section, for the typical examples of the threaded connections subjected to an external load, a numerical method and the analytical results are introduced in the case of the flange coupling or the like under an external load.

3-1. Threaded Connection with Axial Loads

It is unavoidable to use a method of iteration and convergence in order to investigate variations in the axial tension and the contact area which are associated with the external load variation. Particularly, in the special case of a geometrically non-linear problem, it is necessary to reassemble the stiffness matrices in each step of the iterative procedure, so the involved computational time becomes very long. However, the reassembly of the stiffness matrices in the whole region is not always necessary, and sometimes the partial arrangement of the matrices in

Fig. 9 Relation between contact surface and external load

subregion is rather attractive to a certain kind of structure analysis. Especially in the analysis of a contact problem based on small-deformation theory using linear elements, it saves the computational time to introduce a reduced matrix for nodal points of the boundary which are necessary for the calculation at least.

The process to introduce a reduced matrix by improving the method of unit partitioning in series which is widely used is as follows: In the first stage, the stiffness matrix of each unit is rearranged according to the order shown in Tabl. then the reduced matrix consisting of components of Classes 3, 4 and 5 is introduced and then superposed on the stiffness matrix of the following unit. So the stiffness matrix increases the number of components just as the sum of components of Classes 4 and 5 in the preceding unit. In the last unit the reduced matrix obtained for the components of Classes 4 and 5 is the stiffness matrix which is at least for the calculation.

In this study the threaded connection under an axi-symmetric loading is analysed with the element division shown in Fig.8. For the boundary condition the interface of the fastened plates is axially fixed and the initial tension is caused by applying a forced displacement of 10 μm to the bottom of the bolt. The external load acting on the point F shown in Fig.8 is increased in displacement form by an increment of 2 μm to a total of 20 μm . In the process of finding a succession of solution the iteration continues until the convergence is

achieved by releasing the negative load acting on nodes from the contact surfaces. The reduced stiffness matrix obtained in the last unit consists of the nodal components of the bottom of the bolt, the nut bearing surface and the regions which are designed to separate in the interface of the fastened plates. The relation between the contact surface and the external load is shown in Fig. 9, then the relation between the axial tension and the external load is shown in Fig.10. In these figures the co-ordinate axes represent the ratio of the radius of the contact surface to the radius of the bolt hole and the ratio of the load to the

Fig.11 Division of the flange coupling into isoparametric elements

Fig.10 Relation between bolt tension and external load

Fig.12 Deformation and contact surface of the flange coupling

initial tension. It is clear from the figures above mentioned that such a simple lineae relationship show in Fig.10 that has ever been used does not hold between the axial load and the external load. In the model of the threaded connection the axial tension does not show a rapid variation until the external load increases to about 40% of the initial tension of the bolt, but if the external load exceeds this value the axial tension shows a very rapid increase. Especially when the external load remains small the axial tension slightly decreases because the inner side of the nut bearing surface moves downward in the direction of the axis. The radius of the surface of contact rapidly reduces until the external load increases to about 50% of the initial tension of the bolt-nut; if the external load exceeds this range, the slope of the variation curve of the radius becomes gentle. It is found by the analytical results that the whole nut bearing surface always keeps in contact with the fastened plate.

3-2. General Case of Threaded Connection with External Loading

It is a rare case that a system is subjected to axi-symmetric loads as shown in Fig.8. Actually more various types of load act on the system. The fastened plates which have an axi-symmetric shape are so rare that the general case of the threaded connections requires a three dimensional analysis using the finite element method. In the previous report, the authors analysed a flange coupling tightened with bolts, but the present report also shows the analysis of the flange coupling of this type that is subjected to an

external load. The criterion for contact is not so easy in the analysis using high-order elements as one using linear elements, i.e the sign of the nodal forces acting on the interfaces in bodies does not help in judging the faces to be in contact. So this study uses the sign of the pressure between two bodies in contact. The analysis was performed by using the flange coupling with 4-bolt holes which carries an external load of 800 Kgf and was tightened with the bolts by applying a forced displacement of 10 μm to each bottom of the bolts. Figure 11 shows the model used for the analysis, that is one of 8 equally-divided parts and is generated with 20-points isoparametric elements. Figure 12 shows the contact surface between the fastened plates and then the deformation of the model that is obtained from the analysis. In this figure the contact surface is shown as a shaded part whereas the shape of the flange coupling after deformation is indicated by heavy black lines. Figure 13 shows the stress distribution in this coupling. It is found from these results that the contact surface between the flanges remains in the vicinity of the bolt hole.

Furthermore this paper shows another examples of the investigation of the bolted plates subjected to an external load. Use of symmetry is made to reduce the size of problem in the same way as with the flange coupling. The finite element model used for the analysis is shown in Fig.14. The bottom of the bolt axis is displaced by a distance of 10 μm and successive loads of 200 Kgf and 450 Kgf are applied to edge of the plate. When no loads act on the plate, the obtained value of the axial tension is 892 Kgf. The deformation of the

Fig.13 Equivalent stress distribution in the flange coupling

Fig.14 Division of the bolted plate into iso-parametric element

plate and the axial tension of the bolt caused by the load are shown in Figs. 15 and 16. When no loads act on the plate, the stress distribution and the contact surface are symmetric with respect to the bolt axis. When the load remains small, but the contact surface moves in a direction opposite to the point of application of the load, the variation in the axial tension is very small. If the load becomes very great, the contact surface moves farther from the bolt hole, and the axial tension considerably increases together with the external load.

4. Conclusion

Bolts and nuts are one of practically the most useful and indispensable machine elements, but they had many difficult points in analysis because of complexity of their shape and the elasticity of bodies in contact. Especially the threaded connections subjected to an external load have frequently given rise to problems in the design of machines. And it is evident that the traditional simple method of analysis is not applicable to some problems, in the threaded connection. This paper introduces a numerical method to analyse such cases as the threaded connections subjected to a transverse displacement and the bolted joints under an arbitrary type of loading for the typical examples of the threaded connections subjected to an external load. The analytical method used in this paper is applicable not

only to these problems but also to the complicated contact problems like such as pipe threaded or double nuts.

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Fig.15 Deformation state of the bolted plate

Fig.16 Contact surface and equivalent stress distribution