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Transition to Turbulence and Velocity Distribution in an Oscillating Pipe Flow*

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Experiments on transition to turbulence in an oscillating pipe flow were made by using a hot wire anemometer over ranges of Reynolds numbers $Re_{os} = |u_{m,os}|D/\nu$ ($|u_{m,os}|$ is the cross-sectional mean velocity amplitude, $D = 2R$ the pipe diameter, ν the kinematic viscosity) from 600 to 65000 and of dimensionless frequencies $\sqrt{\omega} = R\sqrt{\omega}/\nu$ (ω is the angular frequency) from 2.6 to 41. Critical Reynolds numbers from laminar to transitional and from transitional to turbulent flows agreed well with published experimental results. In the turbulent regime turbulent bursts follow by relaminarization in the same cycle. It is observed that the instantaneous velocity profile in every phase when a turbulence with higher frequency appears is represented by the well-known Blasius 1/7 power law, but in the laminar-like phase it does not follow the laminar theoretical solutions for a steady oscillating pipe flow.

1. Introduction

Transition to turbulence in an oscillating pipe flow has been investigated experimentally by Sergeev⁽¹⁾, Merkli-Thomann⁽²⁾ and Hino, et al.⁽³⁾ and theoretically by Yang-Yih.⁽⁴⁾ Yang-Yih found on the basis of the quasi-steady linear stability theory that the flow is stable against axially axisymmetric small amplitude disturbances for all frequencies and Reynolds numbers, while the experimental evidence shows the occurrence of turbulent bursts.

Sergeev⁽¹⁾ determined the critical Reynolds number from visual observation and power measurement of oscillator. Merkli-Thomann⁽²⁾ observed the transition using smoke visualization method and a hot wire anemometer and presented the velocity wave forms typical of transition. Hino, et al.⁽³⁾, referring to these results along with their own velocity measurements by a hot wire anemometer, classified the flows into four types with respect to Reynolds number as follows:

Region (I) : <laminar flow>

Region (II) : <small amplitude perturbations appear in the early stage of accelerating phase at the central portion of a pipe>

Region (III) : <small amplitude perturbations exist in the phase of higher velocity>

Region (IV) : <turbulent bursts occur in the decelerating phase>

Hino, et al.⁽³⁾ determined the critical Reynolds number from Region (I) to Region (II), from Region (I) to Region (IV), and from Region (III) to Region (IV). Here, the limit between Region (II) and Region (III) is not clear.

Although some experiments on transition to turbulence in an oscillating pipe flow are known as above, there is no agreed opinion on the critical Reynolds numbers because of restricted experimental range of each experiment and very little is known about the velocity distribution in the phase when a turbulence with higher frequency appears. Therefore, in this paper, whether or not the classification of the flows and the critical Reynolds numbers proposed by Hino, et al.⁽³⁾ are adequate will be first examined. Second the relationship between critical Reynolds numbers determined by Sergeev⁽¹⁾ and Merkli-Thomann⁽²⁾ and those of Hino, et al.⁽³⁾ will be investigated on the basis of the present experiments including the range of parameters for previous experiments. Then the existence of the following fifth region is shown.

Region (V) : <turbulent bursts occur in the accelerating phase as well as in the decelerating phase>

Finally it becomes evident that a turbulence appears in most of phases in a cycle except the early stage of accelerating and the latest stage of decelerating phase with an increasing Reynolds number and that the velocity distribution in the phase when a turbulence appears follows the well-known Blasius 1/7 power law.

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2. Nomenclature

Main symbols used in this paper are listed as follows:

- A : quantity defined by Eq.(8)
 D : pipe diameter
 $M_\nu(\sqrt{\omega})$, $\theta_\nu(\sqrt{\omega})$: modulus and phase of the first kind Bessel function of order ν , $J_\nu(i^{3/2}\sqrt{\omega})$
 Re : instantaneous Reynolds number
 $\equiv 2Ru_m/\nu$
 Re_{os} : Reynolds number based on the amplitude of cross-sectional mean velocity $\equiv 2R|u_{m,os,1}|/\nu$
 R : radius of pipe
 r : radial distance
 $r' = r/R$
 u : axial velocity component
 u_m : cross-sectional mean velocity
 ν : kinematic viscosity
 ω : angular frequency of oscillation
 ω' : dimensionless frequency $\equiv R^2\omega/\nu$
 Subscripts
 c : critical value
 m : cross-sectional mean value
 os : oscillating flow
 1 : fundamental wave in the finite Fourier expansion

3. Basic relations and previous experimental results

3.1 Basic relations

Assuming that the axial velocity component u and the cross-sectional mean velocity u_m in an oscillating pipe flow are represented as

$$u = |u_{os,1}| \cos(\omega t + \angle u_{os,1}) \quad (1)$$

$$u_m = |u_{m,os,1}| \cos(\omega t + \angle u_{m,os,1}) \quad (2)$$

an amplitude ratio $|u_{os,1}|/|u_{m,os,1}|$ and a phase difference $\angle u_{os,1} - \angle u_{m,os,1}$ become (5)

$$\frac{|u_{os,1}|}{|u_{m,os,1}|} = \frac{M_0(\sqrt{\omega'})}{M_2(\sqrt{\omega'})} \sqrt{1 + \left\{ \frac{M_0(r'\sqrt{\omega'})}{M_0(\sqrt{\omega'})} \right\}^2 - 2 \frac{M_0(r'\sqrt{\omega'})}{M_0(\sqrt{\omega'})} \cos(\theta_0(r'\sqrt{\omega'}) - \theta_0(\sqrt{\omega'}))} \quad (3)$$

$$\angle u_{os,1} - \angle u_{m,os,1} = \tan^{-1} \left[\frac{1 - \{M_0(r'\sqrt{\omega'})/M_0(\sqrt{\omega'})\} \cos(\theta_0(r'\sqrt{\omega'}) - \theta_0(\sqrt{\omega'}))}{\{M_0(r'\sqrt{\omega'})/M_0(\sqrt{\omega'})\} \sin(\theta_0(r'\sqrt{\omega'}) - \theta_0(\sqrt{\omega'}))} \right] - \theta_2(\sqrt{\omega'}) + \theta_0(\sqrt{\omega'}) + 3\pi/2 \quad (4)$$

Both of these expressions are termed the laminar theoretical solutions and will be used for comparison with experimental results.

3.2 Previous experimental results for critical Reynolds number

Hino, et al. defined the Reynolds number Re_{os} as

$$Re_{os} = 2R|u_{m,os,1}|/\nu \quad (5)$$

and found that the critical Reynolds number $Re_{os,c}$ indicating the limit between Region (II)·(III) and Region (IV) can be expressed as

$$Re_{os,c} = 780\sqrt{\omega'} \quad (2.3 \leq \sqrt{\omega'} \leq 8.8) \quad (6)$$

Additionally, critical Reynolds numbers between Region (I) and Region (II)·(III) and between Region (I) and Region (IV) are shown in Fig.3. Note that Region (II) and Region (III) are combined here and represented as Region (II)·(III) because the limit between them is not clear.

Sergeev⁽¹⁾ employed the same Reynolds number as Eq.(5) and reported $Re_{os,c}$ of the form

$$Re_{os,c} = 710\sqrt{\omega'} \quad (4.0 \leq \sqrt{\omega'} \leq 40) \quad (7)$$

However, it is not obvious which of the limits between four regions proposed by Hino, et al. corresponds to Eq.(7). Merkli-Thomann proposed the following non-dimensional value A as a characteristic parameter describing the transition to turbulence in an oscillating pipe flow and obtained the critical value of $A_c = 400$.

$$A = 2|u_{os,1}|_{r=0}/\sqrt{\nu\omega} \quad (8)$$

It is convenient to rewrite Eq.(8) as shown below for the comparison with experiments in the subsequent sections. Equation (8) can be transformed into

$$A = Re_{os}^*/\sqrt{\omega'}, \quad Re_{os}^* = 2R|u_{os,1}|_{r=0}/\nu \quad (9)$$

Whence the critical value of $A_c = 400$ yields

$$Re_{os,c}^* = 400\sqrt{\omega'} \quad (10)$$

Assuming that the flow is laminar when $Re_{os}^* < Re_{os,c}^*$ (this assumption will be found to be reasonable in Fig.5 shown later), Eq.(3) becomes

$$\frac{|u_{os,1}|_{r=0}}{|u_{m,os,1}|} = \frac{M_0(\sqrt{\omega'})}{M_2(\sqrt{\omega'})} \times \sqrt{1 + \left\{ \frac{1}{M_0(\sqrt{\omega'})} \right\}^2 - 2 \frac{\cos \theta_0(\sqrt{\omega'})}{M_0(\sqrt{\omega'})}} \quad (11)$$

The replacement of the right hand side of Eq.(11) with $\chi(\sqrt{\omega'})$ and the combination of Eqs.(5) and (9) give

$$Re_{os}^* = \chi(\sqrt{\omega'}) Re_{os} \quad (12)$$

Thus Eq.(10) can be rewritten as

$$Re_{os,c} = 400\sqrt{\omega'}/\chi(\sqrt{\omega'}) \quad (13)$$

As the relation $\chi(\sqrt{\omega'}) \approx 1$ is valid in the experimental range of Merkli-Thomann ($42 \leq \sqrt{\omega'} \leq 71$), Eq.(13) takes the form

$$Re_{os,c} = 400\sqrt{\omega'} \quad (42 \leq \sqrt{\omega'} \leq 71) \quad (14)$$

The coefficient in Eq.(14) is about a half of those of Eqs.(6) and (7). Hereafter, Eq.

(5) will be employed as a definition of Reynolds number.

4. Experimental

A schematic layout of the apparatus is shown in Fig.1.

Fig.1 Schematic layout of the experimental apparatus

An oscillating flow was generated by a piston-crank mechanism driven by means of a suitable combination of a geared motor and a gearbox. The variational range of oscillation frequencies is 0.0527 to 6.24 Hz and the Reynolds number Re_{os} can be attained up to about 10^5 . The crankweb is variable from 10 mm to 80 mm at equal intervals of 10 mm and the length of connecting rod 520 mm is larger than crankweb by 5.5 times. Accordingly, the amplitude of harmonics in piston speed is under 8 percent of that of fundamental wave. Whence the change in piston speed can be regarded as sinusoidal with sufficient accuracy.

Mechanical vibrations caused by the piston-crank mechanism were damped out not to transfer the test section by connecting the cylinder and the brass tube (test section) with a rubber sleeve. The observation of velocity wave forms revealed that the results were satisfactory. Velocity measurements were made at 16 or 17 radial points by using a hot wire anemometer. A support diameter of I-probe is 2 mm (DISA 55p11).

An oscillating flow changes its direction alternately and therefore if the disturbances produced by the probe itself return to the probe, there is a great possibility that they may be mistaken as a wall turbulence. However, the preliminary experiments showed that there can be no such a possibility because disturbances produced by the probe and the wall turbulence together disappear when the flow reverses its direction.

It is known that the inlet length of an oscillating laminar pipe flow becomes shorter than that of a steady laminar pipe flow.⁽⁶⁾ On the other hand, this is not clear for the turbulent flow. In order to ascertain this subject the velocity measurements were made at several cross-sections from section ① in Fig.1 to the inlet of the pipe at intervals of 50 cm.

The results show that the turbulent flows are fully developed at distance about 1 m away from the inlet in the present experimental range.

Velocity wave forms recorded on an oscillograph paper with 12 time marks a cycle change almost sinusoidally, so that the amplitude $|u_{os,i}|$ and the phase $\angle u_{os,i}$ were determined by approximating the measured values of u as a finite Fourier expansion. The measured values of cross-sectional mean velocity u_m obtained by graphycal integration method were also approximated as a finite Fourier expansion by using 12 values a cycle, and thus $|u_{m,os,i}|$ and $\angle u_{m,os,i}$ were determined.

5. Experimental results and discussion

5.1 Experiments on the transition to turbulence in a steady pipe flow

In order to check the adequacy of the present experimental method and to examine the ambient conditions surrounding the apparatus, preliminary experiments in a steady pipe flow were carried out. Details of experimental apparatus are given elsewhere.⁽⁷⁾ Figure 2 shows $u_m/u|_{r=0}$ against $Re = 2Ru_m/\nu$. The present measured values are in good agreement with those of Patel-Head⁽⁸⁾ represented by solid line and the critical Reynolds number Re_c is almost equal to the well-known values of 2300 for steady flow. Observation of velocity wave forms (not shown here) leads to the same critical Reynolds number as above.

5.2 Transition to turbulence in an oscillating pipe flow

Previous investigations are mainly concerned with velocity wave forms and visual observations. Here, at first, the classification of flows will be made on the basis of velocity wave forms. Second, the relation between critical Reynolds numbers determined from the observation of velocity distributions and those obtained above will be investigated.

5.2.1 Limits between laminar, transitional, and turbulent flows on the basis of velocity wave forms

Four regions with respect to Reynolds number classified by Hino, et al.⁽³⁾ are seen in the present experiments. After their classification, the present experimental results are arranged in Fig.3. Note that symbols O (laminar), ● (transitional), and ● (turbulent) correspond to Region (I), Region (II)·(III), and Region (IV)·(V) respectively and that Eqs.(6) and (13) are extended outside of their individual applicable range. Furthermore, Region (IV) and Region (V) are combined here and represented as Region (IV)·(V).

It is seen in Fig.3 that Eq.(13) derived from Merkli-Thomann's proposal is in accord with the limit between Region (I) and Region (II)·(III). Merkli-Thomann⁽²⁾ did not classify the velocity wave forms in detail like Hino, et al.⁽³⁾ Therefore, if the disturbances typical of Region (II)·(III) and

Fig.2 Relation between $u/u|_{r=0}$ and Reynolds number

Fig.3 Characteristic figure indicating the existing range of Region (I) through Region (V) (†: see Ref.(9))

a turbulence with higher frequency in Region (IV) are taken together as a single disturbance, the fact that the critical Reynolds number proposed by Merkli-Thomann agrees with that between Region (I) and Region (II)·(III) will not be strange. Sergeev⁽¹⁾, Hino, et al.,⁽³⁾ and the present authors made measurements for the case where the compressibility of fluid is negligible, whereas the compressibility may be controversial which is likely to have affected the results of Merkli-Thomann because one end of their pipe is closed. The fluid will not be regarded as incompressible either in the present apparatus when oscillation frequency is much higher than those adopted here. Effect of compressibility of fluid on the transition to turbulence must be a subject for future study.

Equation (6) proposed by Hino, et al.⁽³⁾ for the limit between Region (II)·(III) and Region (IV) is in fairly good agreement with Eq.(7) by Sergeev⁽¹⁾ and with the present experimental results. Furthermore, the agreement between the present results and those of Hino, et al.⁽³⁾ for the other limits is good. Consequently, referring to the present results together with previous investigations, each of the five regions is distinguished clearly in Fig.3 assigning

the symbols (I) through (V).

5.2.2 Limits between laminar, transitional, and turbulent flows on the basis of velocity distributions

Distributions of $|u_{os,i}|/|u_{m,os,i}|$ and $\angle u_{os,i} - \angle u_{m,os,i}$ are shown in Figs.4(a) through 4(l) for most of the runs. The remaining runs which are not shown here were carried out at $\sqrt{\omega} = 3.74, 6.06, 16.10$, and 23.01 and all of them belong to the laminar regime. In these figures open and shaded symbols denote amplitude ratio and phase difference respectively. The solid lines in each figure represent analytical values calculated from laminar theory, Eqs.(3) and (4). The broken line represents the $1/7$ power law. As Re_{os} increases for each $\sqrt{\omega}$, measured values of $|u_{os,i}|/|u_{m,os,i}|$ deviate from Eq.(3) and asymptotically follow the $1/7$ power law, while those of phase difference $\angle u_{os,i} - \angle u_{m,os,i}$ differ from Eq.(4) and become zero over a cross-section of the pipe. Negative value of phase difference in the vicinity of wall in some runs results from the fact that the velocity wave form of u cannot be approximated precisely as a finite Fourier expansion with fundamental wave because u near the wall changes drastically away from the sinusoidal wave when turbulent bursts first occur. This trend seems to depend on the value of $\sqrt{\omega}$, i.e., the degree of temporal inertial force.

Figure 5 shows $|u_{m,os,i}|/|u_{os,i}|_{r=0}$ and $\angle u_{m,os,i} - \angle u_{os,i}|_{r=0}$ against Re_{os} . Parallel lines indicated by a, b, c, etc. denote laminar analytical values for each $\sqrt{\omega}$. An increasing difference between measured and analytical values with Re_{os} is clearly seen in Fig.5. Here, the curve labelled "Merkli-Thomann" was determined by calculating the ordinate from Eq.(3) and the abscissa from Eq.(13). Since Eq.(13) represents the limit between laminar and transitional flows, Eq.(3) is valid at that limit. Accordingly, the curve labelled "Merkli-Thomann" denotes accurately the limit between Region (I) and Region (II)·(III). On the other hand, the curve labelled "Hino, et al." was determined from Eqs.(3) and (16). As Eq.(16) indicates the limit between transitional and turbulent flows and hence Eq.(3) does not hold, the curve labelled "Hino, et al." does not stand for the exact limit between Region (II)·(III) and Region (IV) in Fig.5.

A clear-cut dependence of Re_{os} on the phase difference is not seen, so that a discussion on amplitude ratio will be made. Measured values of $|u_{m,os,i}|/|u_{os,i}|_{r=0}$ for each $\sqrt{\omega}$ agree favourably with laminar analytical data when Re_{os} is relatively small but gradually deviate from laminar theory near the limit proposed by Merkli-Thomann⁽²⁾ with an increasing Re_{os} . Further increase in Re_{os} makes the measured values approach the curve of Patel-Head⁽⁸⁾ obtained for a steady pipe flow. Thus, it is obvious that both limits between Region (I) and Region (II)·(III) determined on the basis of velocity wave forms and velocity distributions agree well with each other.

Fig.4 Distributions of velocity amplitudes and phase differences

Fig.5 Variations in velocity amplitude and phase difference with Reynolds number

(a) Velocity wave forms

(b) Instantaneous velocity distribution
(— 1/7 power law, --- laminar theory)

Fig.6 Velocity wave forms and velocity distribution in an oscillating pipe flow

5.3 Velocity distributions at the phase when a turbulence appears

As shown above, the present critical Reynolds numbers indicating the limits between Region (I) through Region (V) are in good agreement with those proposed by previous investigators and the distribution of velocity amplitudes for each $\sqrt{\omega}$ approaches the 1/7 power law as Re_{os} increases.

Here, the velocity distribution at the phase when a turbulence appears will be investigated. As the representative examples, velocity wave forms and instantaneous velocity distributions together with variation in u_m for $\sqrt{\omega} = 7.93$ are shown in Figs. 6 and 7. The ordinate of each curve was displaced vertically by equal distances. The broken line in Fig.6(b) was calculated from Eqs.(3) and (4) with help of Eq.(2) which is the finite Fourier expansion with fundamental wave for measured values of u_m . The curves for the cases of $\omega t/(\pi/6) = 6$ through 11 are not shown because they are the same as those at $\omega t/(\pi/6) = 0$ through 5 respectively. Observation of velocity wave forms shows that the portion at which turbulent bursts occur in a cycle increases with Re_{os} and finally a turbulence appears at most of phases except the early stage of accelerating phase and the latest stage of decelerating phase. From Fig.6(b) it is clear that the measured values of u at the laminar-like phase do not follow the laminar theory indicated by the broken line. In Fig.7(b) the velocity distribution at each instant when turbulence appears follows the well-known 1/7 power law.

6. Conclusions

The results obtained in this investigation are summarized as follows:

(1) In addition to the four regions classified by Hino, et al.,⁽³⁾ a fifth type, Region (V), in which turbulent bursts occur in the accelerating phase as well as in the decelerating phase was observed.

(2) As for the limits between the five regions the following interpretations were obtained.

(i) The limit between Region (II) and Region (III) is not well defined. Accordingly, it is appropriate to combine them and represent as Region (II)·(III). Furthermore, the limit between Region (IV) and Region (V) is not clear either. Clarification of this limit must be done from a different standpoint.

(ii) Present results for the limit between Region (I) and Region (II)·(III) agree well with Eq.(13) derived from experiments of Merkli-Thomann⁽²⁾ and with that shown in Fig.3 by Hino, et al.⁽³⁾

(iii) The limit between Region (II)·(III) and Region (IV) presented here is in accord with those determined by Sergeev⁽¹⁾ and Hino, et al.⁽³⁾

(3) When Re_{os} is larger than the critical Reynolds number indicating the limit above which turbulent bursts occur, i.e., when the flow is in Region (IV) or Region (V),

(a) Velocity wave forms

(b) Instantaneous velocity distribution
(— 1/7 power law)

a turbulence appears at most of time except the early stage of accelerating phase and the latest stage of decelerating phase. At the phase when a turbulence appears the instantaneous velocity distribution follows the 1/7 power law which is valid for a steady pipe flow.

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Fig.7 Velocity wave forms and velocity distribution in an oscillating pipe flow