532.551:532.517.6

Paper No. 202-8

Flow Patterns and Frictional Losses in an Oscillating Pipe Flow*

By Munekazu OHMI,** Manabu IGUCHI,***
and Ikuo URAHATA****

Velocity distribution and pressure gradient in an oscillating pipe flow are measured over wide ranges of Reynolds numbers and dimensionless frequencies. Wall shear stress is determined by substituting experimental values of cross-sectional mean velocity and pressure gradient into an unsteady momentum integral equation. From these experimental quantities frictional losses and four characteristic parameters describing the flow pattern are calculated. They are well represented by the known laminar theory in a laminar regime and by the turbulent quasi-steady relations in a turbulent regime. Here, turbulent quasi-steady state is defined as the state in which relationship between cross-sectional mean velocity and wall shear stress for steady turbulent pipe flow holds at any moment in a cycle.

1. Introduction

In a previous paper⁽¹⁾ results of velocity measurements made by using a hot wire anemometer in an oscillating pipe flow demonstrated the presence of three types of flows, i.e., laminar, transitional, and turbulent flows and the limits between these three regimes were determined referring also to the previous experiments.⁽²⁾⁻⁽⁴⁾ Additionally, it was found that the instantaneous velocity distribution in the laminar regime agrees well with the laminar theoretical solution, while in the turbulent regime where turbulent bursts followed by relaminarization in the same cycle occur it obeys the well-known $1/7$ power law in the phase where turbulence appears and the portion of turbulent phase increases with Reynolds number Re_{os} .

As seen above it can be considered that the characteristic parameters describing the flow pattern and the frictional losses in the laminar regime are well represented by their individual laminar theoretical solutions. Therefore, the characteristic parameters and frictional losses in transitional and turbulent regimes will be mainly investigated here.

The quantities required for the calculation of characteristic parameters and frictional losses are pressure gradient, cross-sectional mean velocity, and wall shear stress. The former two can be measured by a semiconductor type difference

pressure transducer and a hot wire anemometer respectively, whereas the latter one must be evaluated by substituting these measured values into an unsteady momentum integral equation. Experiments revealed that four characteristic parameters and two kinds of friction factors tend to agree favourably with their individual turbulent quasi-steady relations with an increasing Re_{os} in the turbulent regime. Measured values of time average friction factor made it possible to clearly distinguish the region where turbulent quasi-steady state approximately holds. Furthermore, characteristics of eddy viscosity in the turbulent phase were described.

2. Nomenclature

Main symbols used in this paper are listed as follows:

D : pipe diameter
 L : length of test section
 $M_\nu(\sqrt{\omega})$, $\theta_\nu(\sqrt{\omega})$: modulus and phase of the first kind Bessel function of order ν , $J_\nu(i^{3/2}\sqrt{\omega})$
 p : pressure
 Δp : pressure drop
 R : radius of pipe
 Re : instantaneous Reynolds number
 $\equiv u_m D / \nu$
 Re_{os} : Reynolds number based on the amplitude of cross-sectional mean velocity $\equiv |u_{m,os}| D / \nu$
 r : radial distance
 T : period of oscillation
 u : axial velocity component
 u_m : cross-sectional mean velocity
 u^* : friction velocity
 $u^+ = u / u^*$
 x : axial distance
 $y = R - r$
 $y^+ = y u^* / \nu$
 ϵ : eddy viscosity
 $\lambda_{q,t}$: turbulent quasi-steady friction factor

* Received 26th March, 1980.

** Professor, Faculty of Engineering, Osaka University.

*** Assistant, Faculty of Engineering, Osaka University, Yamadaoka 2-1, Suita, Osaka.

**** Graduate Student, Faculty of Engineering, Osaka University.

$\lambda_{u,u}$: time average friction factor
 μ : viscosity
 ν : kinematic viscosity
 ν_{Σ} : modified eddy viscosity
 ρ_0 : density
 τ : shear stress
 τ_w : wall shear stress
 $\Phi_{t,l}, \Phi_{\Sigma,l}, \alpha_{t,l}, \alpha_{\Sigma,l}, \Phi'_{t,l}, \Phi'_{\Sigma,l}, \alpha'_{t,l}, \alpha'_{\Sigma,l}$
 : characteristic parameters describing the flow pattern
 ω : angular frequency of oscillation
 ω' : dimensionless frequency $= R^2\omega/\nu$
 Subscripts and others
 os : oscillating flow
 1 : fundamental wave in the finite Fourier expansion
 $-$: short-time-averaged value which permits the relatively low frequencies to remain for the sinusoidal waves under consideration

3. Basic relations

3.1 Equation of motion in an oscillating flow and modified eddy viscosity ν_{Σ}

An approximate equation of motion of an incompressible fluid in an oscillating turbulent pipe flow can be written as follows: (5)

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \bar{u}}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} (r \bar{u}'v') \dots (1)$$

where " $-$ " denotes the short-time-averaged value. As usually done for a steady turbulent pipe flow, it is convenient to express the Reynolds shear stress term $\bar{u}'v'$ in terms of eddy viscosity ϵ .

$$\bar{u}'v' = -\epsilon \frac{\partial \bar{u}}{\partial r} \dots (2)$$

Combination of Eqs.(1) and (2) gives

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \nu_{\Sigma} \frac{\partial \bar{u}}{\partial r} \right) \dots (3)$$

where

$$\nu_{\Sigma} = \nu + \epsilon \dots (4)$$

is termed the modified eddy viscosity and the short-time-averaged shear stress $\bar{\tau}$ is in the following form.

$$\bar{\tau} = -\rho_0 \nu_{\Sigma} \frac{\partial \bar{u}}{\partial r} \dots (5)$$

Here, note that the abbreviation of symbol " $-$ " and substitution of 0 for ϵ in Eqs.(3) and (5) yield the equations for laminar flow.

After integration of Eq.(3) from 0 to r , shear stress $\bar{\tau}$ becomes

$$\bar{\tau} = \frac{\rho_0 r}{2} \left\{ -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \frac{\partial}{\partial t} \left(\int_0^r 2\pi r \bar{u} dr / \pi r^2 \right) \right\} \dots (6)$$

Whence the modified eddy viscosity ν_{Σ} can be evaluated readily from Eqs.(5) and (6), provided that $\bar{u}$ and $\partial \bar{p} / \partial x$ are measured.

Strictly speaking, pressure p , axial velocity component u , and shear stress τ must be expressed as follows:

$$p = \bar{p} + p', \quad u = \bar{u} + u', \quad \tau = \bar{\tau} + \tau' \dots (7)$$

where " $\prime$ " denotes the fluctuating component due to turbulent motion. Hereafter the symbol " $-$ " is omitted and, for example, $\bar{p}$ is simply represented by p so far as confusion does not occur because an oscillating pipe flow accompanied with turbulent bursts is not always turbulent over one cycle. When the flow is laminar, relations, such as $p = \bar{p}$, etc. are valid as a matter of course.

3.2 Characteristic parameters describing the flow pattern

A relationship between pressure gradient $\Delta p / l$, cross-sectional mean velocity u_m , and wall shear stress τ_w can be represented by the following unsteady momentum integral equation

$$\rho_0 \frac{du_m}{dt} + \frac{4}{D} \tau_w = \frac{\Delta p}{l} \dots (8)$$

This equation can be derived by integrating Eq.(3) first over a cross-section of a pipe and subsequently from $x = 0$ to l and it is valid for both laminar and turbulent flows. Since measured values of cross-sectional mean velocity u_m can be well approximated by the finite Fourier expansion having a fundamental wave (sinusoidal wave) in the present experimental apparatus, let's express u_m as follows:

$$u_m = |u_{m,os,1}| \cos(\omega t + \angle u_{m,os,1}) \dots (9)$$

Equation (8) is linear for laminar flow. Accordingly, $\Delta p / l$ and τ_w can also be represented by the following sinusoidal waves, respectively.

$$\frac{\Delta p}{l} = \frac{|\Delta p_{os,1}|}{l} \cos(\omega t + \angle \frac{\Delta p_{os,1}}{l}) \dots (10)$$

$$\tau_w = |\tau_{w,os,1}| \cos(\omega t + \angle \tau_{w,os,1}) \dots (11)$$

The present authors proposed previously⁽⁶⁾ four characteristic parameters $\Phi_{t,1}$, $\Phi_{\Sigma,1}$, $\alpha_{t,1}$, and $\alpha_{\Sigma,1}$ in order to describe the flow patterns, i.e., the types of flows specified by the quantitative relationship between three terms, such as the inertia term $\rho_0 du_m / dt$, the viscous term $4\tau_w / D$, and the pressure gradient term $\Delta p / l$ in Eq.(8). Here, flow patterns can be classified into three categories. These are the quasi-steady region in which the latter two terms are almost balanced, the intermediate region in which three terms are balanced with each other, and the inertia dominant region in which the former two terms are almost balanced. Characteristic parameters are represented for laminar flow as follows:⁽⁶⁾

$$\left. \begin{aligned} \Phi_{t,1} &\equiv \frac{\rho_0 \omega |u_{m,os,1}|}{|\Delta p_{os,1}| / l} = \frac{M_2(\sqrt{\omega'})}{M_0(\sqrt{\omega'})} \\ \Phi_{\Sigma,1} &\equiv \frac{4|\tau_{w,os,1}| / D}{|\Delta p_{os,1}| / l} = \frac{2}{\sqrt{\omega'}} \frac{M_1(\sqrt{\omega'})}{M_0(\sqrt{\omega'})} \\ \alpha_{t,1} &\equiv \angle \frac{\Delta p_{os,1}}{l} - \angle u_{m,os,1} \\ &= \theta_0(\sqrt{\omega'}) - \theta_2(\sqrt{\omega'}) + \frac{3\pi}{2} \\ \alpha_{\Sigma,1} &\equiv \angle \frac{\Delta p_{os,1}}{l} - \angle \tau_{w,os,1} \\ &= \theta_0(\sqrt{\omega'}) - \theta_1(\sqrt{\omega'}) + \frac{3\pi}{4} \end{aligned} \right\} \dots (12)$$

That is, characteristic parameters are functions of ω' alone and if any one of three quantities $\Delta p/l$, u_m , and τ_w is given, the remainder can be readily calculated from Eq.(12).

On the other hand, Eq.(8) is generally nonlinear in turbulent flow. Velocity measurements reported in the previous paper⁽¹⁾ suggest that the wall shear stress τ_w can be well approximated by the following turbulent quasi-steady relation at the phase where turbulence appears.

$$\left. \begin{aligned} \tau_w &= \lambda_{q,t} \rho_0 |u_m| u_m / 8 \\ \lambda_{q,t} &= 0.3164 / |Re|^{1/4} \\ Re &= u_m D / \nu \end{aligned} \right\} \dots\dots\dots (13)$$

Equation (13) implies that Eqs.(10) and (11) are not valid for $\Delta p/l$ and τ_w respectively, even if u_m is in the form of Eq.(9). Therefore, previously proposed four parameters cannot provide a practical advantage for an oscillating turbulent flow. Consequently, it is necessary here to newly propose alternative characteristic parameters of the following forms on the basis of the maxima of $\Delta p/l$, u_m , and τ_w .

$$\left. \begin{aligned} \phi_{t,1}' &\equiv \frac{\rho_0 \omega (u_m)_{\max}}{(\Delta p/l)_{\max}} \\ \phi_{s,1}' &\equiv \frac{(4/D)(\tau_w)_{\max}}{(\Delta p/l)_{\max}} \\ \alpha_{t,1}' &\equiv \angle (\Delta p/l)_{\max} - \angle (u_m)_{\max} \\ \alpha_{s,1}' &\equiv \angle (\Delta p/l)_{\max} - \angle (\tau_w)_{\max} \end{aligned} \right\} \dots\dots\dots (14)$$

Note that Eq.(14) is equivalent to Eq.(12) for a laminar flow.

Consider the case when u_m and τ_w are represented by Eqs.(9) and (13), respectively. By putting $\angle u_{m,os,1} = 0$ (i.e., u_m has its maximum value at $\omega t = 0$) and by defining the phase at which $\Delta p/l$ is maximum by ϕ , then ϕ must satisfy the following relation.

$$\left. \begin{aligned} \frac{\cos^{1/4} \phi}{\sin \phi} &= -\frac{Re_{os}^{3/4}}{14.45 \omega'} \\ Re_{os} &= |u_{m,os,1}| D / \nu \end{aligned} \right\} \dots\dots\dots (15)$$

This relation can be obtained by substitution of Eqs.(9) and (13) into Eq.(8). Whence four characteristic parameters become

$$\alpha_{t,1}' = \alpha_{s,1}' = -\phi \dots\dots\dots (16)$$

$$\phi_{t,1}' = 1 / \left\{ -\sin \phi + \frac{Re_{os}^{3/4}}{25.28 \omega'} \cos^{7/4} \phi \right\} \dots\dots\dots (17)$$

$$\phi_{s,1}' = \frac{Re_{os}^{3/4}}{25.28 \omega'} \phi_{t,1}' \dots\dots\dots (18)$$

These expressions demonstrate that the flow pattern in an oscillating turbulent pipe flow is a function of ω' and Re_{os} .

3.3 Friction factors

Instantaneous friction factor $\lambda_u(t)$ and time average friction factor $\lambda_{u,ta}$ are

defined by⁽⁷⁾

$$\tau_w u_m = \lambda_u(t) \rho_0 |u_m| u_m^2 / 8 \dots\dots\dots (19)$$

$$\int_0^T \tau_w u_m dt / T = \lambda_{u,ta} \rho_0 |u_{m,os,1}|^3 / 8 \dots\dots\dots (20)$$

respectively.

First consider the laminar flow. When u_m is given by Eq.(9), τ_w becomes⁽⁶⁾

$$\tau_w = \frac{4\mu}{R} \left\{ \eta_1(\sqrt{\omega'}) u_m + \frac{\xi_1(\sqrt{\omega'})}{\omega} \frac{du_m}{dt} \right\} \dots\dots\dots (21)$$

$$\left. \begin{aligned} \eta_1(\sqrt{\omega'}) &= \frac{\sqrt{\omega'}}{4} \frac{M_1(\sqrt{\omega'})}{M_2(\sqrt{\omega'})} \cos \{ \theta_1(\sqrt{\omega'}) - \theta_2(\sqrt{\omega'}) + 3\pi/4 \} \\ \xi_1(\sqrt{\omega'}) &= \frac{\sqrt{\omega'}}{4} \frac{M_1(\sqrt{\omega'})}{M_2(\sqrt{\omega'})} \sin \{ \theta_1(\sqrt{\omega'}) - \theta_2(\sqrt{\omega'}) + 3\pi/4 \} \end{aligned} \right\} \dots\dots\dots (22)$$

Substitution of Eqs.(9) and (21) into Eq.(19) and Eq.(20) gives $\lambda_u(t)$ and $\lambda_{u,ta}$, respectively as follows:

$$\lambda_u(t) = \frac{64}{|Re|} \left\{ \eta_1(\sqrt{\omega'}) + \frac{\xi_1(\sqrt{\omega'})}{\omega u_m} \frac{du_m}{dt} \right\} \dots\dots\dots (23)$$

$$\lambda_{u,ta} = 32 \eta_1(\sqrt{\omega'}) / Re_{os} \dots\dots\dots (24)$$

Next for a turbulent flow, assuming that the turbulent quasi-steady state holds over one cycle, comparison of Eq.(13) with Eq.(19) yields

$$\lambda_u(t) = 0.3164 / |Re|^{1/4} \dots\dots\dots (25)$$

and substitution of Eqs.(9) and (13) into Eq.(20) leads to

$$\begin{aligned} \lambda_{u,ta} &= \frac{1}{\sqrt{\pi}} \frac{\Gamma(15/8)}{\Gamma(19/8)} \frac{0.3164}{Re_{os}^{1/4}} \\ &= 0.1392 / Re_{os}^{1/4} \dots\dots\dots (26) \end{aligned}$$

where $\Gamma(x)$ is a gamma function.

4. Experimental

A schematic diagram of the experimental arrangement is shown in Fig.1. It consists of a piston-cylinder and a test section.

Fig.1 Schematic of the flow system

Measurement and arrangement of an axial velocity u are the same as those reported in the previous paper.⁽¹⁾ Pressure drop measurements were made with two semiconductor type pressure transducers connected to a dynamic strain meter. One transducer is fixed to one static pressure tap and the other is set in order at 0.5 m intervals. Then the pressure gradient was determined by means

of the least squares method. Since the pressure gradient at each phase shows a fine linear distribution along the axis of a pipe exclusive of the vicinities of two inlets, it can be concluded that the fluid is regarded as incompressible in the present experimental range. Measured values of $\Delta p/l$ are approximated by the finite Fourier expansion having harmonics up to six by using 12 values which are obtained at the phases with equal interval in a cycle.

Substitution of the measured values of $\Delta p/l$ and acceleration du_m/dt which is obtained by differentiating the finite Fourier expansion of u_m having harmonics up to six into Eq.(8) gives wall shear stress τ_w at every phase. Characteristic parameters are calculated from Eq.(14) by reading maxima of measured values of $\Delta p/l$, u_m and τ_w plotted in figures. Instantaneous friction factor $\lambda_u(t)$ can be evaluated from Eq.(19) and measured values of u_m and τ_w . Substitution of the finite Fourier expansions of u_m and τ_w having harmonics up to six which are determined by using their individual 12 measured values for a cycle into Eq.(20) gives the measured values of $\lambda_{u,ta}$. Eddy viscosity is obtained in the manner as shown in section 3.1. Here, velocity gradient $\partial \bar{u}/\partial r$ in Eq.(5) was determined by graphical differentiation method and integration in the parenthesis in Eq.(6) was performed numerically by means of the Simpson formula and the Newton-Cotes formula. The second term on the right hand side of Eq.(6) was evaluated by differentiating the finite Fourier expansion having harmonics up to six which was determined by using 12 values of the quantity in parenthesis with respect to time.

5. Experimental results and discussion

Table 1 shows an outline of experimental results. Symbols ○ and ● denote transitional and turbulent flows defined in the previous paper,⁽¹⁾ respectively. In the subsequent sections the validity of turbulent quasi-steady state for sufficiently large Reynolds number will be investigated from various view points.

5.1 Examples of measured values of $\Delta p/l$, u_m , and τ_w

Experimental results of three runs with $\sqrt{\omega} = 11.24$ are shown in Figs.2 through 4. Figures 3 and 4 are the cases that turbulence appears in the most of phases except the early stage of accelerating phase and the latest stage of decelerating phase of velocity wave forms (not shown). Although the measured values of u_m can be well approximated by the finite Fourier expansion having a fundamental wave, both of those of $\Delta p/l$ and τ_w deviate from sinusoidal wave with an increasing Reynolds number Re_{os} . The evidence that the measured values of τ_w agree favourably with turbulent quasi-steady values calculated from Eq.(13) and measured values of u_m suggests the validity of turbulent quasi-steady relation in the most of phases of a cycle for sufficient large Reynolds number Re_{os} . The quasi-

steady state is valid for other runs with various ω' whenever Re_{os} is much larger than the critical Reynolds number, $Re_{os,c} \approx 800\sqrt{\omega}$. This fact supports the experimental evidence⁽¹⁾ that the velocity distribution follows the 1/7 power law in the most of phases except the early stage of accelerating and the latest stage of decelerating phase of a cycle.

Emsmann⁽⁸⁾ carried out velocity measurements in an oscillating pipe flow in the ranges of $\sqrt{\omega'} = 50 \sim 116$ at $Re_{os} = 1.8 \times 10^4$ and of $\sqrt{\omega'} = 72 \sim 116$ at $Re_{os} = 3.6 \times 10^4$ and found that wall shear stress τ_w determined experimentally from a velocity gradient at wall is almost equal to the laminar theoretical value. Emsmann⁽⁸⁾ considers that the flow is turbulent, though it can be judged from a characteristic figure⁽¹⁾ showing three flow regimes that his experimental range falls into the laminar or transitional regimes. Therefore, the good agreement of the wall shear stress obtained by Emsmann with laminar theoretical values is not strange.

5.2 Modified eddy viscosity ν_Σ and others

Figure 5(a) through 5(f) are chosen in order to clarify the flow characteristics in the phase when turbulence appears (Run 9) and they show velocity wave forms, velocity distribution (comparison of measured values with 1/7 power law distribution), variations in $\Delta p/l$, u_m , and τ_w over one cycle, shear stress distribution, semilogarithmic plot of measured values of axial velocity component, and modified eddy viscosity distribution respectively. Here, the absolute value of friction velocity u^* is calculated from $\sqrt{|\tau_w|/\rho_0}$ and its sign is the same as that of wall shear stress τ_w . It should be noted that all data points in

Table 1 An outline of experimental results

Run	D mm	f Hz	$\sqrt{\omega'}$	Re_{os}	$\lambda_{u,ta}$	記号
1	25.0	0.300	4.29	8 550	0.0147	●
2	25.0	0.300	4.29	8 920	0.0118	●
3	25.0	0.300	4.29	13 400	0.0130	●
4	25.0	1.026	7.93	6 060	0.0093	○
5	25.0	1.026	7.93	9 650	0.0106	●
6	25.0	1.026	7.93	15 300	0.0108	●
7	25.0	1.026	7.93	22 700	0.0115	●
8	25.0	1.026	7.93	32 100	0.0111	●
9	25.0	1.026	7.93	43 500	0.0105	●
10	25.0	2.083	11.24	10 700	0.0069	○
11	25.0	2.083	11.24	11 900	0.0102	○
12	25.0	2.083	11.24	26 200	0.0101	●
13	25.0	2.083	11.24	41 900	0.0094	●
14	25.0	2.083	11.24	60 800	0.0089	●
15	50.4	2.083	23.37	26 300	0.0084	●
16	50.4	2.083	23.37	32 400	0.0069	●
17	50.4	2.083	23.37	41 400	0.0090	●
18	50.4	2.083	23.37	44 100	0.0093	●
19	50.4	2.083	23.37	51 400	0.0083	●
20	50.4	1.026	16.36	21 400	0.0114	●
21	50.4	1.026	16.36	24 700	0.0093	●
22	25.0	1.026	8.09	26 800	0.0127	●
23	25.0	1.026	8.09	38 200	0.0104	●
24	25.0	2.083	11.41	11 200	0.0072	●
25	25.0	2.083	11.41	24 700	0.0099	●
26	25.0	2.083	11.41	39 000	0.0100	●
27	38.0	2.083	17.69	44 000	0.0083	●
28	38.0	2.083	17.69	49 200	0.0093	●
29	38.0	2.083	17.69	62 300	0.0087	●
30	38.0	2.083	17.69	67 700	0.0083	●

Fig.2 Measured values of $\Delta p/l$, u_m , and τ_w over one cycle (●: measured, Δ: turbulent quasi-steady, □: laminar theoretical)

Fig.3 Measured values of $\Delta p/l$, u_m , and τ_w over one cycle (●: measured, Δ: turbulent quasi-steady)

Fig.4 Measured values of $\Delta p/l$, u_m , and τ_w over one cycle (●: measured, Δ: turbulent quasi-steady)

(a) Velocity wave forms

(c) Variations in $\Delta p/l$, u_m , and τ_w over one cycle

(e) Semilogarithmic plot of measured axial velocity component

(b) Comparison of measured values with 1/7 power distribution

(d) Shear stress distribution

(f) Modified eddy viscosity distribution

Fig.5 An example of an oscillating turbulent pipe flow accompanied with turbulent bursts (Run 9)

in Fig.5(f) are obtained in the turbulent phase. The distribution of v_z seems to depend somewhat on ω' in the moderate Reynolds number range (not shown) and the increase in Re_{os} ($Re_{os} \gg Re_{os,c} \approx 800\sqrt{\omega'}$) gives rise to the distribution of v_z known for steady pipe flow.

Experimental results demonstrate the realization of turbulent quasi-steady state in the phase of $\omega t/(\pi/6) = 2 \sim 5$ and $8 \sim 11$ in which turbulence appears as seen from Fig.5(a) because the velocity distribution agrees favourably with the $1/7$ power law and the universal law ($u^+ = 5.75 \log y^+ + 5.5$) and the shear stress and modified eddy viscosities agree with their individual distributions available for steady turbulent pipe flow.

5.3 Flow patterns

Characteristic parameters describing the flow pattern are shown against Re_{os} in Figs.6 through 9. It should be kept in mind that the relatively large scattering of data points in Figs.8 and 9 is mainly due to reading uncertainty of phase difference from figures showing measured values of $\Delta p/l$, u_m , and τ_w over one cycle. Measured values of every characteristic parameter depart from laminar analytical value with an increasing Re_{os} and finally approach asymptotically the turbulent quasi-steady curve.

In an oscillating pipe flow the inertia term $\rho_0 du_m/dt$, viscous term $4\tau_w/D$, and pressure gradient term $\Delta p/l$ balance with one another as is evident from Eq.(8). At sufficiently large Reynolds number the proportion of inertia term in unsteady momentum integral equation becomes vanishingly small and hence the state that the viscous and pressure gradient terms are balanced, i.e., quasi-steady state holds.

5.4 Friction factors

5.4.1 Instantaneous friction factor $\lambda_u(t)$

As a representative example, measured values of instantaneous friction factor for Run 9 are shown in Fig.10. The solid line denotes the turbulent quasi-steady friction factor

$$\lambda_{g,t} = 0.3164/|Re|^{1/4}$$

Comparison of Fig.10 with Fig.5 reveals that the measured values of $\lambda_u(t)$ agree well with $\lambda_{g,t}$ in the phase of $\omega t/(\pi/6) = 2 \sim 5$ and $8 \sim 11$ in which turbulence appears. This feature can be also seen in any other runs accompanied with the occurrence of turbulent bursts.

5.4.2 Time average friction factor $\lambda_{u,ta}$

Figure 11 shows the comparison of measured values of $\lambda_{u,ta}$ with laminar theoretical values and turbulent quasi-steady ones. With an increasing Re_{os} , measured values of $\lambda_{u,ta}$ approach asymptotically Eq.(26). On the other hand, the present $\lambda_{u,ta}$ data cannot show agreement with laminar theoretical data in the small Re_{os} range, though it can be considered that measured values of $\lambda_{u,ta}$ will agree with them

Fig.6 Relation between $\Phi_{t,1}$ and Re_{os}

Fig.7 Relation between $\Phi_{z,1}$ and Re_{os}

Fig.8 Relation between $\alpha_{t,1}$ and Re_{os}

Fig.9 Relation between $\alpha_{t,1}$ and Re_{os}

on the basis of the agreement of the measured velocity distribution with the laminar theoretical one shown in the previous paper.⁽¹⁾ In the small Re_{os} range the same conclusions as for $\lambda_{u,ta}$ can be drawn for $\lambda_u(t)$ and characteristic parameters.

5.5 Range where turbulent quasi-steady state is valid

The range where turbulent quasi-steady state is valid will be determined based on the comparison of measured with turbulent quasi-steady values of $\lambda_{u,ta}$. By reading the value of Re_{os} at which measured value of $\lambda_{u,ta}$ is almost equal to the turbulent quasi-steady value, $0.1392/Re_{os}^{1/4}$, the values of this limit are shown in Fig.12 for various ω' . This figure represents a realization of turbulent quasi-steady state in the range of $Re_{os} \gg 2800\sqrt{\omega'}$ when $4 \lesssim \sqrt{\omega'} \lesssim 24$.

Hayama, et al.⁽¹⁰⁾ considered that the time-averaged component of a pulsating turbulent pipe flow is vanishingly small compared with the oscillating component near resonance frequency and hence the former is neglected. That is, they⁽¹⁰⁾ regarded the flow as an oscillating turbulent pipe flow and adopted the resistance formula available for steady turbulent pipe flow. The calculated results of resonance pressure amplitude agreed well with their experimental results. The present experiments support the assumption for resistance (wall shear stress) made by Hayama, et al.⁽¹⁰⁾

6. Conclusions

In order to clarify the flow pattern and frictional losses in an oscillating pipe flow, velocity and pressure gradient measurements were made and the results are summarized as follows:

(1) Characteristic parameters and two kinds of friction factors can be well represented by their individual laminar theoretical solutions in a laminar regime.

(2) In a turbulent regime, the turbulent quasi-steady state is valid in the phase where turbulent bursts occur. The portion of turbulent phase increases with Re_{os} and finally turbulence appears in the most of phases except the early stage of accelerating phase and the latest stage of decelerating phase. Accordingly, if a Reynolds number Re_{os} is sufficiently larger than its critical value, $Re_{os,c} \approx 800\sqrt{\omega'}$, in the present experimental range, characteristic parameters and friction factors can be calculated from their individual turbulent quasi-steady relations with sufficient accuracy. For example, time average friction factor is well approximated by its quasi-steady value, $0.1392/Re_{os}^{1/4}$.

(3) The range where the turbulent quasi-steady state is approximately valid over one cycle can be expressed as follows:

$$Re_{os} \geq 2800\sqrt{\omega'} \quad (4 \lesssim \sqrt{\omega'} \lesssim 24)$$

● : experimental

Fig.10 Instantaneous friction factor (Run 9)

Fig.11 Time average friction factor $\lambda_{u,ta}$

Fig.12 Range where turbulent quasi-steady state is valid (hatching denotes the transitional regime)

The authors would like to express their thanks to Prof. Ashino of Fukui University for his kind suggestions. This work was supported by the Japanese Ministry of Education's scientific research grant.

References

- (1) Ohmi, M., et al., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol.47, No. 419, B (1981), p. 1214.
- (2) Sergeev, S. I., *Fluid Dyn.*, Vol. 1 (1966), p. 21.
- (3) Merkli, P. and Thomann, H., *J. Fluid Mech.*, Vol.68, Part 3 (1975), p. 567.
- (4) Hino, M., et al., *J. Fluid Mech.*, Vol. 75, Part 2 (1976), p. 193.
- (5) Ohmi, M. and Usui, T., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol.41, No.347 (1975), p. 2030.
- (6) Ohmi, M., et al., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol.46, No. 405, B (1980), p. 846 : Ohmi, M., et al., *Bull. JSME*, Vol.24, No.187(1981), p. 75.
- (7) Ohmi, M. and Iguchi, M., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol. 46, No. 404, B (1980), p. 628 : Ohmi, M. and Iguchi, M., *Bull. JSME*, Vol. 23, No. 186 (1980), p. 2021.
- (8) Emsmann, S., *Mitt. Inst. f. Wasserbau und Wasserwirtschaft, Techn. Universität Berlin*, Nr. 79, Berlin, (1973).
- (9) Reichart, H., *Z. Angew. Math. Mech.*, Vol. 31, No. 7 (1951), p. 208.
- (10) Hayama, S., et al., *Trans. Jpn. Soc. Mech. Eng.* (in Japanese), Vol.42, No. 364 (1976), p. 3825 : Hayama, S., et al., *Bull. JSME*, Vol. 20, No. 146 (1977), p. 955.