532.542:532.517.6

Paper No. 202-11

Unsteady Flow in a Porous, Elastic, Circular Tube

Part 3. The Tube Contracting or Expanding
in Axial and Radial Directions*

By Morimatsu OHKI**

Linear theory for unsteady flow of a viscous, incompressible fluid through a semi-infinite circular tube with porous, elastic wall whose radius together with its length contracts or expands elastically, is considered together with the effects of injection into the wall.

Laminar, similar solutions with respect to space and time are obtained. Velocities, frictional drag, power and work of fluid are analyzed.

There are three critical conditions of injection into the wall. The conditions of injection together with the conditions of motion of the tube either contracting or expanding, diversify the induced flow fields.

1. Introduction

This paper is concerned with the basic problem of pumping action of an elastic vessel with valves, such as heart, arterial and venous capillaries. In general, the length together with the cross-section of the vessel contracts or expands elastically with time, and the fluid can pass through the tissues of vessels under specific conditions of injection.

To study the above flow problem, the following three types of elastic motions of an occluded circular cylindrical tube with porous walls are introduced on the assumption that the working fluid is incompressible and viscous:

- (a) The length of a tube varies with time but its cross-section remains constant.
- (b) The length of a tube remains constant but its cross-section varies with time.
- (c) The length together with the cross-section of a tube varies with time.

Linear theories are developed in the cases of types (a)⁽¹⁾⁽²⁾ and (b)⁽³⁾, and laminar similar solutions with respect to space and time are obtained. The characteristics of velocity, pressure, frictional drag, power and work of fluid are discussed.

In this paper, unsteady flow in a porous, elastic, circular tube of type (c) is analyzed on the assumption that the convective accelerations are negligible in momentum equations.

Nomenclature

Dimensional quantities are indicated with an asterisk, and corresponding dimensionless ones, without an asterisk. Conventional symbols in Part 1⁽¹⁾ are used in the present paper. Additional symbols are denoted, as follows:

A_0 : constant dependent on k
 A : constant dependent on k and S
 $F(t)$: a function of t denoting the radial wall displacement

$$f(t) = -\frac{1}{F} \frac{dF}{dt}$$

$G(t)$: a function of t denoting the axial wall displacement (assume $G=F$, in the present paper)

$$g(t) = -\frac{1}{G} \frac{dG}{dt}$$

H : function of z together with k and S
 K_0 : constant dependent on k
 K : constant dependent on k and S

$k = \frac{k^* L^{*2}}{\nu^*}$: Dimensionless parameter which indicates the elastic motion of the tube either in contraction ($k < 0$) or in expansion ($k > 0$)

k^* : constant which gives the variation of quantity per unit time

L^* : reference length, the radius of a circular tube at $t^* = t_0^* = 0$

S : constant, injection condition on the wall

$S^*(r^*, \theta^*, z^*)$, $S(R, R\theta, Z)$, $r = R/F$, $z = Z/G$
 : cylindrical co-ordinates

$T = kt + 1$

$X(r)$: function of r

2. Flow model and fundamental equations

The density ρ^* and the kinematic viscosity ν^* of fluid are to be constant. Refer to cylindrical co-ordinates $S^*(r^*, \theta^*, z^*)$ as z^* -axis is aligned with the center line of a porous, elastic, circular tube. $V^*(u^*, v^*, w^*)$ denotes the velocity vector, p^* the pressure and t^* the time.

Introduce, hereafter, dimensionless variables without asterisk which correspond to dimensional ones with asterisk.

* Received 9th July, 1979,
 ** Lecturer,
 Department of Mechanical Engineering,
 Faculty of Engineering,
 Yamagata University, Yonezawa

$$t^* = \frac{L^{*2}}{\nu^*} t, \quad s^*(r^*, r^*\theta, z^*) = L^* s(R, R\theta, Z) \quad \dots\dots\dots (1)$$

$$p^* = \frac{\rho^* \nu^{*2}}{L^{*2}} p, \quad V^*(u^*, v^*, w^*) = \frac{\nu^*}{L^*} V(u, v, w)$$

where L^* is the reference length which is the radius of a circular tube at a moment $t=t_0=0$.

Unsteady flow of a viscous, incompressible fluid induced by either contracting or expanding motion of the length together with the cross-section of a circular tube with permeable wall is analytically developed. The tube is occluded at $Z=0$ by an elastic membrane which prevents an axial flow. Hence the flow is symmetric about the plane $Z=0$. It is assumed that the tube does not twist and that the induced flow is axis-symmetric. The circumferential velocity u does not appeared.

The radius and the length of a tube are assumed to vary with time as functions $F(t)$, and $LG(t)$, respectively, where L denotes the dimensionless length of a tube at a moment $t=t_0$. The fluid is assumed to be able to pass through the porous wall of the tube with a velocity directly proportional to the normal velocity of the wall.

Since the radius and the length of a tube vary with time as $r^*=L^*F(t)$ and $z^*=L^*LG(t)$, respectively, we can introduce new co-ordinates r and z which satisfy the following relations:

$$R=rF(t), \quad Z=zG(t) \quad \dots\dots\dots (2)$$

hence,

$$\frac{\partial r}{\partial t} = rf(t), \quad \frac{\partial z}{\partial t} = zg(t),$$

where

$$f(t) = -\frac{1}{F} \frac{dF}{dt}, \quad g(t) = -\frac{1}{G} \frac{dG}{dt} \quad \dots\dots\dots (3)$$

If we use, the above co-ordinates r and z , the equation of conservation of mass is expressed by

$$\frac{1}{F} \frac{\partial}{\partial r} (ru) + \frac{1}{G} \frac{\partial}{\partial z} (rw) = 0 \quad \dots\dots\dots (4)$$

The stream function Ψ can be introduced as

$$u = -\frac{1}{Gr} \frac{\partial \Psi}{\partial z}, \quad w = \frac{1}{Fr} \frac{\partial \Psi}{\partial r} \quad \dots\dots\dots (5)$$

The equations of conservation of momentum are given by

$$\begin{aligned} \frac{1}{F} \frac{\partial p}{\partial r} &= \frac{1}{F^2} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial ru}{\partial r} \right) + \frac{1}{G^2} \frac{\partial^2 u}{\partial z^2} \\ &- \frac{\partial u}{\partial t} - \left(\frac{\partial r}{\partial t} + \frac{u}{F} \right) \frac{\partial u}{\partial r} - \left(\frac{\partial z}{\partial t} + \frac{w}{G} \right) \frac{\partial u}{\partial z} \\ \frac{1}{G} \frac{\partial p}{\partial z} &= \frac{1}{F^2} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) + \frac{1}{G^2} \frac{\partial^2 w}{\partial z^2} \\ &- \frac{\partial w}{\partial t} - \left(\frac{\partial r}{\partial t} + \frac{u}{F} \right) \frac{\partial w}{\partial r} - \left(\frac{\partial z}{\partial t} + \frac{w}{G} \right) \frac{\partial w}{\partial z} \end{aligned} \quad \dots\dots\dots (6)$$

The boundary conditions are given by

$$\begin{aligned} r=1: & \\ u=u_0 &= (1+S_r) \frac{dF}{dt} + S_z \frac{dG}{dt}, \\ w=w_0 &= z(dG/dt), \\ r=0: & u=0, \quad z=0: \quad w=0, \end{aligned} \quad \dots\dots\dots (7)$$

where $S_i = S_i^* L^* / \nu^*$ ($i: r$ or z) is a dimensionless parameter which defines the constant condition of injection into the normal directions of the wall. S_i prescribes the permeability of mass to the porous wall.

3. The elastic motion of a tube

The displacement functions F and G are arbitrary functions of t which define the elastic motions of a tube in either contraction or expansion in radial and axial directions, respectively. We shall, hereafter, consider the simplest case that the above two displacement functions are equal to each other, i, e .

$$F=G \quad \dots\dots\dots (8)$$

and it is assumed that

$$F(t) = \sqrt{T},$$

where

$$T = \begin{cases} 1+kt & (\text{for } k > 0) \\ 1-|k|t & (\text{for } k < 0) \end{cases} \quad \dots\dots\dots (9)$$

k is the condition of motion which is defined by

$$k = k^* L^{*2} / \nu^* \quad \dots\dots\dots (10)$$

where k^* is assumed to be a constant which prescribes the variation in unit time. The positive k means expanding action both in radial directions and in axial ones, while the negative k does contracting action. The above actions become rapid as the absolute value of k increases. If $k=0$, the wall does not vary with time elastically.

The above F satisfies the following differential equations:

$$\frac{df}{dt} = 2f^2 = -k \frac{f}{F^2} = \frac{k^2}{2T^2}, \quad f = -\frac{k}{2T} \quad \dots\dots\dots (11)$$

4. Laminar, similar solutions with respect to space and time

In our situation, the unsteady flow induced by either contracting or expanding tube is symmetric about $z=0$. Taking the boundary conditions (7) into account, we can introduce the stream function Ψ of the similar solutions with respect to space and time with the function X which depends on r alone, as follows:

$$\Psi = zX(r)F(dF/dt) \quad \dots\dots\dots (12)$$

The velocities are given by

$$\begin{aligned} u &= u_0 \frac{dF}{dt}, \quad w = zw_0 \frac{dF}{dt} \\ \text{where} & \\ u_0 &= -\frac{X}{r}, \quad w_0 = \frac{1}{r} \frac{dX}{dr} \end{aligned} \quad \dots\dots\dots (13)$$

$S = -1$ and 0 as well as $k = 100, 1, -1$ and -100 . In case of the positive k , *i.e.* the expanding process of a tube, the axial velocity has a larger value in the center of the tube, while it decreases close to the wall. This compensating flow becomes more remarkable as the expanding motion is rapid. Examples of the above excellently compensating flow are given in cases of $k = 100$ in Fig. 2. On the other hand, in case of the negative k , *i.e.* the contracting process of a tube, the axial velocity near the wall becomes large, but in the center the flow is highly retarded. As k takes a larger absolute value, the flow field in the tube tends to have a core region in the center with the boundary layer in the vicinity of the wall. Stream lines for $k = -100$ in Fig. 2 follow the above example. In the critical condition of injection S_c , the above characteristics vanish.

5. The classifications of induced flow fields

At a given moment, the axial flow rate Q_a^* across a section $z = z$, and the flow rate Q_w^* in the normal direction of the wall of the tube whose length is z are given by

$$\left. \begin{aligned} \frac{Q_a^*}{\nu^* L^*} &= Q_a = 2\pi F^2 \int_0^1 w r dr = -\pi(1+S)zF \frac{dF^2}{dt} \\ \frac{Q_w^*}{\nu^* L^*} &= Q_w = 2\pi F^2 z u \Big|_{r=1} = -Q_a \end{aligned} \right\} \dots\dots\dots(27)$$

The above two equations give the following integrated equation of conservation of mass:

$$Q_a + Q_w = 0 \dots\dots\dots(28)$$

During the time interval $t = t_0$ to t , the tube whose length is z has a volumetric increment V_m^* , a volume V_w^* passing through the wall and a volume V_a^* flowing across the section $z = z$. These are in the following relationships:

$$\left. \begin{aligned} V_m &= \int_{t_0}^t 2F^2 \frac{dF}{dt} dt = \frac{2}{3} \{ F^3(t) - F^3(t_0) \} \\ V_a &= -(1+S)V_m, \quad V_w = SV_m, \end{aligned} \right\} \dots\dots(29)$$

where

$$V_i^*/V_0^* = V_i, \quad V_0^* = \pi z L^{*3},$$

The subscript i indicates the representative

Fig. 2 Streamlines in cases of $k = 100, 1, -1$ and -100 as well as $S = 1$ and 0. Numerics in figures denote the values of Ψ .

of m , α and w . The sign of V_m should be determined by the condition of motion k . Hence in the case of the positive k , the volume increases as time goes by, while in the case of the negative k , the volume decreases with time.

When the condition of injection has a critical value S_c , the above relations reduce to

$$V_w = S_c V_m, \quad V_a = 0.5 V_m \quad (30)$$

In the expanding process ($k > 0$), $S_{wc} = -1.5$, which means that one and a half times the incremental volume is sucked from the wall and a half times the incremental volume is pumped out in the positive direction across the section $z = z$. On the other hand, the contracting process ($k < 0$) takes $S_{wc} = 1.5$. Hence a half of the decremental volume is sucked from the section $z = z$, and one and a half of the decremental volume passes through the wall of a tube.

The null condition of injection gives

$$S = S_0 = 0 \quad (31)$$

and

$$V_w = 0, \quad V_a = -V_m \quad (32)$$

The contracting process pumps out the decremental volume across the section $z = z$, while the expanding process sucks the incremental volume from the section $z = z$.

There is one more critical condition S_a which causes the volume V_a to vanish. That is to say,

$$S = S_a = \pm S_{wa} = -1 \quad \text{for } k \geq 0 \quad (33)$$

and this gives

$$V_w = -V_m, \quad V_a = 0 \quad (34)$$

The contracting process ($S_{wa} = 1$) injects the decremental volume into the wall, while the expanding process ($S_{wa} = -1$) sucks the incremental volume from the wall. This critical value S_{wa} divides the condition of injection S into two parts. The condition of motion k together with the above condition of injection gives the classification of the pumping action as in Table 1. $V_a > 0$ prescribes a pumping action in which the fluid flows out through the section $z = z$, while $V_a < 0$ does one in which the fluid is sucked into the tube through the section $z = z$. There is no flowing through the section $z = z$ for $V_a = 0$.

6. Pressure distributions

According to Starling's law⁽⁴⁾⁽⁵⁾, the mass flow rate through the porous walls is directly proportional to the pressure difference between inner and outer values on

Table 1. The classification of induced flow fields

k	t	V_m	$S_w < S_{wa}$	$S_w = S_{wa}$	$S_w > S_{wa}$
$k > 0$	$-1 < kt < \infty$	$V_m > 0$	$V_a > 0$	$V_a = 0$	$V_a < 0$
$k = 0$	independent	$V_m = 0$	$V_a = 0$	$V_a = 0$	$V_a = 0$
$k < 0$	$-\infty < k't < 1$	$V_m < 0$	$V_a > 0$	$V_a = 0$	$V_a < 0$

the wall, so the following relationship results:

$$p_w(F, Z, t) - p_c = -p_p = \frac{S}{\kappa} \frac{dF}{dt} \quad (35)$$

where p_w denotes the pressure at $R=F$, p_c the external pressure on the tube, p_p the pressure drop in the walls of the tube and κ^* the filtration constant. The condition of injection S is comparable to the dimensionless filtration $\kappa^* v^*/L^* = \kappa$. Without further discussion of the flows of the fluid in the wall, we should consider the pressure in the tube with the condition of injection S .

On the assumption that the convective terms can be neglected, we have the following pressure equations from Eqs. (11) and (17).

$$\frac{\partial p}{\partial r} = -\frac{kK}{4T} r \quad (36)$$

$$\frac{\partial p}{\partial z} = \frac{kK}{2T} z \quad (37)$$

The constant K is in general a function of k and S . Fig. 3 shows the variations of K_0 (a dashed line) and K (solid lines) against k . For a large negative k , K_0 is nearly equal to $2|k|$, and for $k = 0$, K_0 is 16, while for a large positive k , K_0 tends to have a very small value as almost $k^2/2 \exp(-k/4)$, for example, $K_0 = 0.036$ for $k = 40$ and $K_0 = 6.98 \times 10^{-8}$ for $k = 100$. Hence K varies with S for a negative k and a small positive one, but K is nearly equal to k for $k > 40$.

Integration of Eq. (36) over r from $r = 0$ to r yields the pressure distribution in a section z at an arbitrary moment t .

$$p(r, z, t) - p(0, z, t) = -\frac{kK}{8T} r^2 \quad (38)$$

Therefore, the pressure difference p_d between the inner wall and the center has the same constant, as a function of k, S and t , in every section at any moment, as follows:

$$p_d = p(1, z, t) - p(0, z, t) = -\frac{kK}{8T} \quad (39)$$

At an arbitrary moment, the pressure distribution in an axial direction on the cylindrical surface $r=r$ is given by integration of Eq. (37) over z from $z=0$ to z .

Fig. 3 Variations of K_0 (a dashed line) and K (solid lines) against k as functions of S . Numerics in figures denote the values of S .

$$p(r, z, t) - p(r, 0, t) = \frac{kK}{4T} z^2 \dots\dots\dots(40)$$

Hence the pressure distribution along the axis is expressed by

$$p(0, z, t) - p(0, 0, t) = \frac{kK}{4T} z^2 \dots\dots\dots(41)$$

In conclusion, the pressure at any point and at any moment, is resolved by combination of Eqs. (38) and (41), as follows:

$$\frac{p^* L^{*2}}{\rho^* \nu^{*2}} = p = \frac{kK}{4T} \left(z^2 - \frac{r^2}{2} \right) \dots\dots\dots(42)$$

p is hereafter defined as

$$p = p(r, z, t) - p(0, 0, t) \dots\dots\dots(43)$$

where $p(0, 0, t)$ is the pressure on the center of elastic membrane which occludes one end of a tube at $z=0$. $p(0, 0, t)$ is taken as the reference pressure.

The contours of equi-pressures in a tube are shown in Fig. 4. Numerics in the figure denote the values of $p/(kK/4T)$.

$$p=0 \text{ where } r=\sqrt{2}z \dots\dots\dots(44)$$

The contours near the occluded membrane are almost parallel to the axis z , but those far from the membrane become nearly perpen-

Fig. 4 Contours of equi-pressure. Numerics in the figure denote the values of $p/(kK/4T)$.

Fig. 5 Variations of $kK/2$ against k as functions of S . Numerics in the figure denote the values of S .

dicular to the axis z . The value of the pressure p is resolved by the products of the value of the contour and the time variation T^{-1} together with the constant $kK/4$ which is a function of the condition of injection S on the wall as well as a function of the condition of the elastic motion of the tube k . Figure 5 shows the variations of $kK/2$ versus k as functions of S . In cases of the contracting motion of a tube ($k < 0$) and also the slowly expanding motion ($20 > k > 0$), $kK/2$ changes its value with S , while for $k > 40$ kK is nearly equal to k^2 because of $K \approx k$.

$$\text{For } k=0 \text{ or } k+(S+1.5)K_0=0$$

$$kK=0, \text{ viz. } p=\partial p/\partial r=\partial p/\partial z=0 \dots\dots(45)$$

and

$$w_0=E(r), \quad X=-ru_0=\frac{2}{k}\{e^{k/4}-E(r)\} \dots\dots(46)$$

The flow described in the above solution occurs in practice only when the pressure in the whole space is constant, though the above formulae constitute exact solutions of our problem for the restricted values of the condition of motion together with those of injection which satisfy Eq. (45).

If the working fluid is a liquid whose vapor pressure is p_{vp} , there exists a critical contour plane where the pressure is equal to the vapor pressure of the liquid. As a function of k and S , this critical pressure, i.e. p_c is given by

$$p_c = p_{vp} - p(0, 0, t) = -\frac{kK}{8T} \text{ or } \frac{kK}{4T} z_c^2 \dots\dots(47)$$

where z_c is a critical length and

$$p_{r,p} = p(1, 0, t) \text{ or } p(z_c, 0, t)$$

7. Frictional drag, powers and works

The shear stress along the wall, τ_w^* is given by

$$\frac{\tau_w^* L^{*2}}{\rho^* \nu^{*2}} = \tau_w = \frac{1}{F} \frac{\partial w}{\partial r} \Big|_{r=1} = \tau_0 \frac{z}{F} \frac{dF}{dt} \dots\dots(48)$$

where

$$\tau_0 = \frac{dw_0}{dr} \Big|_{r=1} = \begin{cases} (S+1.5)K_0/2 & (k \neq 0) \\ 8(S+1.5) & (k=0) \end{cases} \dots\dots(49)$$

At a given moment, the frictional drag D_f^* acting on the cylindrical surface of the wall of the tube whose length is z , is obtained by integration of Eq. (48) over the surface, as follows:

$$\frac{D_f^*}{\rho^* \nu^{*2}} = D_f = 2\pi \int_0^z \tau_w F^2 dz = \frac{\pi \tau_0}{2} \frac{dF^2}{dt} z^2 \dots\dots(50)$$

As the wall is moved elastically by its velocity w_w , there arise a frictional power P_{fw}^* at a moment and also a frictional work W_{fw}^* acting on the length z during the time interval $t=t_0$ to t :

$$\left. \begin{aligned} P_{fw} &= 2\pi \int_0^z \tau_w w_w F^2 dz = \pi k \tau_0 \frac{z^3}{3} \frac{dF}{dt} \dots\dots (51) \\ W_{fw} &= \pi k \tau_0 \frac{z^3}{3} F_d, \end{aligned} \right\}$$

where

$$F_d = F(t) - F(t_0)$$

Dimensionless expressions of power and work are defined, hereafter, as follows:

$$\frac{P_i^* L^*}{\rho^* \nu^* s} = P_i, \quad \frac{W_i^*}{\rho^* \nu^* s^2 L^*} = W_i = \int_{t_0}^t P_i dt \dots\dots (52)$$

where the subscript i means representative.

In the critical condition of injection S_c ,

$$\tau_0 = \tau_w = D_f = P_{fw} = W_{fw} = 0 \dots\dots\dots (53)$$

As it is known in Fig. 3, K_0 has a large value for the negative k , but it has a smaller value as the positive k becomes large.

Fig. 6 Variations of A_0 (a dashed line) and A (solid lines) against k as functions of S . Numerics in the figure denote the values of S .

Therefore, shear stress, frictional drag, powers and works in a contracting process acting on the wall are larger than those in an expanding process.

At an arbitrary instant, the power P_w^* of the fluid which flows to the porous wall in its normal direction, the axial power P_a^* acting on a section $z=z$ to maintain the pressure field, and the power P_f^* dissipated by viscous friction in the tube whose length is z , are obtained, as follows:

Fig. 7 Variations of $H_w / (\pi k K / 2)$ (a dashed line) and $-H_a / (\pi k K / 2)$ (solid lines) against z as functions of k in case of $S=0$. Numerics in the figure denote the values of k .

Fig. 8 Variations of $-H_a / (\pi k K / 2)$ against z as functions of S in cases of $k=-100$ and 100 . Numerics in the figures denote the values of S .

$$\left. \begin{aligned} P_w &= 2\pi \int_0^z p_w u_w F^2 dz = H_w \frac{dF}{dt}, \\ P_a &= 2\pi \int_0^1 p_w F^2 r dr = H_a \frac{dF}{dt}, \\ P_f &= -P_a - P_w \end{aligned} \right\} \dots\dots\dots (54)$$

where

$$\left. \begin{aligned} H_w &= \frac{\pi k K}{2} (1+S) \left(\frac{z^3}{3} - \frac{z}{2} \right), \\ H_a &= \frac{\pi k K}{2} \{ Az - (1+S)z^3 \}, \\ A &= (S+1.5)A_0 - \frac{1}{8}, \\ A_0 &= \begin{cases} \frac{2}{k} \left(1 - \frac{K_0}{16} \right) & (k \neq 0) \\ \frac{1}{6} & (k = 0) \end{cases} \end{aligned} \right\} \dots\dots\dots (55)$$

Figure 6 shows the variations of A_0 (a dashed line) and A (solid lines) against k . For a large negative k , A_0 is nearly equal to $1/4$ and for $k=0$ A_0 is $1/6$, but for a large positive k , A_0 is almost $2/k$. Hence, A varies with S for a large negative k and a small positive one, but A converges to $-1/8$ as a positive k becomes large.

During the time interval $t=t_0$ to t , the work W_w of the fluid which should flow to the porous wall, the axial work W_a acting on a section $z=z$, and the work W_f of viscous dissipation in a tube whose length is z , are expressed as

$$W_w = H_w F_d, \quad W_a = H_a F_d, \quad W_f = -W_a - W_w \dots\dots\dots (56)$$

The above given powers P_i and works W_i vary in direct proportion to dF/dt and F_d , respectively. To clarify the characteristics of powers and works, H_w and H_a , which are dependent on kK , A , S and z should be examined. Functional relationships of $kK/2$ to k and S are illustrated in Fig. 5. Figure 7 shows the variations of $H_w/(\pi k K/2)$ (a dashed line) and $-H_a/(\pi k K/2)$ (solid lines) against z as functions of k in case of $S=0$. Besides, the variations of $-H_a/(\pi k K/2)$ versus z as functions of S in cases of $k=-100$ and 100 are shown in Fig. 8. Since the contours of equi-pressures in the neighborhood of the membrane at $z=0$ are nearly parallel to the z -axis, but those far from the membrane are almost perpendicular to the z -axis, the signs of H_w and H_a are altered depending on the length of a tube. The critical length of them are given by

$$\left. \begin{aligned} H_w &= 0 \text{ at } z = z_w = \sqrt{1.5} = 1.2247 \\ H_a &= 0 \text{ at } z = z_a = \sqrt{A/(1+S)}, \text{ for } S \neq -1 \end{aligned} \right\} \dots\dots\dots (57)$$

It should be noted that z_a depends on S and k . When the condition of injection S has a critical value S_a , H_w vanishes and H_a is proportional to z . For a comparatively large z , $H_a/(\pi k K/2)$ is almost directly proportional to $(1+S)z^3$, so $H_a/H_w \approx -3$.

8. Conclusions

A linearized theory on the laminar similar solutions with respect to space and time in case of an unsteady flow induced by the elastic contraction or expansion of the length together with the cross-section of an occluded, porous, elastic, circular tube has been developed. The effects of the inflow or the outflow of a fluid which is assumed to be directly proportional to the velocity of elastic movements of the wall are considered. Velocities, pressure, frictional drag, powers and works of fluid are analyzed exactly.

A rapid elastic expansion of a tube develops a remarkable compensating flow in the center of the tube, while a rapid elastic constriction of the tube induces a flow which has a core region in the center of the tube with the boundary layer in the vicinity of the wall.

The contours of equi-pressures in the neighborhood of the occluded end are nearly parallel to the axial direction of the tube, while those far from the closed membrane are almost perpendicular to the axial direction. They cause the critical lengths of integrated powers and works to vanish, but it should be noted that the local powers and works persist.

Whether the elastic tube discharges the fluid or sucks it can be decided by the conditions of the tube either in contraction or in expansion as combined with the injection conditions which have three critical conditions.

The numerical calculations were performed at the Computation Centers of Tokyo University and Yamagata University.

The author acknowledges with thanks many helpful comments and discussions received from Prof. Ayukawa of Ehime University, Prof. Uchida of Nagoya University, Prof. Tanahashi of Keio University and Prof. Ohmi of Osaka University as well as their reviewing of the manuscripts. Also he would like to express his appreciation to Mr. Jiro Nakamura for his help in tracing figures.

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