Free Convection Heat Transfer near Leading Edge
of Semi-infinite Vertical Flat Plate

(2nd Report, Uniform Heat Generation, \( Pr = 0.72 \))

By Masahide MIYAMOTO** and Takanori AKIYOSHI***

Free convection heat transfer near the leading edge of semi-infinite vertical thin flat plate with uniform heat generation, has been analyzed numerically by the finite-difference method for \( Pr = 0.72 \). For uniform heat flux case, the present Nusselt number distribution of free convection heat transfer is approximated well by the following equation:

\[
\frac{Nu}{X^{1/5}} = 0.4894 + 0.117X^{0.535}, \quad X \geq 1.75
\]

The surface temperature distribution near the leading edge of the flat plate with uniform heat generation, which is placed vertically in air, has been calculated by considering not only free convection heat transfer but also thermal radiation and axial heat conduction in the plate. The effects of axial heat conduction in the plate on the surface temperature distribution near the leading edge are expressed in terms of the dimensionless parameter \( XD \). The calculated temperature distributions agreed well with the experimental results using a Mach-Zehnder interferometer.

1. Introduction

In the first report\(^{(1)}\), the free convection heat transfer near the leading edge of a semi-infinite vertical isothermal flat plate was numerically analyzed by the use of the finite-difference method for \( Pr = 0.72 \). The effects of the leading edge configurations and of the horizontal floor under the leading edge on the free convection heat transfer were investigated in detail. A heat transfer problem is usually analyzed under the condition of uniform surface heat flux as well as uniform surface temperature. Perturbation solutions of higher order boundary layer effects for free convection around a semi-infinite vertical flat plate with uniform surface heat flux were obtained by Berezovski et al\(^{(2)}\) and Mahajan et al\(^{(3)}\). Their results about the local Nusselt number are shown in Table 1. The difference between these two results is evident. Furthermore, both results indicate that the surface temperature distribution changes abruptly near the leading edge. If the convecting fluid is a gas such as air, this abrupt change exerts not a little influence of thermal radiation and axial heat conduction in the plate on the surface temperature distribution. In such a case, a theoretical analysis of free convection heat transfer only under the condition of uniform heat flux becomes far from realistic. The interference effects of thermal radiation and axial heat conduction in the plate with free convection heat transfer had been analyzed \(^{(4)}\) by the use of classical boundary layer equations. There is no publication about the free convection heat transfer near the leading edge subjected to the influence of thermal radiation and axial heat conduction in the plate.

In this second report, the finite-difference method used in the first report has been employed to analyze the problem including these interference effects with the free convection heat transfer near the leading edge, which can not be analyzed by classical boundary layer approximation. The following thermal boundary conditions on the plate have been assumed in the present analysis.

- Case a 3 Uniform surface heat flux: Free convection heat transfer only takes place.
- Case b 3 Uniform-heat-generating plate: Free convection and thermal radiation heat transfer take place.

Table 1 Comparison of local Nusselt number between previous perturbation solutions, case a.

\[
\begin{align*}
\text{Berezovski, Sokovishin (1977) } Pr &= 0.733^{(2)} \\
\frac{Nu}{X^{1/5}} &= 0.4898(1 + \frac{0.6908}{X^{4/5}} - \frac{0.3321}{X^{2/5}} + \frac{0.2388}{X^{6/5}})
\end{align*}
\]

\[
\begin{align*}
\text{Mahajan, Gebhart (1978) } Pr &= 0.733^{(3)} \\
\frac{Nu}{X^{1/5}} &= 0.4898(1 + \frac{0.3327}{X^{4/5}} + \frac{0.2101}{X^{8/5}})
\end{align*}
\]

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** Associate Professor, Faculty of Engineering, Yamaguchi University, Tokiwadai, Ube, 755.
*** Graduate student, Faculty of Engineering, Yamaguchi University.
2. Nomenclature

- $A$: dimensionless radiation parameter
- $a_1, a_2, a_3$: constants defined by Eq. (20)
- $B$: dimensionless radiation parameter
- $B = \gamma_2 q_0 / [g \delta_{0}]^{1/4}$
- $d$: thickness of a flat plate
- $D$: dimensionless thickness, Eq. (6)
- $g$: acceleration due to gravity
- $G$: parameter defined by Eq. (11)
- $h$: height of the leading edge from the horizontal floor
- $H$: dimensionless height, Eq. (6)
- $i, j$: number of finite-difference meshes in $X$ and $Y$ directions
- $I_{X, Y}$: maximum values of mesh number $i$ and $j$
- $K$: ratio of thermal conductivities, $= k_0/k_f$
- $k_0, k_f$: thermal conductivities of the fluid and solid
- $N_{xt}$: local Nusselt number, $= \omega_0 \psi_0 / k_f$
- $Pr$: Prandtl number
- $q_0, q_0$: generated (total) heat flux and convective heat flux on the plate
- $t$: time
- $T, T_w$: temperature, and surface temperature of the plate
- $T_r$: reference temperature defined by Eq. (25)
- $u, v$: velocities in $X$ and $Y$ directions
- $U, V$: dimensionless velocities in $X$ and $Y$ directions, Eq. (7)
- $x$: coordinate along the vertical plate from the leading edge
- $y$: horizontal coordinate from the center of the vertical plate
- $X, Y$: dimensionless coordinates of $x$ and $y$, Eq. (6)
- $X_{max}$, $Y_{max}$: maximum values of $X$ and $Y$ in the calculated region
- $\alpha$: local heat transfer coefficient, $\alpha = q_0 / (T_w - T_0)$
- $\beta$: volume expansion
- $\delta_{X, Y}$: mesh sizes of $X$ and $Y$ directions, $j = 1$ on the plate
- $\Delta X_{min}$, $\Delta Y_{min}$: minimum values of $\Delta X_{X}$ and $\Delta Y_{Y}$
- $\Delta X_{X}$, $\Delta Y_{Y}$: dimensionless coordinates of $X$ and $Y$ directions
- $\gamma$: normal total emissivity of the plate and the floor
- $\theta$: vorticity
- $\theta$: dimensionless temperature difference, Eq. (8)
- $\nu$: kinematic viscosity
- $\zeta$: dimensionless vorticity, Eq. (9)
- $\sigma$: Stefan-Boltzmann constant
- $\tau$: dimensionless time, Eq. (5)
- $\phi$: stream function, Eqs. (4) and (10)
- $\psi$: dimensionless stream function, Eq. (10)
- $\nabla^2$: Laplace operator

3. Basic equations and numerical solutions

3-1. Basic equations and boundary conditions

A semi-infinite vertical flat plate over the horizontal floor is shown in Fig. 1, with the coordinate system. The basic equations governing free convection heat transfer are given by the following Eqs. (1) - (4), written in a dimensionless form. These equations stand on the
following assumptions:
(1) The basic equations are made two-dimensional
(2) Physical properties other than density in the buoyancy term are constant.
(3) Fluid is transparent for thermal radiation.
(4) Viscous dissipation term is ignored.

\[
\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 T}{\partial y^2} + \nabla^2 \xi - \frac{\partial \Phi}{\partial y} \quad (1)
\]

\[
\frac{\partial T}{\partial t} = -\frac{\partial \Phi}{\partial x} + \frac{1}{Pr} \nabla^2 \Phi \quad (2)
\]

\[\xi + \nabla^2 \psi = 0 \quad (3) \quad U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x} \quad (4)
\]

Eq. (1) is a vorticity transport equation. 
Eq. (2) is an energy transport equation. 
Eq. (3) gives the relation between vorticity and stream function.  Eq. (1) ~ (4) are the same as those under isothermal conditions in the first report.  The dimensionless variables in these equations are defined by the following equations:

\[
\tau = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{1/4} t \quad (5)
\]

\[X = G x, \quad Y = G y, \quad D = G d, \quad H = G h \quad (6)
\]

\[U = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{1/4} U, \quad V = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{1/4} V \quad (7)
\]

\[\theta = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{1/2} \left( \theta - \theta_w \right) \quad (8)
\]

\[\xi = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{3/2} (9) \quad \psi = \frac{\Phi}{\nu} \quad (10)
\]

\[Q = \left[ \frac{k}{g \beta \rho v_s^2} \right]^{1/4} (11)
\]

The unsteady terms are included in Eqs. (1) and (2) for the convenience of the numerical solution.  In the following development of the analysis, the same descriptions as those in the first report will not be repeated.

The thermal boundary conditions on the vertical plate and on the floor are given by the following:

[Case a] Uniform surface heat flux:

\[X = \text{const}, \quad \frac{\partial T}{\partial y} = 0, \quad Y = 0: \quad \frac{\partial \Phi}{\partial y} = 0 \quad (12)
\]

[The boundary condition on the plate can be given at X = const, because the plate is negligibly thin.]

\[X = -H, \quad Y = \text{const}: \quad \frac{\partial \Phi}{\partial x} = 0 \quad (13)
\]

[The floor is adiabatic.]

[Case b] Uniform-heat-generating plate:

\[X = \text{const}, \quad \frac{\partial T}{\partial y} = 0, \quad Y = 0: \quad \frac{\partial \Phi}{\partial y} + A[(B0 + 1)^4 - 1] = 0 \quad (14)
\]

\[A = -H, \quad \text{max} \geq Y \geq 0: \quad \frac{\partial \Phi}{\partial y} = 0 \quad (15)
\]

\[X = X_{\text{max}} \geq 0, \quad Y = 0: \quad \frac{\partial \Phi}{\partial x} = 0 \quad (16)
\]

\[X = X_{\text{max}} \geq 0, \quad Y = 0: \quad \frac{\partial \Phi}{\partial x} = 0 \quad (17)
\]

\[X = 0, \quad Y = 0: \quad \frac{\partial \Phi}{\partial x} = 0 \quad (18)
\]

The left hand side of Eq. (18) expresses the heat conduction in the plate, and the first and second terms in the right hand side of Eq. (18) express the free convection heat flux and the thermal radiation heat flux, respectively at the bottom side of the plate.

3-2. Finite-difference equations

Eqs. (1) ~ (4) have been transformed into the finite-difference form using the variable size of mesh in the same way as in the first report.  The obtained finite-difference equations also have been solved numerically by the ADI method.  In the following, the finite-difference form of the boundary conditions will be shown and the method of their solutions will be explained.

[Case a]

Eq. (12) is transformed into the finite-difference equation (19).  The dimensionless surface temperature \( \theta_{\text{s,1}} \) can be determined by the known dimensionless fluid temperature \( \theta_{\text{s,2}} \) and \( \theta_{\text{s,3}} \) at each time step of the ADI method.

\[\theta_{\text{s,1}} = a_1 + a_2 \theta_{\text{s,2}} + a_3 \theta_{\text{s,3}} \quad (19)
\]

\[a_1 = \frac{\Delta Y_2 \theta_1}{\Delta Y_1 \theta_2}, \quad a_2 = \frac{\Delta Y_1 \theta_2}{\Delta Y_1 \theta_2}, \quad a_3 = \frac{\Delta Y_1}{\Delta Y_2 + 2 \Delta Y_1} \quad (20)
\]

[Case b]

Eq. (14) is transformed into Eq. (21).

\[\theta_{\text{s,1}} = \left[ 1 + \frac{\Delta Y_2 \theta_1}{\Delta Y_2 / \Delta Y_2} \right]^2 \frac{\Delta Y_2}{\Delta Y_2} \theta_{\text{s,2}} - \frac{\Delta Y_2}{\Delta Y_2} \theta_{\text{s,3}} - \frac{\Delta Y_1 \theta_1}{\Delta Y_2} + \theta_1 \theta_{\text{s,2}} + \theta_1 \theta_{\text{s,3}} + \theta_1 \theta_{\text{s,1}} \right] \quad (21)
\]

Eq. (21) indicates that \( \theta_{\text{s,1}} \) is determined by successive iteration using a linearised approximation of the radiation term.  The terms included in \( \theta_{\text{s,1}} \) in Eq. (21) express the higher order radiation terms.  \( \theta_{\text{s,1}} \) is substituted by the value of \( \theta_{\text{s,1}} \) obtained at the last time of this iteration.  \( \theta_{\text{s,1}} \)
is determined by Eq. (21) at each ADI time step and the higher order radiation terms may require 2 or 3 times of successive iteration in each computation of Eq. (21).

[Case c]  
The conduction term in Eq. (16) is transformed into the following finite-difference form:

\[
\frac{\partial^2 \theta}{\partial x^2} |_{x=1} = \frac{\Delta x}{\Delta x} \theta_{1-1,1} - \frac{\Delta x + \Delta x - 1}{3 \Delta x^2} \theta_{1,1} + \frac{2}{3 \Delta x^2} \theta_{1+1,1}
\]

The other terms in Eq. (16) are transformed in the same way as in case b. The obtained finite-difference equations are a tridiagonal equation of \( \theta_{1,1} \) which can be solved easily by the known \( \theta_{1,2} \) and \( \theta_{1,0} \) even if the radiation term requires successive iteration.

However, if \( \theta_{1,1} \) in case c is determined by the finite-difference form of Eq. (16) at each ADI time step, a very large number of ADI time steps may be required for the convergence of the whole solution including all the basic equations. The following successive approximation method, efficient for this conjugate free convection heat transfer, which has been proposed first by Galilevich et al.\(^{[24]}\) is used.

1. Basic equations of the free convection are solved for some value of the wall temperature by methods such as finite-difference. The obtained solution makes it possible to calculate the heat transfer coefficient on the wall.

2. The heat conduction equation in the solid wall is solved also by the finite-difference method, using the heat transfer coefficient calculated in step 1, as the boundary condition.

3. If the difference of the wall temperature between the initial approximation in step 1 and the calculated value in step 2 is larger than some threshold value, then the calculations of steps 1 and 2 are repeated using the wall temperature obtained in step 2 as the initial value in step 1.

After a repetition of the above mentioned steps 1, 2 and 3, the converged wall temperature will be obtained. In the present calculation, the solution for case b was applied to the initial approximation in step 1. In step 2, the finite-difference form of Eq. (16) in case c was used as the heat conduction equation. The threshold value in step 3 was about 1%. After about 3 cycles of the calculation steps, the converged wall temperature was obtained.

4. Results and discussions of numerical solutions

Numerical calculations were performed on FACOM M-190 and M-200 at Kyushu University. Throughout the following numerical results, \( Pr = 0.72 \) and the thickness of the plate was so small as shown in the first report that the effects of the leading edge shape on the free convection heat transfer was neglected. (For this study, \( D = 0 \))

[Case a] Uniform heat flux

Numerical solutions for various dimensionless heights, \( H \) were obtained only in this case. The effects of the floor on the free convection heat transfer were examined. A numerical solution required larger CPU time for larger \( H \) and \( X_{max} \). For reference, an example of the computation requiring the maximum CPU time is as follows: \( X_{max} = 85.75 \), \( Y_{max} = 54.75 \), \( T_W = 30 \times 10^6 \), \( \Delta X_{min} = \Delta Y_{min} = 0.5 \). Number of ADI time steps for convergence was about 5000. CPU time was about 130 sec. (on M-190).

Convergence was judged by the same way as in the first report. On the other hand, when \( H = 0 \), \( X_{max} = 85.75 \), \( Y_{max} = 36.75 \) and \( I X_W = 17 \times 18 \). (The other conditions were the same as in the above-mentioned example.) The number of ADI time steps for convergence was about 1800. The accuracy of the numerical solution was examined by comparison of the calculated surface temperature between \( \Delta X_{min} = \Delta Y_{min} = 0.25 \) and 0.5.

The former case (smaller mesh size) gave merely about 2% lower dimensionless surface temperature than the latter case. Throughout numerical results, \( \Delta X_{min} \) and \( \Delta Y_{min} \) were equal to 0.5.

The local Nusselt number and the dimensionless surface temperature distributions for various dimensionless heights \( H \) are shown in Fig.2. The similarity solution of the boundary layer equations and the perturbation solutions are shown, too, for comparison. The following conclusions may be obtained from these results.

The differences of the results between \( H = 85.75 \) and 37.25 are insignificant; hence, the effects of the floor on the surface temperature and the local Nusselt number can be neglected for \( H \geq 40 \). (This relation \( H \geq 40 \), is the same as that for uniform wall temperature.) The lower the \( H \), the lower the local Nusselt number and when \( H = 2.25 \), the local Nusselt numbers near the leading edge are lower than those from the similarity solution. The local Nusselt number when \( H = 85.75 \) is approximated by the following equation within 0.5% error.

\[
\frac{\theta}{\theta_X} = 0.487 X + 0.117 \cdot X^{0.535}
\]

The difference of Eq. (23) from Berezovskii's perturbation solution is within 1% for \( X > 4 \). The difference of Eq. (23) from the similarity solution is 3.5% at 40 of \( X \) and is larger than 1.3% for the uniform wall temperature case.\(^{[17]}\) Throughout the following calculations, \( H \) nearly equals 40. This means that the semi-infinite vertical flat plate in the infinite fluid is treated.

[Case b] Uniform-heat-generating plate

Thermal radiation and free convection heat transfer exist together. The ratios of the convective heat to the heat flux to the heat generation are shown in Fig.3, for \( \varepsilon = 0.2 \) and 1, when the rates of the heat generation are 34.9 and 116.3 \( \text{W/m}^2 \) and the ambient temperature is 20°C. The dotted lines (for \( X = 1 \)) indicate the results from the approximation equations which Fujii et al.\(^{[18]}\) proposed based on the finite-difference solutions of the boundary layer equations. (The boundary layer solution results in \( K_0/K_1 = 1 \) at \( X = 0 \).) The ther-
Fig. 2 Comparison of local Nusselt number and dimensionless surface temperature distribution between various heights of the leading edge from the floor, case a.

Small radiation becomes smaller, nearer to the leading edge. Generally, the approximate equation by Fujii et al. shows a slightly smaller convection heat flux. The difference between Fujii's approximation and the present solutions becomes larger nearer to the leading edge.

The analogous comparisons of the dimensionless surface temperatures are shown in Fig. 4. A dotted chain line giving the highest temperature distribution in the figure indicates a similarity solution. The difference between the similarity solution and the present solution for $\varepsilon = 0$ can be attributed to the errors of the boundary layer approximations. Fujii's approxima-

Fig. 3 Effects of thermal radiation on the ratio of convective flux to generating (total) heat flux, case b.

Fig. 4 Effects of thermal radiation on dimensionless surface temperature distribution, case b.

Fig. 5 Effects of axial heat conduction in the plate on dimensionless surface temperature, case c.

...tions, being analogous to the similarity solution, show a slightly higher temperature than the present solution. The difference of the dimensionless surface temperature between the present solution and Fujii's approximations seems to become smaller for higher ratios of thermal radiation because the surface temperature distribution becomes more uniform for higher ratios of thermal radiation. (The boundary layer solution results in $\theta = 0$ at $X = 0$.)

Case c: Uniform-heat-generating plate
Thermal radiation, free convection heat transfer and axial heat conduction in the plate exist together. The plate thickness is very small but finite in the calculation of the axial heat conduction in the plate. (The effects of the leading edge shape on the free convection heat transfer can be neglected when $D$ is smaller than 0.5.)

The comparisons of the dimensionless surface temperature distributions are shown in Fig. 5. A dotted line indicates the present solution for case b in which thermal radiation and free convection heat transfer exist together. (The indicated solution corresponds to an instance in which thermal radiation is relatively small...
because \( c = 0.14 \) and \( q_w = 233 \text{ W/m}^2 \). The ratio of thermal radiation to generating heat is about 12% at \( X = 50 \). Solid lines show the present solutions taking the axial heat conduction into consideration. The differences between the solid lines and dotted line indicate the effect of the axial heat conduction on the surface temperature. For the larger dimensionless parameter \( K_D \), the heat flow conducted axially from the upper location in the plate increases, and the temperature near the leading edge then swings upward. The calculations for the same value of \( K_D \) (with a different \( K \) and a different \( D \)) show that the temperature at the leading edge is slightly higher for the large value of \( K \). But the differences between these temperatures distributions are very small and the dimensionless surface temperature distributions are well correlated with \( K_D \) only.

The dimensionless length \( X' \) and the dimensionless temperature \( \theta_w \) at the leading edge are shown against \( K_D \) in Fig. 6. \( X' \) corresponds to the value of \( X \) at a point at which the dotted line intersects the solid lines in Fig. 5. \( X' \) indicates a criterion of the range in which the influence of the heat conduction in the plate on the surface temperature is apparent. \( X' \) and \( \theta_w X = 0 \) increase almost linearly with \( K_D \) in the present range of the calculations. For the smaller ratio of thermal radiation to generating heat (corresponding to the solid line in Fig. 6, \( q_w = 385 \text{ W/m}^2 \), \( c = 0.14 \)), the following approximate equation has been obtained.

\[
X' = 0.046 K_D, \quad K_D \leq 400 \quad (24)
\]

On the other hand, if the thermal radiation heat flux increases relatively, the surface temperature distribution of the plate approaches a uniform distribution without the axial heat conduction in the plate (refer to Fig. 4). Hence, the change of the surface temperature caused by the axial heat conduction only is small in spite of the same value of \( K_D \), as it is shown by the dotted lines (corresponding to \( q_w = 34.9 \text{ W/m}^2 \) and \( c = 1.0 \)). \( X' \) in this condition is about 30% smaller than Eq. (24) at \( K_D = 400 \).

5. Experimental apparatus and method

The vertical heating surface was placed at the test section of the Mach-Zehnder interferometer in a veneer test chamber (960x925 and 1085 mm height). The height of its leading edge from the horizontal floor, on which a piece of black paper was attached, was 68 mm. Hence the effects of the floor on the free convection heat transfer around the vertical plate could be neglected. The construction of the heating surface made of polished foil of SUS 304 stainless steel is shown in Fig. 7. Copper-constantan thermocouples with 30 \( \mu \text{m} \) dia. were bonded on the back side of the heating surface to compare the measured surface temperature with N.I., and these coated lead wires were drawn out horizontally. The temperatures of the ambient fluid and of the inside surface of the veneer chamber were measured by copper-constantan thermocouples (0.35 mm dia.) arranged at vertical intervals of 100 mm. All these thermocouples used an ice bath as a reference and were switched through a manual stepping switch. Their thermal e.m.f. was measured by a digital volt meter with a resolution of 0.001 mV. These thermocouple systems were calibrated in advance by comparison with a standard thermometer with a precision of 0.1°C. The stainless foil was uniformly heated by direct current supplied through the copper electrodes at its both sides. The rates of the generating heat were measured by an ammeter and by the electric resistance of the stainless foil which was calculated in advance by the specific electric resistance of SUS 304 at the mean temperature. The effective diameter of the mirrors of the N.I. used was 60 mm. A monochromatic point light source was made by the use of a pin hole with 0.5 mm dia. and a filtered (5461 Å) light source from a mercury lamp. The interference fringe patterns were photographed on mini-copy film and enlarged for finding the value of the fringe shift. The value of 0.14 was used as the emittance of the stainless heating surface being measured by the use
of a radiation thermometer. The reference temperature at which the physical properties of air were evaluated excluding the volumetric expansion, was given by the following equation.

\[ T_r = T_0 - 0.38 (T_0 - T_a) \]  

(25)

The flow visualization by smoke showed that the fluctuation of the flow pattern near the leading edge was very faint.

6. Experimental results and discussions

The heating surface settled to steady state in five minutes after turning on the electric current. The horizontal variation of the inside surface temperature of the veneer chamber was about 0.8°C at the average height of the heating surface. The bulk fluid temperature was nearly equal to the lower one of the inside surface temperature. Regardless of temperature, the variation of the resulting thermal radiation heat flux was less than 1.5%. Then throughout the following results, the bulk fluid temperature at the average height of the heating surface was chosen as \( T_a \). The vertical variation of the bulk fluid temperature was 0.3°C/10 cm (for \( q_w = 200 \text{ W/m}^2 \)) at this height. The five thermocouples arranged in a horizontal line on the heating surface (refer to Fig.7) showed that the horizontal distribution of the surface temperatures was nearly uniform and their variation was within about 2% of the temperature difference between the bulk fluid and the heating surface. It can be seen that the temperature distribution was almost horizontally uniform except near the ends of both sides.

An example of the interferogram of the temperature field near the leading edge is shown in Fig.8. The fringe shift with a wedge fringes was measured using a photograph of cabinet size and was converted to temperature. A larger temperature gradient very near the leading edge caused a refraction error of the interferometer. Therefore, the normal refraction correction was made very near the leading edge.

The comparisons of the dimensionless surface temperature between the numerical solutions and the experiments are shown in Fig.9. In this figure, two solid lines indicating the lower temperature at large \( X \) correspond to the present solutions taking thermal radiation and axial heat conduction in the plate into consideration. The values of \( k \) and \( D \) for these two solid lines are the same as in the experimental results for the corresponding heat generation rate. Concerning the temperature rise near the leading edge caused by axial heat conduction in the plate, and concerning the difference between the dimensionless surface temperature distributions caused by the difference of heat generation rate, the present numerical solutions show a tendency to agree well with the experimental results obtained by the M.Z.I. The difference between the numerical solutions and the M.Z.I. is considered to have been caused mainly by the indistinct location of the heating surface in the analysed photograph. At larger \( X \), the results of the interferometry gave lower dimensionless surface temperatures and a different distribution from that of the thermocouples. A major cause for this difference is considered to be that the effective diameter of the interferometer is too small. At larger \( X \), the thickness of the thermal boundary layer increases, hence the region of the undisturbed fringe indicating the ambient fluid temperature becomes narrower and then the undisturbed fringe becomes more indistinguishable in the small visual field. Consequently the temperature measured by the interferometer tends to be too low (see Fig.8). Generally speaking, it can be seen that the numerical solutions well agree with the experiments for \( X < 20 \).

The comparisons of the dimensionless temperature distributions in the fluid between the numerical and the interferometer are made in Fig.10. The values of \( k \), \( D \) and \( c \) are the same. The agreement of these two results is very good excluding the region very near the plate.

7. Conclusions

![Fig.8 Interferogram of temperature field around the leading edge using wedge fringes, \( q_w = 196 \text{ W/m}^2 \)]

![Fig.9 Comparison of dimensionless surface temperature distribution between numerical solution and experiments.](image-url)
Fig. 10 Comparison of temperature profile around the leading edge between numerical solution and interferometer.

The surface temperature distribution near the leading edge of the vertical flat plate uniformly generating heat in air depends not only on free convection heat transfer but also on thermal radiation and axial heat conduction in the plate. The effects of axial heat conduction in the plate on temperature distribution are more significant especially nearer to the leading edge. The measurements of the temperature field around the leading edge by the M.Z.I. gives results corresponding closely with the numerical solutions. The main results obtained by the present analysis may give the following conclusions.

1. The Nusselt number of free convection heat transfer near the leading edge of the semi-infinite vertical flat plate with uniform surface heat flux is given by Eq. (23) for Fr = 0.72.

2. When thermal radiation and free convection heat transfer exist together under the condition of uniform heat generation, the previous solutions based on the boundary layer approximation give a higher surface temperature and a lower convective heat flux near the leading edge than the present more exact solutions.

3. The effects of the axial heat conduction in the plate on the temperature distribution of the uniform-heat-generating plate correlated well with the dimensionless parameter KD. The larger KD value caused the temperature rise near the leading edge. When thermal radiation heat flux was relatively small (corresponding to small ε and large qw), the range near the leading edge of the plate, in which an apparent rise of the surface temperature was caused by the influence of axial heat conduction, was determined by Eq. (24). On the other hand, when thermal radiation heat flux relatively large (large ε and small qw), the surface temperature distribution became uniform without the effects of axial heat conduction. In such a case, the above-mentioned range became smaller than that given by Eq. (24).

The above-mentioned conclusions are based on the assumption that the dimensionless height H of the leading edge from the floor was about 40. Therefore, the region occupied by the ambient fluid was considered to be not semi-infinite. If H becomes lower than 40, free convection heat transfer near the leading edge is affected by the existence of the floor. If the adiabatic floor approaches the leading edge of the semi-infinite vertical plate with uniform heat generation, the local heat transfer coefficient near the leading edge becomes lower and the surface temperature distribution becomes uniform. In this situation, the interference effects of thermal radiation and axial heat conduction in the plate with free convection heat transfer (including the consideration for the thermal boundary condition of the floor) remain to be investigated.

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