621.833.031

Paper No. 202-26

Bending Stresses of Internal Spur Gear*

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The influences of rim thickness of an internal spur gear on tensile and compressive bending stresses at tooth fillets and bottoms are discussed. The stress measurements by strain gages, the photoelasticity experiments, and stress analyses by use of the finite element method showed that the position where the maximum stress appears in an internal gear is different from that of an external gear. Especially when the rim thickness of an internal gear is thin, the maximum tensile stress appears at the center of the bottom of the tooth space, which is apart from the loaded tooth. As the rim thickness becomes thinner, the maximum compressive stress at the fillet of the loaded tooth increases sharply, while the maximum tensile stress hardly changes. As meshing proceeds, the tooth fillet and bottom stresses of a thin-rimmed internal gear alternate more strongly than those of a solid or thicker rimmed gear.

1. Introduction

The tooth bending strength calculation of internal gears used in epicyclic gearing has been done in the same manner as that of external gears hitherto. However, the stress distribution at the tooth fillet and bottom of an internal gear is different from that of an external gear, because the tooth form of an internal gear differs from that of an external gear and the internal gear com-

monly has a thin rim. Especially, the rim thickness of an internal gear used in epicyclic gearing is often designed very thin in order to distribute uniformly the transmitting power of every planet by an elastic deformation of the thin rim. Because of the large deformation of ring-bulk of an internal gear, it seems impossible to apply the present method for the strength calculation of an external gear directly to an internal gear design.

Some research on tooth fillet and bottom stresses of the internal gear have been published⁽¹⁾⁻⁽⁴⁾, but the stress distribution at the fillet and bottom of the tooth and its quantitative intensity are still not clear. Some studies on tooth stress of the thin-rimmed external gear have been published^{(5),(6)}, but the application of those results to the thin-rimmed internal gear is controversial, because of the differences in tooth form.

It is the purpose of the present paper to clarify the influences of rim thickness on the tooth fillet and bottom stresses by experimental and theoretical stress analyses of the internal gear used for epicyclic gearing with 3 planets.

2. Experiment and method of analysis

Fig.1. Schema of test rig.

* Received 11th July, 1980.

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Fig.2. Fixing position of internal gear, constraint conditions on calculation, and position of loaded tooth.

2.1 Stress measurement of an internal gear in epicyclic gearing

The internal test gear for stress measurement at the tooth fillet and bottom is a standard spur gear of module $m=4$ mm, pressure angle $\alpha_0=20^\circ$, number of teeth $z=66$, total tooth height $h=2.25m$, face width $b=8$ mm, and material S45C. Rim thicknesses t of the internal gears used for the investigation are 36 mm ($9m$), 27 mm ($6.75m$), 18 mm ($4.5m$), 12 mm ($3m$), 8 mm ($2m$). An internal gear of $t=36$ mm was at first used for stress measurement and then the rim thickness was decreased step by step to 27 mm, 18 mm, 12 mm, 8 mm. The stress measurement was carried out with the use of a static loading apparatus shown in Fig.1. This test rig is a simple planetary gearing with 3 planets and has an arm for loading. The rotational movement of the planetary carrier and the internal gear is fixed, and the torque applied on the arm is transmitted from a sun gear shaft to a sun gear, to planetary gears, and then to internal gear teeth. The point that the planetary gear teeth mesh with the internal gear teeth can be selected by adjusting the spacer between the planetary carrier and the internal gear to change the relative rotational position between the two. In order to get uniform load distribution on every planet, a sun gear is floatingly supported: the sun gear shaft has a bear-

ing only at the joint of the arm. Stress measurement was carried out in the state of a single tooth pair meshing by cutting off a tooth of the planetary gear to make clear the load acting on the internal gear tooth to be measured. To examine the influences of fixing methods of the internal gear on the tooth fillet and bottom stresses, stress measurements were carried out for the following two cases: i) fixing the internal gear with 6 bolts, the angle between each bolt being set at 60° intervals (Fig.2 (A)); and ii) fixing the internal gear with 3 bolts at 120° intervals (Fig.2 (B), (D) and (E)). The tooth's normal load P_n was applied on the tip of the tooth of the internal gear. Strain gages of 0.5 mm length were used in measuring the tooth fillet and bottom stresses. The strain gages were attached on tension and compression side fillets of the loaded tooth ($0a$, $0b$ of Fig.3(a) in the case of Fig.2 (A)). In the case of Fig.2 (B) strain gages were set at the fillets ($1a$, $1b$, $-1a$, $-1b$ of Fig.3(a)) of adjacent teeth. Because the maximum tooth fillet stress of the internal gear appears around a place nearer to the tooth bottom, the strain gages were attached at a place 45° tangent to the fillet curve from the tooth center line (cf. Fig.3(b)). In the case of Fig.2 (B) strain gages were set at the center of the bottom of every tooth space to investigate the influence of rim bending on stress distributions. In the case of (D) and (E) stresses were measured at 6 tensile and compressive positions of the tooth fillets and the bottoms of tooth spaces.

Fig.3. Positions of stress measurement.

2.2 Two-dimensional photoelasticity experiment

The internal gear used for the photoelasticity experiment is made of epoxy resin. It has the same shape as the steel internal gear of module $m=3.5$ mm used for the test mentioned in 2.1. The photoelasticity measurement of stress was carried out for rim thicknesses 28.73 mm ($8.2m$), 23.63 mm ($6.75m$), 15.75 mm ($4.5m$), 10.5 mm ($3m$), and 7 mm ($2m$). The load is

applied on a tooth which lies nearly at the middle position between the supporting points of the epoxy internal gear. The supporting points were selected at 3 positions where the radial displacement became zero, that is, at 120° intervals of the internal gear. The load was applied on a point of tooth flank 1 mm apart from the tooth tip edge. The load was appropriately selected to match the rim thickness.

Fig.4. An example of partitioned meshes of internal gear for FEM calculation.

2.3 Calculation of the tooth fillet and bottom stresses by the finite element method

The model in the FEM calculation is the same as the test gear mentioned in 2.1. Fig.4 shows one example of the

Fig.5. Relationship between tooth root (bottom) stress and loaded tooth positions.

division into elements of a triangular shape for the calculation: 5 teeth including the loaded tooth have fine meshes which represent the tooth profile well, and the rest of the teeth have coarse meshes and the teeth have a trapezoidal form. Stresses were calculated by a two-dimensional elastic stress theory, and calculation was carried out for 1/3 of the span of the internal gear because of the periodicity of stress distribution over the whole internal gear. The boundary conditions for the calculation are as follows: in the case of the internal gear fixed by bolts, the displacements of nodal points of FEM meshing corresponding to the centers of bolts are zero, and displacements of nodal points on section AA' and those of the nodal points on BB' which lie at the same radial position as the former points are the same. In the case of the photoelastic stress analysis model (2.2), the radial displacements of nodal points on section AA' and BB' are the same and circumferential displacements of these points are zero (cf. Fig.2 (C)). The loading condition of $Pn/b=9.8$

$t/m=8.2$ $Pn/b=14.5$ N/mm

$t/m=4.5$ $Pn/b=14.5$ N/mm

$t/m=2$ $Pn/b=5.9$ N/mm

Fig.6. Variation of fillet and bottom stresses of loaded tooth with the change of rim thickness.

Fig.7. Relationship between tooth fillet stress and rim thickness.

Fig.8. Relationship between tooth fillet stress and rim thickness.

N/mm (P_n :normal load, b :face width) is employed for the calculation, the load acting on the nodal point of the meshed model corresponding to the loading position of actual test gear. The total numbers of nodal points and elements are varied according to the rim thickness. The shapes and sizes of triangle elements which lie along the tooth profile curve are, however, kept always the same for all the models with different rim thicknesses.

3. Calculated and experimental results and discussions

Fig.5 shows the stress distribution over successive tooth bottoms on both sides of the loaded tooth of an internal gear. Stresses were measured at every center of the bottom of the tooth space by strain gages under conditions shown in Fig.2 (B), (D) and (E). The abscissa in this figure shows positions of teeth and tooth spaces by the angle ϕ measured from the center line of the central tooth along the rim of the internal gear. It is found that the tensile and compressive stresses under constraint condition (B) are greater than those of (D) and (E). It is considered that the constraint condition (B) gives, therefore, results on the safety side for the strength calculation of an internal gear fixed with 3 bolts.

3.1 The influences of rim thickness on tooth fillet and bottom stresses of a loaded tooth

Fig.6 shows the photoelastic isochromatic curves of a loaded tooth. Although the tooth profile and the load application position are the same, the stress distribution at tooth fillets and bottoms changes when the rim thickness is different. When the rim is thick enough the maximum tensile and also the maximum compressive stresses appear at tooth fil-

lets, but as the rim thickness becomes thinner the position of the maximum tensile stress moves toward pitch point and that of the maximum compressive stress moves towards the center of the bottom of the tooth space.

Fig.7 and Fig.8 show the relation between the tooth fillet stress and the rim thickness when the internal gear is fixed by 6 and 3 bolts (Fig.2 (A) and (B)). The tooth fillet stresses at the position of 45° tangent to tooth fillet curve of the loaded tooth were measured by strain gages and calculated by the FEM. It is seen that the compressive stresses under condition (B) are greater than those under condition (A), but the tensile stresses under (A) and (B) are almost the same. As the rim thickness becomes thinner, the tensile stress in-

Fig.10. Stress distributions around loaded tooth in the case of constraint condition (B).

Fig.9. Stress distributions around loaded tooth in the case of constraint condition (A).

Fig.11. Stress distributions around loaded tooth in the case of constraint condition (C).

creases only a little or stays almost constant under both conditions (A) and (B), but the compressive stress increases sharply and the increasing rate under (B) is greater than that of (A). The ratio of the compressive stress to tensile stress increases steeply as the rim thickness becomes thinner, and it gets over 2.5 in the case of the internal gear of $t=2m$ fixed by 3 bolts as shown in Fig.8. The reason for this phenomenon is thought to be as follows: as the rim thickness becomes thinner the deformation of the rim increases and the compressive stress component by rim bending increases at the fillet of the loaded tooth. The stress value at the tension side of a tooth, therefore, hardly increases because the compressive stress due to rim bending and tensile stress due to tooth bending cancel each other; on the contrary, the stress at the compression side of a tooth increases sharply because the rim bending and the tooth bending stress components are added together. This fact is confirmed also by the results of the photoelasticity experiment in Fig.6. When the rim thickness t becomes over $5m$ both the tensile and compressive tooth fillet stresses take almost constant values without any influence of the rim thickness (cf. Figs.7 and 8).

3.2 The position of maximum stress

Figs.9, 10, and 11 show the FEM calculated principal stress distributions around the tooth fillets and the bottoms of tooth spaces near the loaded tooth under constraint conditions of Fig.2 (A), (B) and (C). The normal load $Pn/b=9.8$ N/mm was applied on the tip of the tooth which lies at the center of the rim sections corresponding to 1/3 of the span of

the internal gear. The signs + and - represent tensile and compressive stresses respectively. From these stress distributions it is recognized that the maximum stress at the tooth fillet appears at a position nearer to the tooth bottom unlike with the external gear tooth. Fig.12 shows the tooth fillet stress distribution more exactly: the abscissa shows the position at the tooth fillet by the angle between the tangent line to the fillet curve and the tooth center line. When the rim is thick enough the maximum tensile and compressive stresses appear at the position of 45° tangent angle, but as the rim thickness becomes thinner the position of the maximum stresses moves toward the bottom of the tooth space. It is recognized, therefore, that the position of the maximum stress on the loaded tooth of internal gear appears at a place nearer to the tooth bottom than the position in the external gear tooth.

Fig.13 shows the measured and FEM calculated tooth fillet stresses at a 45° tangent angle position, in the case when the internal gear is fixed with 3 bolts (condition (B)) and the normal load $Pn/b=122.5$ N/mm is applied on the tooth tip. When the rim thickness is as thin as $t=2m$, the tensile stress at the adjacent tooth fillet (1b, 1a of Fig.3(a)) is greater than that at the loaded tooth (0a of Fig.3(a)). This result implies the possibility that the maximum tensile stress appears at another distant place from the loaded tooth when the rim thickness becomes thin enough.

In order to make clear the stress distribution over a wider range of the internal gear rims and teeth, a photoelasticity experiment was carried out over

Fig.12. Stress intensity at fillet and bottom of loaded tooth.

1/3 of the span of the internal gear and the results are shown in Fig.14. When the rim is thick enough the maximum tensile stress appears at the fillet of the loaded tooth, but as the rim thickness becomes thinner a large tensile stress appears at bottom of a tooth space apart from the fillet of loaded tooth. To prove this result quantitatively, stresses over successive bottoms of tooth spaces were measured by strain gages under the constraint condition (B) of the internal gear. The result is shown in

(a) Experimental

(b) Calculated

Fig.13. Comparison of measured and calculated values of fillet stresses of a loaded and its adjacent tooth.

Fig.14. Stress distributions in the internal gear.

Fig.15. The abscissa takes the position of teeth in the same manner as the case in Fig.5, and the ordinate takes stress intensity at the center of the bottom of the tooth space. When the rim thickness becomes thinner than $t=3m$ the maximum tensile stress appears at the bottom of the tooth space apart from the loaded tooth the same as the results obtained from the photoelasticity experiment.

When gearing is in operation, the tooth meshing alternates between a single tooth pair meshing and a double tooth

pair meshing, and it is necessary to confirm the results mentioned above not only for a case of single meshing but also for a case of double meshing. Fig.16 shows the comparisons between measured stress distributions at the bottoms of tooth spaces under the single meshing and under the double meshing. At the single mesh, the load is applied on a tooth tip and at the double mesh, the load is applied on the tooth tip and the adjacent tooth dedendum. The stress intensity around the loaded tooth is different between the case of a single mesh and that of a double mesh, but the qualitative status of stress distribution, that the maximum tensile stress appears at a bottom of a tooth space apart from the loaded tooth, is the same for both cases of a single mesh and a double mesh.

It is considered, therefore, that the maximum tensile stress in the internal gear appears at the fillet of a loaded tooth when the rim is thick enough, but as the rim thickness becomes thinner the position of maximum tensile stress moves toward the bottom of a loaded tooth and then to the center of the bottom of the tooth space apart from a loaded tooth. As the rim thickness becomes thinner, a fairly large tensile

stress appears near the pitch point of a loaded tooth and also around the peripheral part of a rim of a loaded tooth as Hidaka et al. mentioned⁴⁾, but the absolute values of stresses that appear at these places are smaller than the stress value at a center of the bottom of a tooth space apart from the loaded tooth.

Fig. 15. Measured values of stresses at the center of the bottom of tooth spaces (in the case of (B)).

Fig. 16. Comparison of tooth bottom stresses between a single mesh and a double mesh (measured values by strain gages for constraint condition (B)).

Fig. 17. Alternation of tooth fillet stresses when meshing proceeds.

To consider the bending strength of the internal gear it would be therefore enough to check every tooth fillet and bottom stresses with consideration of the rim deformation.

On the other hand, as seen from the results of the photoelasticity experiment (Fig. 14), the position of the maximum compressive stress moves from tooth fillet to tooth bottom of the loaded tooth as the rim thickness becomes thinner, but the position will not move to the bottom of a tooth space apart from a loaded tooth.

3.3 Stress alternation at tooth fillet and bottom

As recognized from Figs. 9-11, Fig. 13 and Fig. 15, when the rim is thick enough a small compressive stress and a small tensile stress appear at the fillet of the two teeth to the left and right of the loaded tooth. As the rim thickness becomes thinner, these compressive and tensile stresses become large. This means that when a definite position of tooth fillet and/or tooth bottom of a driven internal gear, for example, is taken into consideration, the tensile stress appears there before meshing of this tooth and the compressive stress appears after meshing, and these tensile and compressive stresses act alternately. Fig. 17 shows the calculated fillet stress alternation at point 'a' and at point 'b' of No. 0 tooth, in the case when the load moves from No. -2 to No. +2 tooth, and the fillet stress value at 45° tangent position is employed and the fixing condition of the internal gear is (B), i.e., fixed by 3 bolts. The amplitude of the tooth fillet and bottom stress alternation is intensified as the rim thickness becomes thinner, and partial alternate stresses and full alternate stresses act on the internal gear tooth in the cases of thick and thin rim gears respectively. This result must be taken into account to consider the fatigue strength of a thin-rimmed internal gear.

3.4 The influence of supporting methods of internal gear on tooth fillet stresses

Today, according to internal gear application some different kinds of methods are employed to support the internal gear. To clarify the influences of supporting methods on the tooth fillet stresses, 45° tangent fillet stresses of a loaded tooth under different supporting methods of the internal gear are compared in Fig. 18. The supporting conditions (A), (B) and (C) are shown in Fig. 2. As the rim thickness becomes thinner the tensile stresses of the loaded tooth fillet increase a little in

the case of condition (A), but hardly increase in the case of (B). The compressive stresses in both cases of (B) and (C) increase sharply with thinning of rim thickness. In the case of (C), as the rim thickness becomes thinner, the tensile stress tends to decrease and the compressive stress increases very steeply. It is a general tendency that the tensile stresses decrease and the compressive stresses increase at the tooth fillet of a loaded tooth in thin-rimmed internal gears unlike with thick-rimmed internal gears as the constraint condition of the internal gear support becomes softer, for instance, as the number of fixing points decreases, i.e., condition (A) changes to (C).

4. Conclusions

The main conclusions are as follows:

(1) The maximum tensile and compressive stresses of the internal gear with a thick rim appear at the tooth fillet with a 45° tangent angle of a loaded tooth.

(2) As the rim thickness of the internal gear becomes thinner, the position of the maximum tensile stress moves from the tooth fillet with a 45° tangent angle to the bottom of a loaded tooth, and then, moves to a bottom of a tooth space apart from the loaded tooth. On the other hand, the position of the maximum compressive stress moves only from the fillet with a 45° tangent angle to the bottom of the loaded tooth.

(3) As the rim thickness of the internal gear becomes thinner, the stresses at the loaded tooth fillet hardly increase in tension side but increase sharply in compression side.

(4) With the progress of meshing, a strongly alternating stress acts on the tooth fillet and bottom of a thin-rimmed internal gear.

(5) The stress at a tooth fillet and bottom of a thin-rimmed internal gear is influenced strongly by the supporting method of the internal gear. There is a general tendency that the tensile stress decreases and the compressive stress increases with thinning of rim thickness at a tooth fillet and bottom of a loaded tooth as the constraint conditions of the internal gear support become softer.

Acknowledgement

The authors appreciate Dr. A.Kubo,

Fig.18. Relationship of rim thickness and the maximum fillet stress of loaded tooth under some different supporting methods of internal gear (values by FEM calculation).

Associate Professor at Kyoto University for his kind instruction and advice in working out this research.

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