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Analysis of a Jet Attaching to an Offset Parallel Plate*

(2nd Report, Influence of an Opposite Wall)

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One of the authors reported on the analysis for the oscillation of a jet attaching to an offset parallel plate, caused by the infinitesimal periodic fluctuation of bubble pressure.

This paper presents the influence of the geometry of flow passage, especially an opposite wall, on a jet attaching to the side wall and switching phenomenon of the jet caused by traversing the opposite wall quasi-statically.

Some of the conclusions are as follows.

- (1) Flow patterns can be classified into two types depending on the geometry of flow passage.
- (2) An analysis for an attached jet which is influenced by an opposite wall was carried out. The analytical results are in good agreement with the experimental ones.
- (3) Attachment and detachment of the jet show a hysteresis phenomenon when the offset distance of the side wall is varied quasi-statically.

2. Experimental apparatus and procedure

1. Introduction

A study of the flow characteristics of a turbulent jet blowing off parallel to the wall boundary is very important in order to clarify the flow conditions in fluidics, a sudden expansion path of various equipment or a combustion furnace. Many studies ^{1) ~ 9)} have been made on the behavior of this kind of a jet that attaches to a side wall by Coanda effect. Especially, Borque, Newman ²⁾ and Sawyer ³⁾ gave the effective method of the analysis on this flow of a steady, two-dimensional, incompressible, turbulent attached jet under some assumptions.

One of the authors has also reported on the oscillating characteristics of an attached jet caused by the infinitesimal periodic fluctuation of a bubble pressure. ¹⁰⁾ But the behavior of a jet in the flow passage between side wall and opposite one is not made clear enough whereas its configuration is used in a bistable wall reattachment amplifier, etc.

In this paper, the influences of an opposite wall and configuration of flow passage on flow characteristics of an attached jet are made clear and the switching phenomenon of an attached jet caused by traverse of an opposite wall, so-called wall attachment switch ¹¹⁾, is also discussed.

Fig.1 shows the experimental apparatus. Air supplied by the blower is blasted off the rectangular nozzle ① in to the test section through the flow meter, settling chamber and straightener. The test section used is in two types of which width of nozzle, B, is 4.0 mm (aspect ratio : AR = 10) ; lengths of the side and opposite walls, L, are 340 mm, and similarly B = 8.0 mm (AR = 5), L = 120 mm. The test section consists of the side and opposite walls ②, ③, upper plate ⑦ (for static pressure measurements) or ⑧ (for velocity measurements) and lower plate ⑩. In this apparatus, the offset distances, D₁ and D₂ and angle of inclination α' of the side and opposite walls can be set arbitrarily, and the flow pattern can be visualized through the transparent acrylic walls.

Static pressures are measured through static pressure taps ⑤ with diameter 1.0 mm and located at intervals of 5 mm along the center line on the upper plate ⑦.

Velocities on the horizontal plane at the middle depth are measured by a yaw meter, 1.0 mm in diameter, with 0.3 mm hole. The yaw meter is inserted through 1.1 mm holes, which are distributed 5 mm apart along the center line on the upper plate ⑧. These plate ⑦, ⑧ can be traversed in y direction, therefore the static pressure and velocity at any point of y coordinate can be measured.

Visualization of flow was also tried by tuft and oil film method.

Experiments cover the range of $u_0 = 10 \sim 60$ m/s (u_0 : mean velocity at the nozzle exit), namely $Re = u_0 B / \nu \approx 2.5 \times 10^3 \sim 1.5 \times 10^4$ (where, Re : Reynolds number, ν : kinematic viscosity). The under-

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Fig.1 Experimental apparatus

Fig.2 Flow pattern

mentioned results are for the case of $u_0 = 30 \text{ m/s}$ ($Re \approx 7.5 \times 10^3$) except the visualized experiments about the switching process of an attached jet because we can not recognize the difference of flow patterns and characteristics in the aforesaid range.

To study the switching process of an attached jet caused by traversed of the opposite wall, the flow patterns are visualized and photographed using glycerine solution ($\nu = 1.34 \times 10^{-5} \text{ m}^2/\text{s}$ at 15°C) and aluminium powder as the tracer in flow path $B = 8.0 \text{ mm}$, $L = 120 \text{ mm}$.

3. Effects of opposite wall

3.1 Flow patterns

When one wall exists near the nozzle, a jet attaches to the wall in stable state by Coanda effect. But when two walls, side and opposite, exist near the nozzle exit, two kinds of flow patterns appear depending on the width of nozzle, flow passage, D , and length or angle of inclination, α' , of the walls.

Some examples are shown in Fig.2. Numbers in these figures indicate the pressure coefficients $C_p [= (p - p_\infty) / (1/2) u_0^2]$, where, p : pressure at any position, p_∞ : ambient pressure, ρ : density]. The arrows indicate the velocity distribution near the exit of flow passage.

(a) Flow pattern (1): A jet is not influenced very much by the opposite wall (3) and it attaches to the side wall. A distinguishable bubble region (1 in the figures) and inflow from the exit of the flow passage occur [see Fig.2(a)].

(b) Flow pattern (2): The outer edge of a jet attaches to the opposite wall and two distinguishable bubble regions, first and second ones, are formed.

A flow pattern map is shown in Fig.3 by plotting D_1/B against D_2/B . The solid line in the figure indicates the approximate bound of regions in which the flow pattern (1) or (2) occurs. Near the boundary line, either of the two occurs by disturbing the flow [see Fig.2(a), (b)].

3.2 Pressure distribution

The pressure distributions in the flow passage of $D_2 = 40 \text{ mm}$, $D_1 = 80 \text{ mm}$ and those on the side and opposite walls are shown in Figs.2(a), (b) and 4, respectively, for both flow patterns of (1) and (2). We recognize that the pressure in the flow passage for the flow pattern (2), without the inflow from the exit, decreases further than (1) in spite of the same configura-

Fig.3 Flow pattern (1), (2)

tion of the flow passage. The two positions of the minimum pressures on the side wall in x-direction coincide with those of the centers of the first and second bubble regions. Therefore these figures predict that the position of the first bubble is independent of the offset distance of an opposite wall, D_1 , the position of second bubble approaches the nozzle exit with a decreasing D_1 or the width of flow passage, D , and that the pressure on the wall decreases with a decrease of D_1 or D .

Fig.5 shows the relation between D_1 and the minimum pressure, on the side and opposite walls, p_1' and p_2' , for $B = 8.0$ mm,

Fig.4 Pressure distribution
($B = 4$ mm, $L/B = 85$)

Fig.5 Bubble pressure
($\alpha' = 0$)

$L = 120$ mm. The solid lines in the figure show the pressures of the first and second bubbles, p_1 and p_2 , which are calculated later. These pressures decrease as D_2 and D_1 decrease, but the pressure difference across the jet, $(p_2 - p_1)$ or $(p_2 - p_1)$, is constant and independent of D_1 if D_2 keeps a constant distance. Those values are $C_p \approx -0.11, -0.09$ and -0.08 for $D_2/B = 2, 4$ and 6 , respectively, and decrease with an increase of D_2 .

3.3 Reattachment distance

Fig.6 represents the first and second reattachment distances, X_R and X_R' , for the flow patterns (1) and (2). X_R is not affected by the opposite wall, it agrees well with Sawyer's results²⁾ for the case of the opposite wall lacking ($D_1 = \infty$). X_R' increases in proportion to the width of the flow passage, D , and can be expressed as follows.

$$X_R'/B \approx 5.0 D/B \dots\dots\dots(a)$$

Vertical arrows in the figure indicate the experimental results by Aoki and Ando⁶⁾ for $B = 20$ mm ($AR = 8$), $L = 2000$ mm, $Re = 2.8 \times 10^4$. These results are also approximately expressed by Eq.(a).

Abott and Cline⁷⁾ carried out detailed measurements of the reattachment distance in an open channel having equal but various offset distances of a side and opposite walls. They have obtained similar results to those mentioned above, namely the reattachment distance increases with an increase of the offset distance.

Nakayama and others⁸⁾ also showed that the reattachment distance increased with an increase of Reynolds number for the region of relatively low Reynolds number ($Re \leq 2 \times 10^3$).

Fig.7 represents the relation between X_R and D_1 for $B = 8.0$ mm, $L = 120$ mm and

Fig.6 Reattachment distance
($B = 4$ mm, $L/B = 85$)

$D_2/B = 2$. The solid lines in the figure represent the theoretical results. We know that X_R increases as the angle of inclination of a side and opposite walls, α' , increases, but it has a constant value which is independent of D_1 if D_2 and α' are fixed. These constant values are equal to those of the reattachment distances without an opposite wall. The reason why the first reattachment distance, X_R , is not influenced by an opposite wall may be as follows: there exists equilibrium state between the pressure difference across the jet and the jet momentum [Eq.(21)] and further the pressure difference holds a constant value independently of the opposite wall if D_2 and α' are fixed. The calculated results agree well with the experimental ones.

Fig.8 represents the relation between X_R and α' for $D_1 = D_2$. This result is similar to that of the case without an opposite wall. Solid lines in the figure indicate the results of calculation which almost agree with the experimental ones.

3.4 Velocity distribution

Figs.9 and 10(a) show the velocity distributions in attached jets without and with an opposite wall, respectively. The parameter s in these figures is the dis-

Fig.7 Reattachment distance

Fig.8 $X_R/B - \alpha'$

tance from the nozzle exit along the center line of a jet. The profiles of the distribution without an opposite wall exhibit a considerable asymmetry, but they are almost symmetrical when an opposite wall exists.

Fig.10(b) represents the velocity distributions in the presence of an opposite wall by the dimensionless quantities, u/u_{max} and $y/\bar{y}$. Where, u_{max} means the maximum velocity of a jet profile and $\bar{y}$ is the value of y corresponding to $u_{max}/2$. The solid line in the figure is Goertler's velocity profile for a two-dimensional free jet and shows good agreement with the experimental results. Similar results for $D_1 = 32$ mm, 16 mm and $D_2 = 24$ mm are obtained, too.

Fig.9 Velocity distribution of an attached jet ($L/B = 15$, $D_1 = \infty$)

(a) $u - y$

(b) $U/U_{max} - y/y(U_{max}/2)$

Fig.10 Velocity distribution of an attached jet ($L/B = 15$)

Fig.11 shows the velocity distribution of an attached jet at the exit of the flow passage for various values of D_1 at $D_2 = 16$ mm. The maximum velocity decreases with a decrease of D_1 and the profile reduces in similar form. Consequently, the inflow from the exit decreases in proportion to the decrease of D_1 . Fig.11(b) is a redescription of (a) using the dimensionless quantities, $y/\bar{y}$ and $u/u_{\max}$. Those profiles are expressed by a single curve regardless of D_1 and show good agreement with the theoretical result by Glauert¹²⁾ for a wall jet. Thus, jet profile at the exit after reattachment shows the same form as a wall jet.

3.5 Approximate theory and comparison with experimental results

Borque¹⁾, Sawyer²⁾ and others have made theoretical analysis using the momentum theory for a two-dimensional attached jet without an opposite wall.

In this section, an approximate analysis for the flow model shown in Fig.12 involving the influence of an opposite wall to obtain a reattachment distance, bubble pressure, etc. is presented.

Following assumptions are made :

- (1) The jet flow is incompressible and two dimensional.
- (2) The pressure within the bubble region is uniform.
- (3) The center line of the jet is a circular arc of radius R .
- (4) The force due to skin friction within a control volume, $W'X'Y'Z'$ in Fig.12, is negligible.
- (5) The velocity distribution parallel to the axis of an attached jet is given by Goertler's velocity profile for a two-dimensional free jet.

Fig.11 Velocity distribution at the exit

$$u = \frac{\sqrt{3}}{2} \sqrt{\frac{\sigma J}{\rho x}} \operatorname{sech}^2 \frac{\sigma y}{x} \quad (x=s+s_0) \dots\dots(1)$$

where, J is jet momentum per unit span of nozzle, ρ density of working fluid, s_0 the distance from the nozzle exit to the hypothetical origin of a jet and y the distance measured perpendicular to the jet axis. σ is a spread parameter and has a constant value of 7.67 for free turbulent jet taken by Reichardt.

(a) Flow rate : The volume flow rate between the center line of jet ($y = 0$) and the reattaching stream line ($y = -y_1$) is expressed as follows.

$$\int_{-y_1}^0 u dy = \{3J(s+s_0)/(4\rho\sigma)\}^{1/2} T_1 \dots\dots(2)$$

where,

$$T_1 = \tanh\{\sigma y_1/(s+s_0)\} \dots\dots\dots(3)$$

This volume flow rate equal to $(JB/\rho)^{1/2}/2$. Thus the equation of the stream line is

$$T_1 = \{\sigma B/3(s+s_0)\}^{1/2} \dots\dots\dots(4)$$

where,

$$s = R\theta \dots\dots\dots(5)$$

$$s_0 = \sigma B/3 \dots\dots\dots(6)$$

θ is the angle which envelopes the center of curvature of a jet. Thus,

$$y_1 = \{(3R\theta + \sigma B)/3\sigma\} \tanh^{-1} T_1 \dots\dots\dots(7)$$

(b) Geometrical relationship : The following geometrical relationships are obtained from Fig.12.

$$D_2 + \frac{B}{2} = R - (R - y_{10})(\cos \theta_0 + \sin \theta_0 \tan \alpha') \dots\dots\dots(8)$$

Fig.12 Mathematical model of an attached jet

where, y_{10} is the value of y at $s = R\theta_0$ and θ_0 is the angle at which the reattaching stream line impinges on the side wall. Substituting relations (3), (6) and (8) in Eq.(7) gives the curvature of an attached jet, R .

$$R = \frac{D_2 + (B/2) - (B/3)(\tanh^{-1} T_{10})(\cos \theta_0 + \sin \theta_0 \tan \alpha')}{1 - (\cos \theta_0 + \sin \theta_0 \tan \alpha') [1 - (\theta_0/\sigma) \tanh^{-1} T_{10}]} \quad (9)$$

where, T_{10} is the value of T_1 at $y = y_{10}$. Reattachment distance, X_R , is given as follows.

$$X_R = (R - y_{10}) \sin \theta_0 / \cos \alpha' \quad (10)$$

(c) Momentum : The momentum principle applies to the control volume $W'X'Y'Z'$ in Fig.12. Z' is an intersecting point of the center line of jet and the side wall. The momentum equation in x direction is given as follows.

$$J - (J_1 - J_3) \cos \alpha' = P_1 \left(D_1 + \frac{B}{2} \right) - P_2 \left(D_2 + \frac{B}{2} \right) \quad (11)$$

J_1 is the flow momentum proceeding downstream along the side wall and J_3 is the flow momentum reentering the flow passage along the opposite wall.

$$J_1 = \int_0^\infty \rho u^2 dy + \int_{-y_{10}}^0 \rho u^2 dy = \frac{J}{2} + \frac{J}{4} (3T_{10} - T_{10}^3) \quad (12)$$

$$J_3 = \rho b (Q_3/b)^2 \quad (13)$$

where, Q_3 is the volume flow rate of reentering into the flow passage along the opposite wall, but a flowout takes place again from the exit after entanglement in a jet. Q_3 is given approximately as follows.

$$Q_3 = \int_{y_{10}}^\infty u dy = \{3J(s + s_0)/(4\rho\sigma)\}^{1/2} (1 - T_{10}) \quad (14)$$

where,

$$s + s_0 = \sigma B / (3T_{10}^2) \quad (15)$$

b is the width which Q_3 traverses and is given approximately as follows.

$$b = R + D_1 + \frac{B}{2} + (R + y_1') \times (\sin \theta_0 \tan \alpha' - \cos \theta_0) \quad (16)$$

y_1' is the distance between center line at $s = R\theta_0$ and outer edge (the position at $u/u_{\max} = 0.1$) of a jet.

$$y_1' = \{B/(3T_{10}^2)\} \tanh^{-1} \sqrt{0.9} \quad (17)$$

(d) Bubble pressures, p_1, p_2 : The pressure in the first bubble region, p_1 , is given by Bernoulli's equation as follows.

$$P_1 - P_\infty = -\rho v_3^2 / 2 \quad (18)$$

where, v_3 is the mean flow velocity of entering through the width of b of $Y'Z'$ -plane, and expressed by

$$v_3 = Q_3 / b \quad (19)$$

Assuming $p_\infty = 0$,

$$P_1 = -JB(1 - T_{10}^2)/(8b^2 T_{10}^2) \quad (20)$$

The pressure difference across the jet, $(p_1 - p_2)$, exists so equilibrium with the momentum of jet holds as follows :

$$P_1 - P_2 = J/R \quad (21)$$

Thus, p_2 is derived as

$$P_2 = \frac{-JB}{8b^2 T_{10}^2} (1 - T_{10}^2) - \frac{J}{R} \quad (22)$$

Now, the curvature of an attached jet R , reattachment distance X_R , the position of the reattaching stream line, the bubble pressures p_1, p_2 can be determined as a function of D_1, D_2 and α' . The theoretical results in the figures above mentioned are analytical ones for the flow pattern (1). They show good agreement with experimental results. Further, if the value of the spread parameter, σ , is suitably chosen, similar analysis for the flow pattern (2) will be possible.

4. Influence of the traverse of an opposite wall

4.1 Hysteresis for attachment

As published previously¹¹⁾, when the opposite wall (I) traverses quasi-statically from far up to downward, an attached jet switches twice from one side to the other (see Fig.13). And let their offset distances of an opposite wall, D_1 , be $D_{\min 1}$ and $D_{\min 2}$, respectively. Then $D_{\min 2}$ takes a negative value because the end of the opposite wall locates below the nozzle tip. And then, when the opposite wall moves in the reverse direction, the jet switches from the state (2) to (1) in the figure. Let their value of D_1 be $D_{\max 2}$ and $D_{\max 1}$, respectively, and $\Delta X_1 = (D_{\max 1} - D_{\min 1})$, $\Delta X_2 = (D_{\max 2} - D_{\min 2})$. There occurs a hysteresis, in this manner in the range of D_1 of the attachment of jet and this phenomenon is influenced by the offset distances, D_1 and D_2 , the angle of inclination, α' , and the width of the control port, D_c .

Fig.14 shows the relation between the hysteresis range of attachment, ΔX_1 , and α', D_2 . Both $D_{\max 1}$ and $D_{\min 1}$ increase in proportion to D_2 and also with α' . In this case, ΔX_2 does not appear.

Fig.15 represents the change of the mean pressure in the bubble region located on the side wall during ΔX_1 for $D_c = 0$ mm and $D_2 = 8$ mm. Arrows on the curved lines in the figure indicate the traversing direction of the opposite wall. Vertical arrows indicate the switching of attachment;

for example, downward arrow represents switching from the side wall to the opposite one.

The mean pressure decreases as α' and D_1 decrease, but the pressure difference across the jet (Δp in the figure) is held constant for any α' independently of D_1 just before the switching occurs.

Fig.16 shows the relation between ΔX_1 , ΔX_1 and D_2 for $\alpha' = 10^\circ$. ΔX_1 increases with an increase of D_2 and makes an abrupt change at a certain D_2 , and it takes a

Fig.13 Flow patterns of wall attachment switch ¹¹⁾

Fig.14 $\Delta X_1 - D_2/B$

Fig.15 Bubble pressure in the hysteresis region ΔX_1

larger value as D_c becomes larger. ΔX_2 takes a smaller value as D_c becomes larger.

4.2 Switching process of attachment

A visual observation of the flow was made at low Reynolds number in order to make clear the qualitative characteristics of the switching process caused by traverse of an opposite wall. The results are shown in Fig.17 by some photographs which were taken at every 5 seconds for $D_2 = 16$ mm, $L = 120$ mm and $Re = 75$. Glycerin solution and aluminium powder are used as the working fluid and the tracer. In this case, the value of D_{min1} at which switching occurs is 14.5 mm. This value was determined by preliminary experiments. Two distinguishable bubble regions at the both sides of a jet are developed and they move up- and downstream as the time proceeds. Thus, the attachment of a jet switches from the side wall to the opposite one.

Fig.18 shows changes in the reattachment distance X_R , the area of the first bubble region A, the curvature of the center line of a jet R against the time t . Curves in the figure make an almost exponential change as the time proceeds. This result is for low Reynolds number, but the switching process for relatively high Reynolds number in the previous section may be clarified qualitatively by this explanation.

5. Conclusions

The results obtained in this study are summarized as follows:

(1) Flow pattern of a jet from the nozzle with the side and opposite walls is classified into two kinds (1) and (2) depending on the nozzle width, the width of the flow passage and the length and the angle

Fig.16 $\Delta X_1, \Delta X_2 - D_2/B$

of inclination of the two walls. As to the flow pattern (2), the inflow from the exit disappears, and the pressure in the flow passage further reduces than (1).

(2) The pressure in the flow passage decreases with a decrease in the offset distances of the side and opposite walls, but the pressure difference across the jet remains constant regardless of the offset distance of the opposite wall if the offset distance of the side wall, D_2 , is held constant. And it diminishes as D_2 becomes larger.

Fig.17 Switching process of an attached jet ($D_1 = 14.5$ mm, $D_2 = 16$ mm)

Fig.18 R, X_R , A - t

(3) The first reattachment distance increases with D_2 independently of D_1 . The second reattachment distance increases with the width of the flow passage, D , and the relation is given by

$$X_R'/B \approx 5.0 D/B$$

(4) The velocity profiles of an attached jet without an opposite wall indicate a considerable asymmetry, but they are almost symmetric when an opposite wall exists and agree well with Goertler's velocity profile. The velocity profiles at the exit of the flow passage are well expressed by Glauert's velocity profile for a wall jet. The maximum velocity decreases with a decrease of D_1 and the profile reduces in similar form. Consequently, the inflow from the exit decreases in proportion to the decrease of D_1 .

(5) The characteristics of the switching process caused by traversing of an opposite wall are made clear by visual observation. There occur hysteresis phenomena on the attachment of a jet and they are influenced by the offset distances, D_1 and D_2 , the angle of inclination, α' , the width of the control port. The hysteresis ranges ΔX_1 and ΔX_2 increase with an increase of D_2 . The mean pressure in the bubble region during ΔX_1 diminishes as α' and D_1 decrease, but the pressure difference across the jet remains constant independently of D_1 just before the switching occurs.

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