Influence of Compressibility of Oil on the Step

Response of a Hydraulic Servomechanism

3rd Report

The Response under Inertial Loading*

By Eizo URATA**

Step response of a hydraulic servomechanism under inertial loading is studied. Conclusions obtained from this study are summarised as follows: (1) If the hydraulic servomechanism is geometrically symmetric, free from external leakage, and if compressibility of oil is constant, then nondimensional pressures in cylinder chambers satisfy the relation $P_1 + P_2 = 1$. This relation is lost when a cavitation appears in one of the cylinder chambers. (2) The stability becomes inferior with smaller bulk modulus and larger inertial load. (3) The stability becomes superior with internal leakage and viscous and/or Coulomb friction. (4) Asymmetry $A_1 \neq A_2$ causes asymmetric response depending on input direction. (5) The step response is influenced significantly by air bubbles dispersed in the oil. Effective compressibility of the oil is reduced by the air bubbles. The form of basic equations used today to study the hydraulic servomechanism is conserved in spite of presence of the air bubbles if the tangential bulk modulus is used.

1. Introduction

The response of the hydraulic servomechanism has a wide range of its appearances when an inertial load is applied. Some phenomena of different natures, such as instability of the system and cavitation, may be induced by the inertial loads. The influence of the inertial load is thus not limited to the form of the response.

Influence of frictional force, internal leakage flow and other physical factors must be reviewed when an inertial load is added to the hydraulic servomechanism with compressible fluid.

The same basic equations given in the first part of this study are used in this paper. Though the symbols are common throughout the first(1), second(2) and present parts, a list of basic symbols is carried below for convenience of readers.

Nomenclature

- a_2 : Parameter of viscous load
 a_s : Parameter of inertial load
 c_i : Coefficient of internal leakage
 P_1, P_2 : Nondimensional pressures in cylinder chambers
 P_c : Nondimensional cavitating pressure
 s : Asymmetry of cylinder areas
 $= (A_1 - A_2) / (A_1 + A_2)$

Y : Nondimensional displacement of cylinder

λ : Parameter of oil compressibility

τ : Nondimensional time t/T

ϕ : Volumetric ratio of air bubbles contained in oil under atmospheric pressure.

2. Basic response under inertial load

Basic response of a symmetric system satisfying conditions $A_1 = A_2$ and $V_{10} = V_{20}$ is studied in this chapter. When the piston moves, the relation $V_1 = V_2$ cannot be satisfied in a strict sense. Since the magnitude of the step input is sufficiently small in comparison with full stroke of the servocylinder, we can assume $V_1 = V_2$ in the step response.

If we assume no cavitation and no external leakage for such a symmetric system, we can prove the following relation;

$$P_1 + P_2 = 1 \dots\dots\dots (1)$$

In this case only three linearly independent variables are required to express the dynamics of the system, or in other words, the system is expressed by a 3rd order differential equation. A proof is given in the following.

It is mathematically sufficient if the proof is given for $x > 0$. From Eqs. (15), (16), (19) and (20) in the first report, we have

$$\frac{v}{\lambda} \frac{d(P_1 + P_2)}{d\tau} = \sqrt{2}(1-Y)(\sqrt{1-P_1} - \sqrt{P_2}) \dots\dots\dots (2)$$

Then we have the following equations for an arbitrary positive integer n .

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** Associate Professor, College of Engineering, Kanagawa University, Rokkakubashi 3-27, Kanagawa-ku, Yokohama, 221 Japan.

$$\frac{d^n}{d\tau^n}(\sqrt{1-P_1}) = \frac{d^n}{d\tau^n}(\sqrt{\zeta_{n+1-P_1}}) = f\left(\frac{d^n \zeta}{d\tau^n}, \frac{d^{n-1} \zeta}{d\tau^{n-1}}, \dots, \zeta\right) \dots\dots\dots (3)$$

$$\frac{d^n}{d\tau^n}(\sqrt{P_2}) = \frac{d^n}{d\tau^n}(\sqrt{\zeta_{n+P_2}}) = f\left(\frac{d^n \zeta}{d\tau^n}, \frac{d^{n-1} \zeta}{d\tau^{n-1}}, \dots, \zeta\right) \dots\dots\dots (4)$$

If the following relations hold for all N in $N \leq n$,

$$\left. \frac{d^N P_1}{d\tau^N} \right|_{\tau=0} = - \left. \frac{d^N P_2}{d\tau^N} \right|_{\tau=0}, \quad P_2|_{\tau=0} = 1 - P_1|_{\tau=0} \dots\dots\dots (5)$$

then we have

$$\left. \frac{d^N}{d\tau^N}(\sqrt{1-P_1}) \right|_{\tau=0} = \left. \frac{d^N}{d\tau^N}(\sqrt{P_2}) \right|_{\tau=0} \dots\dots\dots (6)$$

By differentiation of Eq.(2), we have

$$\begin{aligned} \frac{v}{\sqrt{2}\lambda} \frac{d^{n+1}(P_1+P_2)}{d\tau^{n+1}} &= (\sqrt{1-P_1} - \sqrt{P_2}) \frac{d^n Y}{d\tau^n} + \dots + \left(\frac{n}{i}\right) \frac{d_i}{d\tau^i} (\sqrt{1-P_1} - \sqrt{P_2}) \frac{d^{n-i} Y}{d\tau^{n-i}} + \dots \\ &+ n \frac{d^{n-1}}{d\tau^{n-1}} (\sqrt{1-P_1} - \sqrt{P_2}) \frac{dY}{d\tau} + \frac{d^n}{d\tau^n} (\sqrt{1-P_1} - \sqrt{P_2}) (Y-1) \dots\dots\dots (7) \end{aligned}$$

Now, if Eq.(5) holds when $N \leq n$, then both sides of Eq.(7) vanish. Hence Eq.(6) must hold also when $N=n+1$. The left side of Eq.(2) is zero when $P_1+P_2=1$, namely Eq.(5) holds when $N=1$. Then (by the principle of mathematical induction) Eq.(5) must hold for each positive integer N . Hence if any derivatives of equation for the system have no discontinuity (e.g. Cavitation), Eq.(1) holds at all times.

Figure 1 shows displacement responses at different compressibility parameters. Figures 2 and 3 show both pressures P_1 and P_2 with and without cavitation respectively. We can observe from the pictures that the relation $P_1+P_2=1$ is lost with generation of the cavitation. Under the effect of the Coulomb friction in this mathematical model, pressures P_1 and P_2 are unequal for $\tau \rightarrow \infty$. When the response of a hydraulic servomechanism is stable, the effect of compressibility of oil diminishes for sufficiently large τ (since $dP_1/d\tau \rightarrow 0$, $dP_2/d\tau \rightarrow 0$). Hence in this state the system behaves like an incompressible fluid system. Final values of pressures are determined by

$$|P_1 - P_2| = P_m, \quad F_s/A < P_m < F_c/A, \quad P_1 + P_2 = 1 \dots\dots\dots (8)$$

The differential pressure P_m is determined by a calculation of the step response.

An oscillatory response is obtained when the compressibility parameter λ becomes small. A further decrease of λ induces in the system an instability. Other parameters which affect the stability are the inertia parameter a_5 , the Coulomb friction, the internal leakage coefficient c_l and the cylinder velocity at valve closing.

3. Effects of Coulomb friction and leakage flow

The mathematical model of the Coulomb friction in this report is given by

$$F_c = F_d + \frac{F_s - F_d}{1 + (k_c dy/dt)^2} \dots\dots\dots (9)$$

where F_d is dynamic friction, F_s is static friction and k_c shall be determined experimentally from a relation between F_c

and velocity. The following nondimensional parameters are used in numerical calculations.

$$\left. \begin{aligned} \frac{F_c}{Ap_s} &= f_c, & \frac{F_s}{Ap_s} &= f_s, \\ \frac{F_d}{Ap_s} &= f_d, & \frac{k_c z_0}{T} &= C_0 \end{aligned} \right\} \dots\dots\dots (10)$$

Systems with inertial loading without the Coulomb friction and viscous resistance become unstable. This fact will be explained later. An offset occurs when both the Coulomb friction and leakage flow exist. The numerical examples shown in Fig.4 reveal that the Coulomb friction is effective to obtain stability. Introduction of the Coulomb friction can only allowed technically when the leakage flow is sufficiently small.

Now let us find stability condition with regard to the equilibrium point of the system. The load of the system is assumed as

$$F = M \frac{d^2 y}{dt^2} + c_f \frac{dy}{dt} \dots\dots\dots (11)$$

The variational equations do not change if the load proportional to the square of velocity is ignored. Moreover the Coulomb friction acts to stabilize the system. Hence our estimate on the stability will be practically done on the safe side. Equation (11) is transformed into the following nondimensional set of equations.

Fig.1 Displacement response of symmetric system

$$\frac{dY}{d\tau} = u \dots\dots\dots(12)$$

$$\frac{du}{d\tau} = \frac{P_1 - P_2 - a_2 u}{a_5} \dots\dots\dots(13)$$

If the piston repeats small movement around the equilibrium point, we can assume

$$P_{1\infty} = P_{2\infty} = 1/2, u_{\infty} = 0, Y_{\infty} = 1 \dots\dots\dots(14)$$

The variational equation around the equilibrium point are

$$d(\Delta Y)/d\tau = \Delta u \dots\dots\dots(15)$$

$$d(\Delta u)/d\tau = \{-a_2 \Delta u + (\Delta P_1 - \Delta P_2)\}/a_5 \dots\dots\dots(16)$$

$$d(\Delta P_1)/d\tau = \{-\Delta Y - \Delta u - (c_1 + e_1) \Delta P_1 + c_1 \Delta P_2\} \lambda / v_1 \dots\dots\dots(17)$$

$$d(\Delta P_2)/d\tau = \{\Delta Y + \Delta u + c_1 \Delta P_1 - (c_1 + e_2) \Delta P_2\} \lambda / v_1 \dots\dots\dots(18)$$

Hence the characteristic equation becomes

$$s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4 = 0 \dots\dots\dots(19)$$

$$\left. \begin{aligned} b_1 &= (v_1 v_2 a_2 + a_5 \lambda (e_1 v_2 + e_2 v_1 + c_1)) / a_5 v_1 v_2 \\ b_2 &= [\lambda \{1 + a_2 (e_1 v_2 + e_2 v_1 + c_1)\} + a_5 \lambda^2 \{e_1 e_2 + c_1 (e_1 + e_2)\}] / a_5 v_1 v_2 \\ b_3 &= [\lambda + \lambda^2 (e_1 + e_2) + \lambda^2 a_2 \{e_1 e_2 + c_1 (e_1 + e_2)\}] / a_5 v_1 v_2 \\ b_4 &= \lambda^2 (e_1 + e_2) / a_5 v_1 v_2 \end{aligned} \right\} \dots\dots\dots(20)$$

A general stability condition of this equilibrium point is

$$b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 > 0 \dots\dots\dots(21)$$

An important case is given at $e_1 = e_2$. The system becomes 3rd order in this case. A few important examples are given in the following.

(a) $e_1 = e_2 = c_1 = a_2 = 0$; unstable

(b) $e_1 = e_2 = c_1 = 0, a_2 \neq 0$; stable if $a_2/a_5 - 1 > 0$, namely stable if the following inequality holds.

$$c_f A / (M c w \sqrt{p_s / \rho}) > 1 \dots\dots\dots(22)$$

(c) $e_1 = e_2 = 0, c_1 \neq 0, a_2 = 0$; stable if $\lambda c_1 > v_1 v_2$. Since $v_1 + v_2 = 1$, then $v_1 v_2 \leq 1/4$. Using this relation, we have stability condition for arbitrary position of the piston;

$$4 \{c_1 K A / (V_1 + V_2)\} / (c w \sqrt{p_s / \rho}) > 1 \dots\dots\dots(23)$$

(d) $e_1 = e_2 = 0, c_1 \neq 0, a_2 \neq 0$; stable if $(1 + a_2 c_1)(v_1 v_2 a_2 + \lambda a_5 c_1) - a_5 v_1 v_2 > 0$. This relation is rewritten as

$$\lambda c_1 a_5 (1 + a_2 c_1) > v_1 v_2 (a_5 - a_2 (1 + a_2 c_1)) \dots\dots\dots(24)$$

Fig.3 Pressure response of symmetric system ($\lambda=1$, other parameters are the same as Fig.1: with generation of cavity)

Fig.2 Pressure response of symmetric system ($\lambda=10$, other parameters are the same as Fig.1: no generation of cavity)

Fig.5 Influence of leakage on displacement response

Fig.4 System stabilization by the Coulomb friction

Fig.6 Influence of leakage on P_1 (parameters are the same as Fig.5)

Thus the system is stable when

$$a_5 \leq a_2(1+a_2c_1) \dots\dots\dots(25)$$

holds. If the inequality (25) does not hold, the system is stable if

$$\frac{\lambda c_1 a_2 (1+a_2 c_1)}{a_5 - a_2 (1+a_2 c_1)} > v_1 v_2 \dots\dots\dots(26)$$

holds. Using also here $v_1 v_2 \leq 1/4$, stability condition at an arbitrary position of the piston is,

$$(4\lambda c_1 + a_2/a_5)(1+a_2 c_1) > 1 \dots\dots\dots(27)$$

The inequality (27) is rewritten using physical parameters as follows;

$$\frac{A}{cw\sqrt{p_s/\rho}} \left(1 + \frac{c_f c_1}{A} \right) \left(\frac{4Kc_1}{V_1 + V_2} + \frac{c_f}{M} \right) > 1 \dots\dots\dots(28)$$

To discuss qualitative effect of the Coulomb friction, we introduce an equivalent linear resistance coefficient. To simplify the discussion, $C_0 \gg 1$ and $F_0 = F_d$ during movement of the piston are assumed. The equivalent linear resistance coefficient c_{fe} may give the same energy loss as the Coulomb friction. Then we take

$$c_{fe} = 4F_d/\pi v_m; (v_m: \text{velocity amplitude}) \dots\dots\dots(29)$$

From this relation we can see that the system comes to rest if v_m is smaller than a certain value. When this is not the case, namely if v_m is greater than the certain value, c_{fe} decreases with time, and the system can become asymptotically unstable. In a step response, the maximum velocity during the step response may be selected as a value of v_m . Thus we can calculate the stability criterion with some margin for instability. Although the true value of the maximum velocity can be obtained only by a numerical calculation, an approximation for the velocity in a practical system is possible when we consider capacity of fluid supply system or saturation of flow through the valve. The valve flow saturation characteristics have in principle no influence upon the characteristics of the equilibrium point where the valve takes small opening.

However, when the maximum velocity of the piston is restricted by the flow saturation through the relation shown in Eq. (28), the stability can be affected by the valve saturation. Further discussion about this phenomenon will be in the next report.

The internal leakage has severe influence upon the stability and the step response of the system. Equation (28) shows that the system becomes stable when c_1 is greater than a certain value even if c_f is zero. Figures 5 and 6 show effect of c_1 on the displacement response and P_1 respectively. When cavitation is absent, P_1 is given by $1 - P_1$. An offset resulting from both the Coulomb friction and the leakage flow is observed when τ becomes sufficiently large. At this stage, however, the compressibility and inertial load have little influence upon flows in the system. Hence the offset is given by Eq.(41) of the first report of this study. Thus we have the offset by substituting P_m in place of a_1 of the Eq.(41).

4. Response of Asymmetric system

An actual hydraulic servomechanism contains asymmetries of various kinds. The most basic asymmetry is geometrical asymmetry expressed by $V_1 \neq V_2$ and $A_1 \neq A_2$. Such asymmetries break symmetric characters in phenomena as expressed by $P_1 + P_2 = 1$. Ports of the pilot valve are not symmetric if strictly examined. But the asymmetry is within manufacturing tolerance, hence it is usually a very small quantity. It is obvious that the influence of the ports asymmetry appears only when the valve opening is small. Therefore the offset appearing in the last stage of the step response may be influenced by the port asymmetry. At this stage, however, the effects of oil compressibility and inertia load have already diminished. Hence this paper does not discuss on the ports asymmetry.

When the compressibility K is a function of pressure, $P_1 + P_2 = 1$ can not hold even if the geometrical symmetry is maintained. This problem will be treated in Chap.5.

Fig.7 Influence of asymmetry on displacement response

Fig.8 Pressure response of asymmetric system (parameters except λ and S are the same as Fig.7)

Fig.9 Pressure response of asymmetric system (parameters except λ and S are the same as Fig.7)

A special feature in the step response of the asymmetric system is that the symmetry of pressures is broken. As a result of this, the displacement response changes with signs of input. When the compressibility is neglected, it has a sign change only, corresponding to a change of input sign if $A_1=A_2$ holds while V_1 is not equal to V_2 .

The step response suffers influence of V_1/V_2 when the oil is compressible (see 2nd report). The form of response corresponding to positive input with $v_1=v_0$ is the same with that corresponding to negative input with $v_1=1-v_0$. Hence calculation corresponding only to positive or negative input is required when $A_1=A_2$ while V_1 is not equal to V_2 . This feature disappears when $A_1 \neq A_2$. Hence we must perform calculations corresponding to both positive and negative inputs with each value of v_1 (which varies with piston position). Figure 7 shows form of displacement response corresponding to positive and negative input under influence of the compressibility parameter. Pressure responses corresponding to the displacement responses appearing in the figure are shown in Figs. 8 to 11. From these figures we can observe that the symmetry in change of pressures P_1 and P_2 relative to 1/2 shown in Fig. 2 is lost in the present condition even if the cavitation does not exist.

Change of cylinder chamber volumes are considered in the calculation presented in this picture. Relations

$V_1 = V_{10} + A_1 y$, $V_2 = V_{20} - A_2 y$ which correspond to Eqs. (11) and (12) of the first report, are used. Calculations assuming $V_1 \equiv V_{10}$ and $V_2 \equiv V_{20}$ are also made. The difference between both calculations is very small and does not appear on the picture, because the input is far smaller than full stroke of the cylinder.

Fig. 10 Pressure response of asymmetric system (parameters except λ and S are the same as Fig. 7)

Fig. 11 Pressure response of asymmetric system (parameters except λ and S are the same as Fig. 7)

To treat the effects of input direction, the system will be identified depending on whether head side piston area is A_1 or A_2 . When we put

$$(A_1 + A_2)/2 = A, \quad \frac{A_1}{A} = 1 + S, \quad \frac{A_2}{A} = 1 - S \quad \dots\dots\dots(30)$$

then $S > 0$ corresponds to positive input, and $S < 0$ corresponds to negative input. When S is positive the displacement response becomes a little faster than the response by negative S . When the response is oscillatory, positive S gives a larger transient amplitude than that by negative S . This fact can be interpreted to mean that the effective time constant is smaller with $S > 0$ than with $S < 0$. This interpretation was analytically demonstrated for an incompressible fluid system (3).

When the compressibility parameter λ is made smaller or the inertia parameter a_5 is made larger, the step response becomes oscillatory, and in its extremity, an instability is induced. Influence of these parameters on the stability of the system appears in combination with other parameters, c_f , c_l and the Coulomb friction. Great volume of numerical calculation will be required to obtain the stability criterion with the Coulomb friction. For the calculation, influence of some other factors not yet appearing in this study will have to be considered. When a hydraulic servomechanism becomes unstable, the displacement amplitude is bounded below a certain limit (limit cycle). According to the author's calculations the limit cycle is characterized in that the pressures change from 0 to 1 (these are nondimensional pressures at exhaust and supply ports). Inverse flow and cavitation may be obtained temporarily in the transient but they will disappear at the state of the limit cycle.

5. Influence of entrained air on the step response

In discussions up to Chap. 4 the parameter K introduced in Eqs. (1) and (2) of the first report (1) is treated as a constant. Usually four bulk moduli, namely adiabatic or isothermal combined with tangential or mean (secant) bulk modulus, can be defined (4). By a direct measurement the mean bulk modulus is obtained. This is defined by

$$K = V_0 \frac{p - p_0}{V_0 - V} \quad \text{or} \quad \frac{dV}{K} = \frac{d\rho}{\rho_0} \quad \dots\dots\dots(31)$$

where V_0 is the volume of oil at pressure p_0 , ρ_0 is the density at the state, V and ρ are the volume and the density of the oil respectively at the pressure p . When V is the volume of a cylinder chamber, the equation of continuity is

$$\frac{\partial(\rho V)}{\partial t} + \Sigma \rho Q = 0 \quad \dots\dots\dots(32)$$

where Q represents outflow from the cylinder chamber. Rearranging Eq. (32); we have

$$\frac{V}{\rho} \frac{d\rho}{dt} + \Sigma Q + \frac{dV}{dt} = 0 \quad \dots\dots\dots(33)$$

By the definition of Eq. (31), above equation becomes

$$\frac{V}{K(1 - p/K)} \frac{dp}{dt} + \Sigma Q + \frac{dV}{dt} = 0 \quad \dots\dots\dots(34)$$

Definition of tangential bulk modulus K is

$$\frac{dp}{K_t} = \frac{d\rho}{\rho} \dots\dots\dots(35)$$

Hence Eq.(33) becomes

$$\frac{V}{K_t} \frac{dp}{dt} + \Sigma Q + \frac{dV}{dt} = 0 \dots\dots\dots(36)$$

This relation has the same form with Eqs.(1) and (2) of the first report. Difference between Eqs.(34) and (36) is negligible when $\Delta p/\bar{K} \ll 1$, which holds usually. Therefore in the past, equations of continuity were expressed in a simple form as Eq.(1) and (2) appearing in the first report. Constancy of K was also usually assumed. The bulk modulus of mineral oils is about 1.4 GPa, hence there was no controversy on selection, whether $\bar{K}$ or K_t should be used.

The hydraulic fluid usually contains, however, a little air in the form of bubbles, which severely lowers the apparent bulk modulus of the fluid. In this state we must select the definition of the bulk modulus either by Eq.(34) or by Eq.(35). To hold the form of Eq.(36) as the equation of continuity, we must define the apparent bulk modulus of fluid containing air bubbles by the following equation;

$$dp/K_a = d\rho/\rho \dots\dots\dots(37)$$

If an isothermal state is assumed for the oil and for the air bubbles, we have

$$K_{at} = \frac{1 - p/K + \phi p_{at}/(p + p_{at})}{1 + \phi \bar{K} p_{at}/(p + p_{at})^2} \bar{K} \dots\dots\dots(38)$$

If an isothermal state is assumed for the oil and the adiabatic state is assumed for the air bubbles, we have

$$K_{ea} = \frac{\{1 - p/\bar{K} + \phi(1 + p/p_{at})^{-1/\kappa}\} \bar{K}}{1 + \phi(\bar{K}/\kappa p_{at})(1 + p/p_{at})^{-(1+1/\kappa)}} \dots\dots\dots(39)$$

where p_{at} is the atmospheric pressure (absolute pressure), κ is the ratio of the specific heat at constant pressure and volume, ϕ is volumetric ratio of the air bubbles and the oil at the atmospheric pressure, namely $\phi = \text{total air volume}/(\text{total volume of fluid-total volume of air})$.

An effect of the air bubbles can be recognized when $\lambda_{ei} = K_{ei}/K$ is substituted for

λ in the mathematical model. If the fluid were simple air ($\phi \rightarrow \infty$), we would have $K_{ea} = \kappa(p + p_{at})$, $K_{at} = p + p_{at}$. These lead to misunderstanding that the adiabatic change shows generally a larger bulk modulus, but this is valid only in low pressure range. Pressures usually appearing in a hydraulic servomechanism give $K_{ea} < K_{ei}$.

Comparing Fig.13 with Fig.16 we can say that the assumption of adiabatic change of air bubbles leads to a more spongy behavior of the system than of isothermal change. Figures 14 and 15 demonstrate the change of pressures influenced by the rate of air content. A decrease of λ induced by an increase of bubble volume is generally observed. When the pressure becomes low, expansion of the air bubbles resists a fall of the pressure. Hence the curve of the pressure change rounds out towards the lower side even when the air content is as small as 10^{-3} . Thus the symmetry of the pressure change is

Fig.12 Apparent bulk modulus

Fig.13 Influence of entrained air on the step response (adiabatic change of air is assumed)

Fig.14 Influence of entrained air on P_1 (adiabatic change of air is assumed: parameters except ϕ is the same as Fig.13)

Fig.15 Influence of entrained air on P_2 (isothermal change of air is assumed: parameters except ϕ is the same as Fig.13)

remarkably disturbed when air bubbles are entrained in the oil.

Fig.16 Influence of entrained air on the step response (isothermal change of air is assumed)

6. Conclusions

The step response of the hydraulic servomechanism under influence of an inertial load is studied. The major results obtained from this study are:

- (1) When the hydraulic servomechanism is geometrically symmetric, has no external leakage and has a constant λ , then the pressure in the cylinder chambers satisfies the relation $P_1 + P_2 \equiv 1$. This relation is lost when a cavity is generated in one of the cylinder chambers.
- (2) The stability of the system becomes inferior when the fluid becomes more compressible. The internal leakage is ef-

fective to stabilize the system.

(3) Greater inertial load degrades the stability of the system. Viscous load and the Coulomb friction improve the system stability.

(4) If the hydraulic servomechanism includes an asymmetry like $A_1 \neq A_2$, its response differs depending on the direction of input.

(5) Though bulk modulus of the hydraulic fluid is changed by entrained air bubbles, basic equations of usual form can be generally used if the tangential modulus is adopted.

(6) The curve of the pressure change in the cylinder chamber rounds out toward lower side of the pressure if the oil entrains air bubbles. In this case $P_1 + P_2 \equiv 1$ does not hold despite the geometrical symmetry of the system.

(7) The displacement response shows little difference regardless of whether adiabatic or isothermal change of air bubble is assumed, still a little inferior stability will be caused by the adiabatic change.

References

- (1) Urata, E. and Koyama, T., Bull. JSME, Vol.24, No.194, (1981), 1413.
- (2) Urata, E., Bull. JSME, No.194, (1981), 1419.
- (3) Urata, E., Trans. Japan Soc. Mech. Engrs., Vol.40, No.337, (1974), 2609 (in Japanese).
- (4) Ishihara, T., Oil Hydraulics (in Japanese), (1968), 19, Asakurashoten Co.