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An Analysis of Cyclic Deformation Based on Dislocation Dynamics in Copper*

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A new approach was made for the cyclic deformation in terms of dislocation dynamics which is derived from modifying Johnston-Gilman's theory, and the change in the flow stress during cyclic deformation was analyzed in annealed copper. The analysis was quantitatively in fairly good agreement with test results as well as qualitatively. On the other hand, the Bauschinger effect could not be expressed well by the present analysis, and therefore, the stress-strain hysteresis loop estimated had a "fatter" shape than the actual one. This suggests that more detailed study will be needed for the evaluation of the internal stress produced by the reversed strain.

1. Introduction

It is well known that the fatigue of metallic materials is induced by the cyclic plastic strain, which produces slip marks on the specimen surface in microscopic way and also forms a stress-strain hysteresis loop in macroscopic way. Many researches have been made from the viewpoint of the latter as well as from that of the former, because they are so effective for understanding the fatigue.

The profile of the stress-strain hysteresis loop has been analyzed by using some parameters determined by experiment (1)-(3). An other analysis has been made by assuming that the material is constituted of many elements which have respective yield strains different from each other(4). The former, however, might not be in common use because of many empirical factors involved, and the latter has also problems in that the mechanical property of each element could not be related to the internal substructure in the material, and so the physical meaning in the analysis is not always clear.

On the other hand, there have been many reports on the internal substructure induced by the cyclic deformation. Nevertheless, the complexity of fatigue phenomena has prevented us from relating quantitatively the cyclic deformation property to the substructure, except that some qualitative explanations have been made

between them.

In view of the situation mentioned above the present analysis was made for the cyclic deformation behavior of copper in terms of the dislocation dynamics, along the same line with the previous report(5); Johnston-Gilman's theory(6)-(8) was modified to make it applicable to the cyclic deformation. From this it can be expected that the macroscopic mechanical behavior during cyclic deformation is interpreted in terms of the microscopic dislocation motion with a physically satisfying condition. We should note that the present analysis is characterized by a semi-technical approach; the precise discussion is excluded from the analysis in a strict sense.

2. Analysis

2.1 Derivation of Basic Equations

The macroscopic plastic strain rate $\dot{\epsilon}_p$ is related to the dislocation motion by the following equation.

$$\dot{\epsilon}_p = gb\rho_m\bar{v} \quad \dots\dots\dots(1)$$

where, g is a constant which is geometrical and relates the microscopic to the macroscopic flow direction, b Burgers vector, ρ_m the density of mobile dislocations and $\bar{v}$ the mean velocity of mobile dislocations. As ρ_m and $\bar{v}$ can be represented as functions of plastic strain ϵ_p and flow stress σ , and σ , respectively, the following Johnston-Gilman's formula is derived by substituting them for ρ_m and $\bar{v}$ in eq.(1).

$$\dot{\epsilon}_p = gbv^*(\rho_{t0} + M\epsilon_p)\exp\left(-\frac{D+H\epsilon_p}{\sigma}\right) \quad \dots\dots\dots(2)$$

where, ρ_{t0} is the initial total dislocation density at zero plastic strain, M the dislocation multiplication coefficient which designates a degree of the increase in dislocation density with the progress of plastic deformation, D the characteristic drag stress which is closely related with the yield stress, v^* the terminal velocity of dislocations at an infinitely great stress,

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and H the work hardening coefficient.

Authors derived previously the following equation by modifying eq.(2) in order to analyze the wave and the frequency effects on fatigue of copper⁽⁵⁾.

$$\dot{\epsilon}_p = gbv^*(\rho_{t0} + M\epsilon_p) \exp\left(-\frac{D + H_1\epsilon_p + H_2\epsilon_p^{1/n}}{\sigma}\right) \quad (3)$$

where H_1 and H_2 are work hardening coefficients and n a material constant.

In the present study the total strain range is controlled during test, and therefore, the total strain $\epsilon_t = \epsilon_t(t)$ which is given in the form of a function of time t has to be introduced in eq.(3).

$$\dot{\epsilon}_t = \dot{\epsilon}_e + \dot{\epsilon}_p \quad (4)$$

$$\dot{\sigma} = E\dot{\epsilon}_t - E\dot{\epsilon}_p \quad (5)$$

are obtained when ϵ_e and E are elastic strain and Young's modulus, respectively. Substituting eq.(5) for $\dot{\epsilon}_p$ in eq.(3)

$$\dot{\sigma} = E\dot{\epsilon}_t - Egbv^*(\rho_{t0} + M\epsilon_p) \times \exp\left(-\frac{D + H_1\epsilon_p + H_2\epsilon_p^{1/n}}{\sigma}\right) \quad (6)$$

is established. Equation(6) is for the monotonic tensile deformation in the region of not so great plastic strains, because a term giving the dislocation multiplication ($\rho_{t0} + M\epsilon_p$) is reasonable only under this condition.

Generally, copper cyclically hardens rapidly in the early stage of the cyclic deformation process, and then the hardening rate decreases gradually. Such a transition hardening stage is followed by the saturation hardening one, where the hardening rate becomes nearly equal to zero. This means that the multiplying rate of dislocations is large in the early stage of the cyclic deformation process, lessens gradually in the transition hardening one, and then becomes almost zero in the saturation hardening one. Furthermore, it is known that the cumulative plastic strain required for saturation hardening to be attained, $\sum |\epsilon_p|$ becomes small with an increase in the applied plastic strain range. The cyclic deformation is also characterized by the Bauschinger strain occurring at the strain reversal. The dislocation density might hardly increase in the deformation process yielding the Bauschinger strain, which may be caused by the reversible motions of the bowing dislocations before the critical radius is attained and of the piled up dislocations. In the previous paper⁽⁹⁾, the authors pointed out that the bend point appeared on the $\log\sigma$ - $\log\epsilon_p$ curve estimated along the profile of the stress-strain hysteresis loop, as shown in Fig.1. Moreover, they also suggested that the plastic strain below the bend point might not be so effective to the fatigue damage, while the plastic strain beyond this point could be considered the effective plastic strain, ϵ_{pe} , because the former might be mainly due to the reversible motion of dislocations, and the latter might result from great increases in the velocity and the density of the

mobile dislocations⁽⁹⁾.

Taking account of the above-mentioned characteristics of cyclic deformation, the following is assumed in the present analysis; only the effective plastic strain, ϵ_{pe} , contributes to the multiplication of dislocations, and the multiplication coefficient M decreases in accordance with an exponential function of the cumulative effective plastic strain, which increases with the progress of the cyclic deformation process. As the result M is expressed by the following equation.

$$M = M_0 \exp(-m_0 \sum |\epsilon_{pe}|) \quad (7)$$

where, M_0 and m_0 are material's constants. In the analysis M is assumed to remain a constant during one strain reversal. Moreover, the total dislocation density in the deformation interval during N th cycle, ρ_{tN} is given by

$$\rho_{tN} = \rho_{t(N-1/2)} + M_{(N-1/2)} \epsilon_{pe} = \rho_{t(N-1/2)} + M_0 \exp(-m_0 \sum^{N-1/2} |\epsilon_{pe}|) \epsilon_{pe} \quad (8)$$

taking $\rho_{t(N-1/2)}$, which is the total dislocation density at the end of $(N-1/2)$ th cycle, as an initial value. The increment of the total dislocation density between $(N-1/2)$ th and N th cycles, $\Delta\rho_{tN}$ is derived from eq.(8) as follows;

$$\Delta\rho_{tN} = \rho_{tN} - \rho_{t(N-1/2)} = M_0 \exp(-m_0 \sum^{N-1/2} |\epsilon_{pe}|) \Delta\epsilon_{pe} \quad (9)$$

where, $\Delta\epsilon_{pe}$ is the amount of the effective plastic strain produced between $(N-1/2)$ th and N th cycles. When the number of strain cycles N_s required for the saturation hardening stage to be attained is sufficiently large, the total dislocation density at this stage, ρ_{ts} is represented by

$$\rho_{ts} = \rho_{t \text{ ann.}} + \int_0^{N_s} \sum |\epsilon_{pe}| M_0 \exp(-m_0 \sum |\epsilon_{pe}|) d\epsilon_{pe} = \rho_{t \text{ ann.}} + M_0/m_0 \{1 - \exp(-m_0 \sum^{N_s} |\epsilon_{pe}|)\} \quad (10)$$

where, $\rho_{t \text{ ann.}}$ is the total dislocation density of the annealed copper before test, and $\sum^{N_s} |\epsilon_{pe}|$ the effective plastic strain accumulated before the hardening attains a stationary state.

On the other hand, using the ratio λ_s of the effective plastic strain ϵ_{pe} to the plastic strain range $\Delta\epsilon_{ps}$ in the stationary state, which is introduced in the previous paper⁽⁹⁾,

$$\epsilon_{pe} = \lambda_s \Delta\epsilon_{ps} = \Delta\epsilon_{ps} [1 - \exp\{-A(\Delta\epsilon_{ps} - \Delta\epsilon_{pw})\}] \quad (11)$$

is obtained. Hereupon, $\lambda_s = 1 - \exp\{-A(\Delta\epsilon_{ps} - \Delta\epsilon_{pw})\}$ is an empirical equation evaluated from an analysis of the profile of the stress-strain hysteresis loop⁽⁹⁾, and $\Delta\epsilon_{pw}$ the plastic strain range yielding at the fatigue limit. Therefore, $\lambda_s = \epsilon_{pe} = 0$ is established at $\Delta\epsilon_{ps} = \Delta\epsilon_{pw}$.

Fig.1 Change in profile of stress-strain hysteresis loop with increase in number of strain cycles (1MPa=0.102 kgf/mm²).

When N_s nearly equates to half the number of strain cycles to failure, $1/2 N_f$ we derive

$$N_s = (1/2)(c/\Delta\epsilon_{ps})^{1/\alpha} \quad (12)$$

from Manson-Coffin's law of $\Delta\epsilon_{ps} \cdot N_f^\alpha = C$. Combining eqs.(11) and (12), and substituting eq.(13) for $\sum |\epsilon_{pe}|$ in eq.(10) we have

$$\sum |\epsilon_{pe}| = 2N_s \Delta\epsilon_{ps} = \Delta\epsilon_{ps} [1 - \exp(-A(\Delta\epsilon_{ps} - \Delta\epsilon_{pw}))] \times (c/\Delta\epsilon_{ps})^{1/\alpha} \quad (13)$$

and

$$\rho_{ts} = \rho_{t \text{ ann.}} + M_0/m_0 [1 - \exp(-m_0 \Delta\epsilon_{ps} [1 - \exp(-A(\Delta\epsilon_{ps} - \Delta\epsilon_{pw}))]) (c/\Delta\epsilon_{ps})^{1/\alpha}] \quad (14)$$

, respectively.

Now, it has been known that the shear stress in the monotonic tensile deformation, τ is related with the total dislocation density, ρ_t by the following equation^{(10)~(15)}.

$$\tau = \alpha \mu b \sqrt{\rho_t} \quad (15)$$

where, μ is the shear modulus, and α a material's constant nearly equal to 0.2~0.4. The same formula is established also between the stress amplitude and the total dislocation density in fatigue of α -iron by Vingsbo et al⁽¹⁶⁾. Accordingly, in the present study the equation(15) was applied to the cyclically deformed copper, determining $\alpha=0.4$. As the shear stress τ is related to the tensile stress σ by $\tau=g\sigma$, eq.(15) is transformed into

$$\left. \begin{aligned} \Delta\sigma_s &= 2\alpha \mu b \sqrt{\rho_{ts}}/g = 0.8 \mu b \sqrt{\rho_{ts}}/g \\ \text{or} \\ \rho_{ts} &= (\Delta\sigma_s g / 0.8 \mu b)^2 \end{aligned} \right\} \quad (16)$$

when $\Delta\sigma_s$ is the stress range at the stationary state of cyclic hardening. On the other hand, the following empirical equation has been approved by many researchers⁽¹⁷⁾⁽¹⁸⁾.

$$\Delta\sigma_s = B \Delta\epsilon_{ps}^{n_1} \quad (17)$$

where, B and n_1 are material's constants. Then, the following equation,

$$\Delta\epsilon_{ps} = \Delta\epsilon_t - \Delta\sigma_s/E \quad (18)$$

is easily proved. When $\Delta\epsilon_{ps}$, $\Delta\sigma_s$ and ρ_{ts} are eliminated by combining eqs.(14), (16), (17) and (18), the relation between the total strain range $\Delta\epsilon_t$ and m_0 comes out. The values of m_0 evaluated as a function of $\Delta\epsilon_t$ are plotted in Fig.2, and the constants used for the estimation are presented in the following;

$$\begin{aligned} A &= 60.8^{(9)}, \quad \Delta\epsilon_{pw} = 7.8 \times 10^{-5}^{(9)} \\ g &= 1/3.06^{(20)}, \quad b = 2.556 \text{ \AA} \\ E &= 1.088 \times 10^2 \text{ GPa}, \quad \mu = 4.609 \times 10 \text{ GPa} \\ B &= \begin{cases} 1.105 \times 10^3 \text{ MPa} & (\Delta\epsilon_t < 5 \times 10^{-3}) \\ 3.93 \times 10^2 \text{ MPa} & (\Delta\epsilon_t \geq 5 \times 10^{-3}) \end{cases} \\ n_1 &= \begin{cases} 0.243 & (\Delta\epsilon_t < 5 \times 10^{-3}) \\ 0.0595 & (\Delta\epsilon_t \geq 5 \times 10^{-3}) \end{cases} \end{aligned}$$

Fig.2 m_0 in eq.(14) plotted against total strain range $\Delta\epsilon_t$.

According to Fig.2, the $\log m_0 - \log \Delta\epsilon_t$ relation might be expressed as a line consisting of two straight portions with a bend point between them. The empirical equation of $m_0 = C_m \Delta\epsilon_t^Z$, therefore, holds between m_0 and $\Delta\epsilon_t$, considering C_m and Z material's constants, and the following equations are derived.

$$\left. \begin{aligned} \Delta\epsilon_t < 5 \times 10^{-3} \text{ のとき } m_0 &= 0.403 \Delta\epsilon_t^{-0.482} \\ \Delta\epsilon_t \geq 5 \times 10^{-3} \text{ のとき } m_0 &= 2.95 \Delta\epsilon_t^{-0.106} \end{aligned} \right\} \quad (19)$$

We can find out ρ_{ts} for each total strain range from eq.(14), by substituting eq.(19) for m_0 in eq.(14).

By the way, the internal stress is a very significant factor controlling the Bauschinger strain caused by the strain reversal in the cyclic deformation. Generally, the applied stress σ is composed of the effective stress, σ_e and the internal stress, σ_i , which directly contribute to the motion of the dislocation and are originated from a microscopic substance like a pile up of the dislocations, respectively and therefore, the following equation holds⁽²¹⁾.

$$\sigma = \sigma_e + \sigma_i \quad (20)$$

This means that the positive and the negative plastic deformations occur under the condition of $\sigma > \sigma_1$ ($\sigma_e > 0$) and of $\sigma < \sigma_1$ ($\sigma_e < 0$), respectively, and that the maximum plastic strain is reached at $\sigma = \sigma_1$ ($\sigma_e = 0$). As detailed information on the internal stress has not been given yet, it is too difficult to evaluate the internal stress precisely. Accordingly, the authors assume in the present analysis that the internal stress depends on the plastic strain and the dislocation density, and is expressed by the following equation.

$$\sigma_i = Q\eta^{Kq}\epsilon_p^R \quad (21)$$

where, Q , Kq and R are the material's constants, and then $\eta = \rho_i / \rho_{t\text{ann.}}$ is introduced. A schematic diagram for the stress-strain hysteresis loop and the internal stress is presented in Fig.3.

Fig.3 Schematic representation of hysteresis loop and setting up of coordinate system, $\sigma_A - \sigma_A - \epsilon_A$.

The effective plastic strain ratio, λ_s was given by eq.(11) in the saturation hardening stage. Now, the ratio λ of the effective plastic strain to the plastic strain range changes gradually in the cyclic deformation process before the saturation hardening is attained. The $\log\sigma - \log\epsilon_p$ relations along the loop profile are plotted for $\Delta\epsilon_{ps} \approx 0.77\%$ in Fig.1. According to this figure, the bend point of the profile moves in the direction of a larger plastic strain as the cyclic hardening progresses. The effective plastic strain ratio λ estimated from this figure is plotted against $\Sigma|\epsilon_p|$ in Fig.4. When a coefficient C_λ , which becomes 1.0 at infinitely large $\Sigma|\epsilon_p|$, is introduced, the $\lambda - \Sigma|\epsilon_p|$ relation is represented by the following empirical equation.

$$\lambda = \{1.43 \exp(-0.371 \Sigma|\epsilon_p|) + 1.0\} \times [1 - \exp(-A(\Delta\epsilon_{ps} - \Delta\epsilon_{pw}))] \quad (22)$$

After all, the equation

$$\dot{\sigma} = E\dot{\epsilon}_t - Egbv^* \{ \rho_{t0} + M_0 \exp(-m_0 \Sigma|\lambda\epsilon_p|) \} \times \exp\left\{ -\frac{D + H_1\epsilon_p + H_2\epsilon_p^{1/n}}{\sigma - Q(\rho_i/\rho_{t\text{ann.}})^{Kq}\epsilon_p^R} \right\} \quad (23)$$

is derived from eq.(6) as follows; σ and $M\epsilon_p$ are replaced by σ_e and $M\epsilon_{pe}$, respectively, and then eqs.(7) and (21) are substituted for M and σ_1 in eq.(6). We can analyze the cyclic deformation through eq.(23), by introducing eqs.(19) and (22) for m_0 and λ , and by giving some constants reasonable values.

Fig.4 Change in effective plastic strain ratio λ with increase in cumulative plastic strain.

2.2 Determination of Constants

The constants E , b , v^* , M_0 and $\rho_{t\text{ann.}}$ included in eq.(23) are inherent in copper, and they and the geometrical factor g were given except $v^* = 2.27 \times 10^3 \text{ m/s}^{(5)}$ in the previous section. The other constants D , H_1 , H_2 , n , Q , Kq and R were determined as follows.

Tensile tests were performed for materials pre-deformed cyclically up to the rapid, the transition and the saturation hardening stages. The above constants were fixed so that the tensile stress-strain curves could be represented well by the numerical analysis of eq.(23). In advance of the analysis, M and ρ_{t0} in eq.(23) were evaluated from $M_0 \exp(-m_0 \Sigma|\lambda\epsilon_p|)$, and from

$$\rho_{t0} = \rho_{tN} = \rho_{t\text{ann.}} + \int_0^{\Sigma|\epsilon_p|} M_0 \exp(-m_0 \Sigma|\lambda\epsilon_p|) d\epsilon_{pe} \\ = \rho_{t\text{ann.}} + M_0/m_0 \{1 - \exp(-m_0 \Sigma|\lambda\epsilon_p|)\}$$

using eqs.(7) and (10), respectively, for each hardening stage. The evaluated values of M and ρ_{t0} are shown for the materials annealed and pre-fatigued up to the above three hardening stages in Table 1. The constants Q , Kq and R were determined as 10.0MPa, 0.59 and 0.1, respectively, considering the shape of the tensile stress-strain curves and the studies⁽²²⁾⁽²³⁾ on the internal stress.

After all, the tensile stress-strain curves are analyzed by integrating eq.(23) numerically, using the constants shown in the previous section and fixed in this way, and giving the tensile test condition of $\dot{\epsilon}_t = 10^{-3}$ and initial condition of $\sigma = 0$ at $t = 0$ as follows. Given suitable values of H_1 ,

H_2 , n and D , the trial and error are repeated until the analysis fits the stress-strain curve of annealed copper, and as the result the necessary constants are decided for the annealed material first. Then, the stress-strain curves of materials deformed cyclically up to the three hardening stages are evaluated through eq.(23), setting H_1 , H_2 and n at the same values as those in the annealed material and varying only D . Several trials gave the following constants.

$$H_1 = 1.57 \times 10^4 \text{ MPa}, H_2 = 9.8 \times 10^2 \text{ MPa}, n = 3$$

$$D = \begin{cases} 196 \text{ MPa} & \text{for material annealed} \\ 784 & \text{for material pre-fatigued up to the rapid hardening stage} \\ 834 & \text{for material pre-fatigued up to the transition hardening stage} \\ 932 & \text{for material pre-fatigued up to the saturation hardening stage} \end{cases}$$

The stress-strain curves analyzed by using the above constants are shown with the test results in Fig.5. It is known from the figure that the analysis is in good agreement with the experimental result in annealed material, but there exists some difference between them in pre-fatigued materials.

Table 1 Dislocation multiplication coefficient M and initial total dislocation density ρ_{t0} estimated for annealed and pre-fatigued coppers.

Materials	$M, 1/\text{cm}^2$	$\rho_{t0}, 1/\text{cm}^2$
Annealed	5×10^{10}	1.0×10^8
Pre-fatigued up to rapid hardening stage	4.66×10^{10}	3.05×10^9
Pre-fatigued up to transition hardening stage	4.02×10^{10}	6.92×10^9
Pre-fatigued up to saturation hardening stage	0	1.03×10^{10}

Fig.5 Tensile stress-strain curves in annealed and pre-fatigued coppers.

2.3 Analysis of Cyclic Deformation

The equation(23) is applied to the analysis of the cyclic deformation. In this case there exists the internal stress caused by the localized distribution of dislocations even at applied stress, $\sigma=0$. Because the dislocation motion is directly governed by the effective stress $\sigma_e = \sigma - \sigma_i$, the analysis of the cyclic deformation should be made by solving eq.(23) after the origin in the coordinate system is transformed from 0 to O_A in Fig.3, where σ_e is equal to zero. Then, eq.(23) is integrated from $O_A(N-1/2)$ to $O_A(N)$ along the loop profile every half cycle, and the $\sigma_A - \epsilon_A$ relation is determined, where the stress and the strain based on the origin O_A are suffixed by A. The $\sigma - \epsilon$ relation is attained when the $\sigma_A - \epsilon_A$ relation is transformed with respect to the former origin 0.

The term of the internal stress, $Q(\rho_t/\rho_{tann})^{Kq} \cdot \epsilon_p^R$ in eq.(23) is transformed σ_{iA} which is the internal stress from the origin of O_A , and is represented by the following equation.

$$\sigma_{iA} = 2^{1-R} Q \eta^{Kq} \epsilon_{pA} \dots \dots \dots (24)$$

Moreover, the characteristic drag stress D varies every half strain cycles and therefore, this is expressed by a function of η_0 , which is the value of η at the beginning of each strain reversal. For that purpose the values of D for materials annealed and pre-fatigued up to three hardening stages were examined in connection with ρ_{t0} of these four materials. The results are given in the form of a relation between D and $\eta_0 = \rho_{t0}/\rho_{tann}$ in Fig.6. This figure shows that $\log D$ is approximately expressed by a straight line against $\log \eta_0$, and therefore, the following empirical equation is established:

$$D = 196 \eta_0^{0.338} \dots \dots \dots (25)$$

As the cyclic deformation tests were performed with a sine wave of the angular velocity, ω rad./sec., the total strain ϵ_{tA} and the total strain rate $\dot{\epsilon}_{tA}$ from the origin, O_A are represented by

$$\epsilon_{tA} = 1/2 \Delta \epsilon_t \{ \cos \varphi \omega - \cos \omega(t + \varphi) \} \dots \dots \dots (26)$$

and

$$\dot{\epsilon}_{tA} = 1/2 \omega \Delta \epsilon_t \sin \omega(t + \varphi) \dots \dots \dots (27)$$

respectively, using the phase difference in time, φ sec.

Fig.6 Characteristic drag stress D plotted against dislocation density ratio η_0 .

When the tensile strain is applied first in the cyclic deformation test, the phase difference at the first compressive strain reversal, $N=1$ is given by

$$\varphi = 1/\omega \cos^{-1}(2\varepsilon_{1/2}/\Delta\varepsilon_t) \dots\dots\dots (28)$$

where, $\varepsilon_{1/2}$ is the total strain at $\sigma_A(1/2)$ from the origin 0 and is decided by giving the value of $\Delta\varepsilon_t$ and by analyzing the tensile stress-strain curve through eq.(23).

In conclusion, the following equations are obtained for the analysis of the cyclic deformation.

(i) for $0 < \varepsilon_{pA} < (1-\lambda)\Delta\varepsilon_p$

$$\dot{\sigma}_A = E\dot{\varepsilon}_{tA} - Egbv^* \rho_{t0} \exp\left(-\frac{D + H_1\varepsilon_{pA} + H_2\varepsilon_{pA}^{1/n}}{\sigma_A - \sigma_{tA}}\right) \dots\dots\dots (29.a)$$

(ii) for $(1-\lambda)\Delta\varepsilon_p \leq \varepsilon_{pA}$

$$\dot{\sigma}_A = E\dot{\varepsilon}_{tA} - Egbv^* \rho_{t0} + M_0 \exp(-m_0 \sum |\varepsilon_{pe}|) \lambda \varepsilon_{pA} \times \exp\left(-\frac{D + H_1\varepsilon_{pA} + H_2\varepsilon_{pA}^{1/n}}{\sigma_A - \sigma_{tA}}\right) \dots\dots\dots (29.b)$$

where,

$$\varepsilon_{pA} = \varepsilon_{tA} - \sigma_A/E \dots\dots\dots (30)$$

Substituting the needed constants determined beforehand and eqs.(19), (22)~(27) and (30) for the relevant terms in eq.(29), this equation is successively integrated from $\sigma_A(1/2)$, changing the initial condi-

tion every strain reversal. The stress-strain hysteresis loop is analyzed in this way. The actual calculation was made through numerical analysis by Runge-Kutta method.

3. Experimental Procedures and Test Materials

The cyclic deformation tests were performed under the controlled total strain range using an electrohydraulic type push-pull testing machine, with a sine wave of the frequency 0.2Hz. The detail of the other test procedures is as stated in the previous papers⁽⁵⁾⁽⁹⁾⁽²⁴⁾⁽²⁵⁾. Table 2 shows the mechanical properties of a commercially pure copper (99.94% purity) annealed, which was used in the present tests.

4. Results Analyzed and Discussions

Figure 7 shows an analyzed variation in the stress-strain hysteresis loop with the progress of the cyclic deformation under $\Delta\varepsilon_t=1.6\%$. The part of the loop profile in the first 1/4 cycles just agrees with the tensile stress-strain curve estimated. The analysis describes successfully such mechanical behavior of the annealed copper that the stress range $\Delta\sigma$ increases and the plastic strain range $\Delta\varepsilon_p$ decreases as the cyclic deformation process progresses. The estimated $\Delta\sigma$ and $\Delta\varepsilon_p$ are plotted against the number of strain cycles N and the cumulative plastic strain $\sum |\varepsilon_p|$ in Figs. 8(a) and (b), respectively. The figures show that $\Delta\sigma$ and $\Delta\varepsilon_p$ increase and decrease as N and $\sum |\varepsilon_p|$ increase, respectively, and that these reach the stationary values inherent to the amount of $\Delta\varepsilon_t$ after some period in the cyclic deformation process. The estimated $\Delta\sigma$ - $\sum |\varepsilon_p|$ relation is compared with the test result in Fig.9. They are in fairly good agreement qualitatively and also quantitatively. Furthermore, the period required for the stationary state to be attained tends to be shorter with an increase in the applied total strain range, as usually recognized in the test results. Such facts might prove the appropriateness of the assumption that the dislocation multiplication is produced only by the effective plastic strain, ε_{pe} .

Figure 10 shows the tensile part of the hysteresis loop which has attained a stationary state under the condition of $\Delta\varepsilon_t=1.29\%$, where the stress and strain are normalized by the maximums, σ_{max} and ε_{pmax} , respectively. Some differences in the loop shape are recognized between the experiment and the analysis. In the former, the plastic strain begins to occur at the stress nearly equal to zero, and the plastic strain in the negative direction is produced even in the stress range of $\sigma > 0$ in the course of unloading, while in the latter the plastic strain yields only in the stress range of $\sigma \geq 0.3\sigma_{max}$, and the reversed plastic strain is hardly accompanied with unloading, being less than 0.0001% at perfectly unloaded state. In this way, a "fat" hysteresis loop is produced in the analysis, against a "thin" one in the experiment. Such a difference

Table 2 Mechanical properties of copper used.

0.2% proof stress, MPa	Tensile strength, MPa	Fracture ductility, %
25	213	131

Fig.7 Stress-strain hysteresis loops obtained by present analysis for $\Delta\varepsilon_t=1.6 \times 10^{-2}$.

(a) Relation between flow stress and number of strain cycles.

(b) Relation between flow stress and cumulative plastic strain.

Fig.8 Analysis of cyclic hardening behavior of copper ($1\text{MPa}=0.102\text{kgf/mm}^2$)

rence in the loop shape is due to the unsatisfied evaluation of the Bauschinger effect in the present analysis.

The Bauschinger effect was not described successfully in the present analysis, as stated before. This suggests that σ_1 is not expressed sufficiently well in the small strain region by eq.(21). As the exponent of ϵ_p , R is assumed to be 0.1 in eq. (21), ϵ_p^R has a large value for a small plastic strain. The effective stress acting on dislocations $\sigma_e(=\sigma-\sigma_1)$, therefore, hardly increases until the applied stress becomes large to some extent. This might be the reason why the Bauschinger effect scarcely appeared in the present analysis. The effective stress and the plastic strain induced by this increase so much in the region of the plastic strain where the differential coefficient $d\sigma_1/d\epsilon_p$ lessens disproportionately. The critical plastic strain at which this situation is attained is small because of the exponent R being small. Such a character in the analysis is reflected in the tensile stress-strain curves given in Fig.5. For instance, the analyzed tensile stress in the specimen pre-fatigued up to the saturation hardening stage goes up nearly linearly to $\sigma \approx 98\text{MPa}$ against the

strain, and then a sudden increase in the plastic strain occurs. Meanwhile in the actual stress-strain curve, the plastic strain begins to occur at a pretty lower stress than 98MPa , but a rapid increase in the plastic strain is not produced until a fairly higher stress is attained, than in the analysis. The above consideration leads to the conclusion that R should be made much larger. A larger R , however, magnifies the difference between the analysis and the experiment in the stress-strain curve in the larger strain region. After all, it is concluded that more detailed consideration will be needed for the estimation of σ_1 , in order to express the Bauschinger effect successfully in future.

The cyclic stress-strain curves are compared between the analysis and the experiment in Fig.11. The analyzed curve crossing the test results at $\Delta\epsilon_p \approx 1.0 \times 10^{-2}$ is caused by the constants, H_1 , H_2 and n which are set at the values for the annealed material, as stated before; the analysis of the tensile stress-strain curve of the pre-fatigued

Fig.9 Comparison in cyclic hardening behavior between analysis and experiment.

Fig.10 Comparison in shape of hysteresis loop under $\Delta\epsilon_t=1.9 \times 10^{-2}$ between analysis and experiment.

Fig.11 Comparison in cyclic stress-strain curve between analysis and experiment.

material, therefore, tends to indicate a higher stress than in the experiment with an increase in the strain, as shown in Fig. 5. On the whole the analysis is in pretty good agreement with test results.

5. Conclusions

A new analysis was made on the cyclic hardening behavior of the annealed copper, where Johnston-Gilman's formula was modified by taking account of the internal stress and the changes in the dislocation multiplication coefficient and dislocation density, dependent on the effective plastic strain, which was previously defined by authors. An experiment was also performed to verify the analysis.

The change in the flow stress accompanied with increases in the number of strain cycles and in the cumulative plastic strain was analyzed in annealed copper. The analysis is quantitatively in fairly good agreement with test results as well as qualitatively. On the other hand, there exists some difference in the shape of the stress-strain hysteresis loop between the analysis and the experiment, which results from the Bauschinger effect not being expressed well by the present analysis. It is suggested from this that more detailed study will be needed for the evaluation of the internal stress produced by the reversed strain.

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