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Instability of a Transversely Isotropic Elastic Strip under Tension*

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The instability problem stated in the title is analysed on the basis of Biot's mechanics of incremental deformations. The strip, made of a hypothetical transversely isotropic, compressible, elastic material, is assumed to be in the plane-stress state and stronger in the direction of its width than in its axial and thickness directions. The analysis shows that, no matter what the anisotropy strength of the strip is or its width is, it can become unstable under axial tension. The critical load is lower for the stronger anisotropy; that is, the reinforcement in the transverse or "wrong" direction weakens the strip with respect to its stability. Furthermore, a weakly anisotropic strip can become unstable only after the axial resultant force reaches its maximum, while a strongly anisotropic strip can lose its stability before the force reaches its maximum. The whole instability aspect is similar to that of a slab in the plane-strain state.

Key Words: Instability, Transverse Isotropy, Strip, Rubber, Tensile Load, Critical Load

1. Introduction

By "instability" of an elastic body we understand here a situation when for certain prescribed boundary loads there exist several adjacent states of displacement that are in equilibrium with these loads. The

smallest load (stress) at which this state is possible is called the "critical" load.

Recently, the author⁽¹⁾ studied the instability of a transversely isotropic, compressible, elastic slab subjected to uniform axial loads, compressive and tensile. The slab was assumed to be stronger in the direction of its width than in its axial and thickness directions, and to be in the plane-strain state both prior to and after becoming unstable. A special form of the strain-energy function for a transversely isotropic, compressible, elastic material was proposed and used for the analysis. The stress-deformation relations resulting from the proposed form of the strain-energy function contained the following two special cases: those of an isotropic Blatz-Ko foam rubber⁽²⁾ and an idealized fiber-reinforced Blatz-Ko foam rubber⁽³⁾.

The present paper deals with the instability problem of an infinite thin strip, instead of a slab, subjected to axial tension. All the other situations are the same as those in the previous paper⁽¹⁾. Therefore, no introductory remarks about the related works and the object of the paper are presented here.

The instability problem of the transversely isotropic strip is solved on the basis of Biot's mechanics of incremental

deformations⁽⁴⁾. The strip is assumed to be in the plane-stress state both prior to and after becoming unstable. An out-of-plane unstable deformation may possibly occur, but this case lies beyond the scope of the present paper.

Some numerical results show that the transversely isotropic, compressible, elastic strip under axial tension loses its stability for all wavelength-to-width ratios λ/h and all anisotropy parameters k . The critical load decreases and becomes asymptotic to that of an idealized fiber-reinforced strip⁽⁵⁾ with an increase in the anisotropy strength; in other words, the fiber reinforcement in the transverse or "wrong" direction weakens the strip under tension with respect to its stability. Furthermore, the critical load does not depend on the wavelength-to-width ratios for the anisotropy strength parameter $k > k_0 \approx 0.66$, but does depend on these ratios for the anisotropy parameter $k < k_0$. Finally, for the anisotropy parameter $k < k_0$ including the isotropic case ($k=0$) the strip loses its stability after the axial resultant force reaches its maximum, while for the parameter $k > k_0$, the strip becomes unstable before the resultant force reaches its maximum. Qualitatively, the whole instability aspect of the strip is similar to that of a slab in the plane-strain state.

2. Governing Equations

2.1. Incremental Deformations

For examining the stability of a finitely deformed elastic body, a small-deformation-superposed-on-large-deformation analysis represents the usual method of approach, and is employed in this paper. There are many mathematical formulations for this analysis, and we shall choose Biot's one⁽⁴⁾ as in the previous paper. The equilibrium equations for incremental stresses due to the superposed infinitesimal deformation are

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$$\partial s_{ij}/\partial x_j + T_{jk} \partial \omega_{ik}/\partial x_j + T_{ik} \partial \omega_{jk}/\partial x_j - e_{jk} \partial T_{ik}/\partial x_j = 0, \quad (1)$$

where s_{ij} are components of an incremental stress tensor $\tilde{s}$ defined by

$$\tilde{s} = \frac{\partial \tilde{T}}{\partial \Delta t}, \quad (2)$$

$\frac{\partial \tilde{T}}{\partial \Delta t}$ being Jaumann rate of Cauchy stress tensor $\tilde{T}$ and Δt the time increment. e_{ij} and ω_{ij} are components of incremental strain and rotation tensors, e and ω , defined in terms of incremental displacement vector u as

$$e = \frac{1}{2} \{ (\partial u / \partial x) + (\partial u / \partial x)^T \}, \quad (3)$$

$$\omega = \frac{1}{2} \{ (\partial u / \partial x) - (\partial u / \partial x)^T \}, \quad (4)$$

where superscript T denotes the transposition of the tensor. The incremental boundary force per unit initial area is

$$\Delta f = (\tilde{s} + \omega^T + T \operatorname{tr} e - T e) n, \quad (5)$$

where n is the unit normal vector to the initial boundary surface.

2.2. Transversely Isotropic Material

Let us consider a transversely isotropic, compressible, elastic material that has a strain-energy function. Let H_a ($a=1, 2, 3$) denote three mutually orthogonal unit vectors in the reference configuration, and take one of these vectors, say H_c (c :fixed), along the anisotropy axis of the material. Strain-energy function W should depend on the deformations only through the five invariants of right Cauchy-Green deformation tensor C (6)

$$I = \operatorname{tr} C, \quad II = \frac{1}{2} \{ (\operatorname{tr} C)^2 - \operatorname{tr} C^2 \}, \quad III = \det C, \\ IV = \operatorname{tr} (CH_c \otimes H_c), \quad V = \operatorname{tr} (CH_a \otimes H_c) \operatorname{tr} (CH_a \otimes H_c), \quad (6)$$

(no sum on c)

under rotations about the preferred direction H_c ; that is,

$$W = W(I, II, III, IV, V). \quad (7)$$

In Eqs.(6) the double suffix notation for summation is applied to the indices but not to the index c . Symbol $\otimes$ denotes the tensor product. The stress-deformation relations for the material under consideration are given as

$$T = \Phi_0 I + \Phi_1 B + \Phi_{-1} B^{-1} + \Phi_2 h_c \otimes h_c + \Phi_3 (B h_c \otimes h_c + h_c \otimes B h_c) \quad (8)$$

with

$$h_c = F H_c, \quad (9)$$

where F and B are the deformation gradient and the left Cauchy-Green deformation tensor relative to the reference configuration. Tensor I and superscript -1 denote the identity tensor and the inverse of the tensor. Response coefficients $\Phi_j = \Phi_j(I, II, III, IV, V)$, ($j=-1, 0, 1, 2, 3$) are expressed in terms of strain-energy function W as

$$\Phi_0 = (2/\sqrt{III}) (II^2 - III^2) W, \quad \Phi_1 = (2/\sqrt{III}) \partial_I W, \\ \Phi_{-1} = -2/\sqrt{III} \partial_{II} W, \quad \Phi_2 = (2/\sqrt{III}) \partial_{IV} W, \quad \Phi_3 = (2/\sqrt{III}) \partial_V W. \quad (10)$$

Here and henceforth, $\partial_I, \partial_{II}$, etc. denote the partial differentiation with respect to the respective invariants. If the material is initially unstressed,

$$\partial_I W + 2 \partial_{II} W + \partial_{III} W = 0, \quad \partial_{IV} W + 2 \partial_V W = 0, \quad (11)$$

when $I-3=II-3=III-1=IV-1=V-1=0$.

The relations between the incremental stresses and the incremental strains, which were obtained in the previous paper, are as follows:

$$\tilde{s} = \Phi_1 (eB + Be) - \Phi_{-1} (eB^{-1} + B^{-1}e) + \Phi_2 (eh_c \otimes h_c + h_c \otimes eh_c) \\ + \Phi_3 \{ (eB + 2Be) h_c \otimes h_c + h_c \otimes (eB + 2Be) h_c \\ + B h_c \otimes eh_c + eh_c \otimes B h_c \} + 2 \operatorname{tr} (eB) \partial_I E \\ + 2 \{ II \operatorname{tr} e - III \operatorname{tr} (eB^{-1}) \} \partial_{II} E + 2 III (\operatorname{tr} e) \partial_{III} E \\ + 2 \operatorname{tr} (eh_c \otimes h_c) \partial_{IV} E + 2 \operatorname{tr} (B h_c \otimes eh_c + eh_c \otimes B h_c) \partial_V E, \quad (12)$$

where $E(I, II, III, IV, V)$ is given by

$$E = \Phi_0 I + \Phi_1 B + \Phi_{-1} B^{-1} + \Phi_2 h_c \otimes h_c + \Phi_3 (B h_c \otimes h_c + h_c \otimes B h_c). \quad (13)$$

2.3. A Special Form of the Strain-Energy Function

In the previous paper (1), we considered a Blatz-Ko foam rubber reinforced by "extensible" fibers in the preferred direction H_c and treated this rubber as a transversely isotropic material possessing rotational symmetry relative to the direction H_c . As one of the simplest forms of the strain-energy function for such a transversely isotropic rubber, we chose the following special one:

$$W = \frac{1}{2} \mu \{ (II/III - 3) + 2(\sqrt{III} - 1) + \frac{1}{2} k (IV - 1)^2 \}, \quad (14)$$

where μ is the shear modulus for an infinitesimal deformation near the natural state and $k \geq 0$ is an additional material constant characterizing the reinforcement by extensible fibers or the strength of the anisotropy. This constant is referred to as an anisotropy parameter. This strain-energy function satisfies natural state condition (11). It follows from Eqs.(14) and (8)-(10) that

$$T = \mu \{ (1 - \frac{1}{\sqrt{III}} B^{-1} + \frac{k}{\sqrt{III}} (IV - 1) h_c \otimes h_c) \}. \quad (15)$$

In the limit $k \rightarrow 0$, this stress-deformation equation coincides with that of an isotropic Blatz-Ko rubber (2). When $k \rightarrow \infty$, this equation agrees with that of an idealized fiber-reinforced Blatz-Ko foam rubber under some provision (3).

For the specified strain-energy function in (14), the incremental stress-strain relations (12) and (13) reduce to

$$\tilde{s} = (\mu/\sqrt{III}) [eB^{-1} + B^{-1}e + B^{-1} \operatorname{tr} e + k \{ 2 h_c \otimes h_c \operatorname{tr} (eh_c \otimes h_c) \\ + (IV - 1) (eh_c \otimes h_c + h_c \otimes eh_c - h_c \otimes h_c \operatorname{tr} e) \}]. \quad (16)$$

3. Instability Problem

3.1. Deformation and Stress of the Strip before Buckling

Let (X, Y, Z) and (x, y, z) denote, respectively, the material coordinates and the spatial coordinates, with the bases (i, j, k) , of a point referring to the same Cartesian system. Consider an infinite elastic strip of width $2H$ in its natural state and having a width $2h$ when subjected to the uniform axial stress T_0 at infinity, as shown in Fig.1. The strain-energy function of its material has the form of Eq.(14), and the symmetry axis is in the X -direction. That is,

$$H_c = i. \quad (17)$$

Furthermore, the strip is assumed to be free from tractions on its $x=\pm h$ boundaries and to

be in a plane-stress state.

Denote the principal stretches in the X-, Y-, and z-directions as λ_1 , λ_2 , λ_3 , respectively, and the deformation gradient is clearly

$$\mathbf{F} = \lambda_1 \mathbf{i} \otimes \mathbf{i} + \lambda_2 \mathbf{j} \otimes \mathbf{j} + \lambda_3 \mathbf{k} \otimes \mathbf{k}, \quad (18)$$

from which follow

$$\mathbf{C} = \mathbf{B} = \lambda_1^2 \mathbf{i} \otimes \mathbf{i} + \lambda_2^2 \mathbf{j} \otimes \mathbf{j} + \lambda_3^2 \mathbf{k} \otimes \mathbf{k} \quad (19)$$

and

$$\mathbf{h}_1 = \begin{vmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{vmatrix}, \quad \mathbf{h}_2 = \begin{vmatrix} 0 & \lambda_2 & 0 \\ \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_3 \end{vmatrix}, \quad \mathbf{h}_3 = \begin{vmatrix} 0 & 0 & \lambda_3 \\ 0 & \lambda_2 & 0 \\ \lambda_1 & 0 & 0 \end{vmatrix}. \quad (20)$$

Five principal invariants of Cauchy-Green deformation tensor (19) under rotations about the X-axis are given by using Eqs. (6) as

$$\begin{aligned} I &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad II = (\lambda_1 \lambda_2)^2 + (\lambda_2 \lambda_3)^2 + (\lambda_3 \lambda_1)^2, \\ III &= (\lambda_1 \lambda_2 \lambda_3)^2, \quad IV = \lambda_1^2, \quad V = \lambda_1^4. \end{aligned} \quad (21)$$

The use of kinematical relations (19)-(21) in Eq. (15) then gives

$$\begin{aligned} T_{xx} &= \mu \{ (1 - \lambda_1^{-3} \lambda_2^{-1} \lambda_3^{-1} + k \lambda_1 \lambda_2^{-1} (\lambda_1^2 - 1)) \}, \\ T_{yy} &= \mu (1 - \lambda_1^{-1} \lambda_2^{-3} \lambda_3^{-1}), \\ T_{zz} &= \mu (1 - \lambda_1^{-1} \lambda_2^{-1} \lambda_3^{-3}), \end{aligned} \quad (22)$$

which should satisfy the loading condition, the boundary condition, and the plane-stress condition. From these conditions follow, respectively,

$$T/\mu_0 = 1 - \lambda_1^{-1} \lambda_2^{-3} \lambda_3^{-1}, \quad (23)$$

$$\lambda_3 = \lambda_1^{-3} \lambda_2^{-2} - k \lambda_1 \lambda_2^{-1} (\lambda_1^2 - 1), \quad (24)$$

$$\lambda_1 \lambda_2 \lambda_3^3 = 1. \quad (25)$$

From these equations, we can determine the values of λ_1 , λ_2 , λ_3 numerically, if T/μ_0 and k are given.

3.2. Incremental Deformation and Buckling Condition

In order to examine whether the finite deformation and the stress of the strip determined in the previous subsection are statically stable or not, let us superpose on that finite deformation an infinitesimal deformation whose incremental strain tensor is given as

$$\mathbf{e} = \begin{vmatrix} e_{11} & e_{12} & 0 \\ e_{12} & e_{22} & 0 \\ 0 & 0 & e_{33} \end{vmatrix}. \quad (26)$$

Substituting Eqs. (19)-(22) and (26) into Eq. (12), we can obtain the incremental stress-incremental strain relations for the plane incremental deformations of the transversely isotropic elastic strip having a strain-energy function (14). These relations should comply with the assumption of the plane-stress state ($s_{33} = 0$), from which follows

$$e_{33} = -\frac{1}{3}(e_{11} + e_{22}). \quad (27)$$

Eliminating e_{33} from those relations by using Eq. (27) yields

$$\begin{aligned} s_{11} &= \phi_{11} e_{11} + \phi_{12} e_{22}, \\ s_{22} &= \phi_{21} e_{11} + \phi_{22} e_{22}, \quad s_{12} = \phi_0 e_{12}, \end{aligned} \quad (28)$$

where

Fig.1 Transversely isotropic strip before loading and under tension

$$\begin{aligned} \phi_{11} &= (2\mu/3) (\lambda_1 \lambda_2 \lambda_3)^{-1} \{ 4\lambda_1^{-2} + k\lambda_1^2 (5\lambda_1^2 - 2) \}, \\ \phi_{12} &= (2\mu/3) (\lambda_1 \lambda_2 \lambda_3)^{-1} \{ \lambda_1^{-2} - k\lambda_1^2 (\lambda_1^2 - 1) \}, \\ \phi_{22} &= (2\mu/3) (\lambda_1 \lambda_2 \lambda_3)^{-1} (4\lambda_2^{-2}), \\ \phi_{21} &= (2\mu/3) (\lambda_1 \lambda_2 \lambda_3)^{-1} (\lambda_2^{-2}), \\ \phi_0 &= \mu (\lambda_1 \lambda_2 \lambda_3)^{-1} \{ (\lambda_1^{-2} + \lambda_2^{-2}) + k\lambda_1^2 (\lambda_1^2 - 1) \}. \end{aligned} \quad (29)$$

Substituting Eqs. (3), (4) and (28) into Eqs. (1), we obtain a set of governing equations in terms of the incremental displacement components $u_1 = u(x, y)$ and $u_2 = v(x, y)$:

$$\begin{aligned} \psi_{11} \partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 + \psi_{12} \partial^2 v / \partial x \partial y &= 0, \\ \psi_{22} \partial^2 v / \partial x^2 + \partial^2 v / \partial y^2 + \psi_{21} \partial^2 u / \partial x \partial y &= 0, \end{aligned} \quad (30)$$

where

$$\begin{aligned} \psi_{11} &= 2\phi_{11} / (\phi_0 + T_0), \\ \psi_{12} &= (2\phi_{12} + \phi_0 - T_0) / (\phi_0 + T_0), \\ \psi_{22} &= (\phi_0 - T_0) / (2\phi_{22}), \\ \psi_{21} &= (2\phi_{21} + \phi_0 + T_0) / (2\phi_{22}). \end{aligned} \quad (31)$$

Since the strip is free from tractions on its edges, the incremental boundary force (5) should vanish at $x = \pm h$. Using Eqs. (3), (4), and (28), we obtain

$$\begin{aligned} \psi_{11} \partial u / \partial x + \psi_{12} \partial v / \partial y &= 0 \quad \text{at } x = \pm h, \\ \partial u / \partial y + \partial v / \partial x &= 0 \quad \text{at } x = h, \end{aligned} \quad (32)$$

where

$$\psi_0 = 2\phi_{12} / (\phi_0 + T_0). \quad (33)$$

Differential equations (30) have the following formal solutions for the antisymmetric mode:

$$u = \{ A_1 \cos(\zeta_1 \pi x / \ell) + A_2 \cos(\zeta_2 \pi x / \ell) \} \cos(\pi y / \ell), \quad (34)$$

$v = \{ A_1 \kappa_1 \sin(\zeta_1 \pi x / \ell) + A_2 \kappa_2 \sin(\zeta_2 \pi x / \ell) \} \sin(\pi y / \ell)$, where A_1 and A_2 are integral constants, and ℓ is the half wavelength in the axial direction. Constants ζ_1 and ζ_2 are given by

$\zeta_\alpha = \{[-b - (-1)^\alpha (b^2 - 4a)^{1/2}] / (2a)\}^{1/2}$, ($\alpha=1,2$) (35)
which are two of the four roots for the algebraic equation

$$a\zeta^4 + b\zeta^2 + 1 = 0 \quad (36)$$

with coefficients

$$a = \psi_{11}\psi_{22}, \quad b = \psi_{11}\psi_{22} - \psi_{12}\psi_{21}. \quad (37)$$

Furthermore, constants κ_1 and κ_2 are given by

$$\kappa_\alpha = (\psi_{11}\zeta_\alpha^2 + 1) / (\psi_{12}\zeta_\alpha), \quad (\alpha=1,2). \quad (38)$$

With reference to boundary conditions (32), the solutions in (34) yield homogeneous linear algebraic equations with respect to unknowns A_1, A_2 . For a nontrivial set of solutions of these equations, the determinant of the coefficients must vanish, resulting in the following buckling condition:

$$(\psi_{11}\zeta_1 - \psi_{01}\kappa_1)(1 - \kappa_1\zeta_1)^{-1} \tan(\zeta_1\pi h/\ell) - (\psi_{11}\zeta_2 - \psi_{01}\kappa_2)(1 - \kappa_2\zeta_2)^{-1} \tan(\zeta_2\pi h/\ell) = 0. \quad (39)$$

The counterparts of Eqs. (34) and (39) for the symmetric mode are

$$u = \{A_1 \sin(\zeta_1 \pi x / \ell) + A_2 \sin(\zeta_2 \pi x / \ell)\} \cos(\pi y / \ell), \\ v = -\{A_1 \kappa_1 \cos(\zeta_1 \pi x / \ell) + A_2 \kappa_2 \cos(\zeta_2 \pi x / \ell)\} \sin(\pi y / \ell), \\ \text{and} \quad (40)$$

$$(\psi_{11}\zeta_1 - \psi_{01}\kappa_1)(1 - \kappa_1\zeta_1)^{-1} / \tan(\zeta_1\pi h/\ell) - (\psi_{11}\zeta_2 - \psi_{01}\kappa_2)(1 - \kappa_2\zeta_2)^{-1} / \tan(\zeta_2\pi h/\ell) = 0, \quad (41)$$

where the constants $\zeta_1, \zeta_2, \kappa_1, \kappa_2$ are given by Eqs. (35) and (38) again.

Solutions (38) and (40), and therefore the equations in (39) and (41) as well, are formal expressions and may be rewritten in other forms depending on the type differential equations (32) are classified into. This classification can be carried out according to whether the two roots ζ_1 and ζ_2 are both real, real and imaginary, both purely imaginary, or both complex. This is summarized in Table 1 and shown in Fig. 2; the k - λ_2 plane is divided into several regions, marked E-1, E-2, E-H, and H, by solid curves.

Table 1 Classification of Eqs. (30)

	ζ_1 and ζ_2	Conditions	Type of eqs. (23)
E-1	Both complex	$b^2 - 4a < 0$	Elliptic
E-2	Both purely imaginary	$b^2 - 4a > 0, a > 0, b > 0$	Elliptic
H	Both real	$b^2 - 4a > 0, a > 0, b < 0$	Hyperbolic
E-H	Real and purely imaginary	$a < 0$	Mixed hyperbolic-elliptic

4. Numerical Computations

From Eqs. (24) and (25) it follows that λ_2 is a single-valued function of λ_1 , if λ_3 is eliminated. The use of this in Eq. (39) [or (41)] then gives an equation for λ_1 which has multiple roots. After these are found for the given ℓ/h and k , we choose a root such that λ_2 is nearest to unity (minimum for tension), say λ_{2c} . The axial load corresponding to this root obtained through Eq. (23) is a critical, or buckling load, such T_c .

However, when $k > k_0 \approx 0.66$, a set of roots λ_2 corresponding to the multiple roots λ_1

does not have a minimum but an infimum, as is proved by examining Eq. (39) [or (41)] in the region E-H in detail. This infimum is given

Fig. 2 Classification of Eqs. (30) in a k - λ_2 plane and critical axial stretch curves $\lambda_2 = \lambda_{2c}$ represented by dashed curves

by equation $a=0$ instead of Eq. (39) [or (41)] and falls on the E-1 and E-H boundary in Fig. 2. That is, this boundary is a locus of points of accumulation of roots λ_2 of Eq. (39) [or (41)]. We regard the corresponding load for this infimum as a critical value, such as T_c again.

In Fig. 2, the dashed curves represent the $\lambda_2 = \lambda_{2c}$ curves for some wavelength-to-width ratios. These curves are on the E-1 and E-H boundary, within the ellipticity region E-2, or even within the hyperbolicity region H.

Within this hyperbolicity region, there might be hyperbolic solutions possibly carrying weak discontinuities or strong discontinuities, though we have assumed continuously differentiable solutions from the outset. These possible discontinuities lie beyond our scope. The vertical line marked λ_{2c} in the upper part of the figure is an asymptote of the E-1 and E-H boundary curve and at the same time an asymptote of the $\lambda_2 = \lambda_{2c}$ curve. The approximate value of λ_1 is given as 1.622⁽⁵⁾.

Figs. 3 and 4 show how the critical load depends on the anisotropy strength k and the wavelength-to-width ratio ℓ/h , respectively. In spite of the tensile load, the strip becomes unstable for all $\ell/h \geq 0$ and for all $k \geq 0$. For $k > k_0 \approx 0.66$, the critical load does not depend on ℓ/h but does depend on ℓ/h for $k < k_0$. This dependence of ℓ/h is shown in Fig. 4; the incipient instability occurs for the antisymmetric mode for some values of ℓ/h and for the

Fig.3 Critical load as a function of anisotropy parameter k

symmetric mode for other values of ℓ/h . When $\ell/h \rightarrow 0$, the critical loads coincide for both the symmetric and antisymmetric modes. For the larger ℓ/h , the symmetric mode is incipient, if $k < k_0$. Furthermore, the critical load decreases with an increase in k ; in other words, the reinforcement in the transverse or "wrong" direction weakens the strip in regard to its stability. The asymptotic critical load in the limit $k \rightarrow \infty$ is $T_{0c} = \mu(1 - \lambda^{-8/3})$ (5).

The chain curve in Fig.3 shows a value the axial stress assumes when the tensile resultant force

$$F = 2H\lambda_1 T_0 \quad (42)$$

reaches its maximum, say T_m . For $k < k_0$, $T > T_m$. This means that, for the anisotropy that is weaker than when $k = k_0$, the instability can arise only after the resultant force reaches its maximum. This is the same conclusion as that reached by Wesolowski⁽⁷⁾ and Hill and Hutchinson⁽⁸⁾. In contrast to that conclusion, for $k > k_0$, $T < T_m$. That is, for the anisotropy that is stronger than when $k = k_0$, the instability can occur before the resultant force reaches its maximum.

Fig.4 Critical load as a function of wavelength-to-width ratio ℓ/h

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