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EVALUATION OF UNSTABLE PROBABILITY OF ROTORS HAVING STIFFNESS ERRORS OF ASYMMETRY*

By Takuzo IWATSUBO** Kunihiro TOMITA***

Ryoji KAWAI****

The bending stiffness of a shaft in a two pole generator and so on is regarded as a random variable, thus the amplitude and the stability of a rotor system are statistically analyzed to seek the influence of the error of the bending stiffness on the system. That is, the variance of amplitude of the rotor and the mode function due to the error are obtained. The probability of unstable vibration of the rotor system is predicted and compared with the results of the simulation of Monte Carlo method. The variation of the amplitude of the rotor appears remarkable near the critical speed of the symmetrical rotor and a half of that speed. The probability of unstable vibration is in good agreement with results of the simulation, so the statistical analysis serves as a reference in designing or producing a rotary machine.

1. INTRODUCTION

The vibration of the rotor systems having asymmetric stiffness, such as represented by a two pole generator, is one of the industrially important problems, and it has been studied by many workers. Most of those studies were devoted to the derivation of an unstable region of the system versus the asymmetry ratio. However, in the actual rotors, it is difficult to determine the vibrational form at the stage of designing or producing them by utilizing the available results, because a lot of slits are provided in them to correct the asymmetry of the bending stiffness of the shaft and the asymmetry is randomly distributed in the axial direction. For these reasons, an unstable vibration may sometimes occur. Therefore it seems to be important to know the influence of those errors on the system and determine the allowable value of the errors at the designing or producing stages of rotors.

In the present study, the bending stiffness is regarded as a random variable, that is, errors and the influence of these errors in the bending stiffness on the stability are investigated. At first, the distribution of the bending stiffnesses of a rotor system is assumed to be normal and the variance of the response of the amplitude of the rotor is obtained, and compared with that of a symmetric one.

Furthermore, the variance of the vibrational mode of the rotor system, of which the bending stiffness is randomly distributed in the axial direction, is obtained by the perturbation method. Next we pay attention to the stability of the system, that is, the unstable probability (it is sometimes called the risk) caused by the relation between errors of the bending stiffness and external damping is obtained by the maximum likelihood method. The unstable probability of a continuous system which has the mass distribution in the axial direction is evaluated in every mode. After the statistic analysis, these results are compared with the simulation results obtained by Monte Carlo method in order to evaluate appropriateness of the risk.

NOTATIONS

- m: mass of a rotor
- c: external damping coefficient of the rotor system
- k_1, k_2 : spring constants of the shaft in ξ, η directions respectively
- EI_1, EI_2 : bending stiffnesses of the shaft in ξ, η directions respectively
- e : eccentricity of the center of gravity
- λ : eigenvalue
- ζ : external damping ratio
- μ : nondimensional value which represents asymmetric ratio
($\mu = \frac{k_1 - k_2}{k_1 + k_2}$)
- ω_1, ω_2 : eigenfrequencies in ξ, η directions respectively
- O- $\xi\eta$ s: rotating coordinate system
- ω : angular velocity of the shaft
- σ, ρ : standard deviation and correlation coefficient respectively
- ν, K : constants of uncertainty
- γ : small constant
- ($\cdot$): expected value of ($\cdot$)

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** Associate Professor, The Faculty of Engineering, Kobe University, Rokko, Nada, Kobe.

*** Engineer, Sumitomo electric co.

**** Professor, The Faculty of Engineering, Kobe University, Rokko, Nada, Kobe.

2. EQUATIONS OF THE ROTOR SYSTEM AND THEIR SOLUTIONS

In this section, the equations of motion and their solutions for a simple rotor system and a continuous rotor system are given to be applied for the reliability analysis.

2-1 EQUATIONS OF MOTION OF A SIMPLE ROTOR SYSTEM AND THEIR SOLUTIONS

A simple rotor is composed of a mass M and a massless shaft of the bending stiffnesses k_1 and k_2 in ξ , η directions respectively. The shaft is simply supported by bearings. Neglecting the internal damping of the system, the equations of motion of this system are represented in rotating coordinate $O-\xi\eta$ as;

$$\left. \begin{aligned} m(\ddot{\xi} - 2\omega\dot{\eta} - \omega^2\xi) + c(\dot{\xi} - \omega\eta) + k_1\xi &= m\epsilon\omega^2 + mg\cos\omega t \\ m(\ddot{\eta} + 2\omega\dot{\xi} - \omega^2\eta) + c(\dot{\eta} + \omega\xi) + k_2\eta &= m\epsilon\omega^2 - mg\sin\omega t \end{aligned} \right\} \quad (1)$$

The characteristic equation becomes as follows,

$$\lambda^4 + 4\delta\lambda^3 + (\omega_1^2 + \omega_2^2 + 2\omega^2 + 4\delta^2)\lambda^2 + 2\delta(\omega_1^2 + \omega_2^2 + 2\omega^2)\lambda + (\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) + 4\delta^2\omega^2 = 0 \quad (2)$$

Applying Routh-Hurwitz criterion to eq.(2) in order to seek the stable condition of the system, we get

$$\zeta^2 - \frac{\zeta^4}{4} > \mu^2 \quad (3)$$

where

$$\frac{1}{2}(\omega_1^2 + \omega_2^2) = \Omega^2, \quad \frac{k_1 - k_2}{k_1 + k_2} = \mu, \quad \frac{2\delta}{\Omega} = \zeta, \quad \hat{q} = \frac{c}{m} \quad (4)$$

Multiplying i ($i = \sqrt{-1}$) by the second expression of eq.(1) and adding it to the first one, we get the following expression;

$$\begin{aligned} \ddot{\phi} + 2(\delta + i\omega)\dot{\phi} + \left(\frac{\omega_1^2 + \omega_2^2}{2} - \omega^2 + 2i\omega\delta \right) \phi + \frac{\omega_1^2 - \omega_2^2}{2} \phi \\ = \omega^2(\epsilon + i\epsilon) + g e^{-i\omega t} \end{aligned} \quad (5)$$

where

$$\phi = \xi + i\eta, \quad \bar{\phi} = \xi - i\eta$$

Neglecting the gravity term of eq.(5) to know the vibration caused by the eccentricity, the solutions are assumed as follows:

$$\phi_1 = A + iB, \quad \bar{\phi}_1 = A - iB \quad (6)$$

Substituting eq.(6) into eq.(5), we get the solution as follows:

$$R_1 = |\phi_1| = \sqrt{A^2 + B^2} = \omega^2 \frac{\sqrt{[\epsilon(\omega_2^2 - \omega^2) + 2\delta\omega\epsilon]^2 + [\epsilon(\omega_1^2 - \omega^2) - 2\delta\omega\epsilon]^2}}{|(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) + 4\delta^2\omega^2|} \quad (7)$$

Neglecting the eccentric term to know the vibration caused by the gravity force, the solutions are assumed to have the following expressions;

$$\phi_g = U e^{i\omega t} + V e^{-i\omega t}, \quad \bar{\phi}_g = \bar{U} e^{-i\omega t} + \bar{V} e^{i\omega t} \quad (8)$$

Substituting eq.(8) into eq.(5), we get the solutions as follows:

$$U = U_1 + iU_2, \quad V = V_1 + iV_2 \quad (9)$$

$$U_1 = \frac{1}{2} \frac{g}{D} (\omega_2^2 - \omega^2) [\omega_1^2 \omega_2^2 - 2\omega^2(\omega_1^2 + \omega_2^2)] \quad (10)$$

$$U_2 = \frac{g}{D} \delta \omega (\omega_1^2 - \omega_2^2) (\omega_1^2 + \omega_2^2) \quad (11)$$

$$V_1 = \frac{g}{D} \left\{ \left[\frac{1}{2} (\omega_1^2 + \omega_2^2) - 4\omega^2 \right] [\omega_1^2 \omega_2^2 - 2\omega^2(\omega_1^2 + \omega_2^2)] + 8\delta^2 \omega^2 (\omega_1^2 + \omega_2^2) \right\} \quad (12)$$

$$V_2 = \frac{g}{D} 2\delta\omega \left\{ \left[\frac{1}{2} (\omega_1^2 + \omega_2^2) - 4\omega^2 \right] (\omega_1^2 + \omega_2^2) - 2[\omega_1^2 \omega_2^2 - 2\omega^2(\omega_1^2 + \omega_2^2)] \right\} \quad (13)$$

where U_1 , U_2 , V_1 and V_2 are real numbers, and

$$D = [\omega_1^2 \omega_2^2 - 2\omega^2(\omega_1^2 + \omega_2^2)]^2 + 4\delta^2 \omega^2 (\omega_1^2 + \omega_2^2)^2 \quad (14)$$

2-2 EQUATIONS OF THE CONTINUOUS ROTOR SYSTEM AND THEIR SOLUTIONS

Consider a shaft which has a uniform cross section and is simply supported at the both ends. Neglecting the internal damping and gyromoment, the equations of motion in the rotating coordinate shown in Fig.1 become as follows;

$$\left. \begin{aligned} EI_1 \frac{\partial^4 \xi}{\partial s^4} + m(\ddot{\xi} - 2\omega\dot{\eta} - \omega^2\xi) + c(\dot{\xi} - \omega\eta) &= m\epsilon\omega^2 + mg\cos\omega t \\ EI_2 \frac{\partial^4 \eta}{\partial s^4} + m(\ddot{\eta} + 2\omega\dot{\xi} - \omega^2\eta) + c(\dot{\eta} + \omega\xi) &= m\epsilon\omega^2 - mg\sin\omega t \end{aligned} \right\} \quad (15)$$

A solution to a homogeneous equation in eq.(15) is assumed as follows;

$$\xi = A \sin \frac{n\pi}{l} s e^{it}, \quad \eta = B \sin \frac{n\pi}{l} s e^{it} \quad (16)$$

Substituting the solution (16) into the homogeneous equation we get the following characteristic equation;

$$\begin{aligned} \lambda^4 + 4\delta\lambda^3 + \left(\frac{EI_1}{m} \frac{n^4\pi^4}{l^4} + \frac{EI_2}{m} \frac{n^4\pi^4}{l^4} + 2\omega^2 + 4\delta^2 \right) \lambda^2 \\ + 2\delta \left(\frac{EI_1}{m} \frac{n^4\pi^4}{l^4} + \frac{EI_2}{m} \frac{n^4\pi^4}{l^4} + 2\omega^2 \right) \lambda \\ + \left(\frac{EI_1}{m} \frac{n^4\pi^4}{l^4} - \omega^2 \right) \left(\frac{EI_2}{m} \frac{n^4\pi^4}{l^4} - \omega^2 \right) + 4\omega^2\delta^2 = 0 \end{aligned} \quad (17)$$

Applying Routh-Hurwitz criterion to eq.(17), we get the stable condition of the system over the whole region of ω as follows:

$$\zeta_n^2 - \frac{\zeta_n^4}{4} > \mu^2 \quad (18)$$

where

$$\frac{1}{2}(\omega_{1n}^2 + \omega_{2n}^2) = \Omega_n^2, \quad \frac{EI_1 - EI_2}{EI_1 + EI_2} = \mu, \quad \frac{2\delta}{\Omega_n} = \zeta_n \quad (19)$$

3. EVALUATION OF STATISTICS OF AMPLITUDE OF ROTOR

In this section, the variation of the amplitude caused by the errors of the bending stiffness is obtained by the partial derivative method under the assumption of the normally distributed errors. For the continuous system, the errors are assumed to be randomly distributed in the axial direction and the variance of the mode is obtained by the perturbation method.

3-1 VARIANCE OF AMPLITUDE OF ROTOR

The bending stiffness of the shaft is assumed to be a random variable and is given by using the mean value $\bar{k}$ and the standard deviation σ_k .

$$k_1 = (\bar{k}_1, \sigma_{k_1}), \quad k_2 = (\bar{k}_2, \sigma_{k_2}) \quad (20)$$

The amplitude obtained from eq.(7) and eqs.(10)-(13) varies with k_1 and k_2 . Therefore the amplitude is expressed as follows:

$$R_1 = f_1(k_1, k_2), \quad U = f_2(k_1, k_2), \quad V = f_3(k_1, k_2) \quad (21)$$

In general, if the random variable f is a function of n random variables k_i ($i = 1, 2, \dots, n$) of which distribution is normal, the variance of f is obtained by the following expression.

$$\sigma_f^2 = \sum_{i=1}^n \left(\frac{\partial f}{\partial k_i} \right)^2 \sigma_{k_i}^2 + \sum_{i=1}^n \sum_{j=1}^n \frac{\partial f}{\partial k_i} \frac{\partial f}{\partial k_j} \sigma_{k_i} \sigma_{k_j} \rho_{k_i k_j} \quad (22)$$

where $\rho_{k_i k_j}$ is a correlation coefficient. Then substituting the relation of eq.(21) into eq.(22), we get the variances of $|R|$, $|V|$, and $|U|$.

Even in the continuous model, we get the variances by the same method as above.

3-2 VARIANCE OF THE MODE

The equations of motion of the continuous system, of which the bending stiffness is randomly distributed in the axial direction, are given as follows:

$$\left. \begin{aligned} \frac{\partial^2}{\partial s^2} \left[\overline{EI} (1 + \gamma \kappa(s)) \frac{\partial^2 \xi}{\partial s^2} \right] + m(\xi - 2\omega \eta - \omega^2 \xi) &= 0 \\ \frac{\partial^2}{\partial s^2} \left[\overline{EI} (1 + \gamma \nu(s)) \frac{\partial^2 \eta}{\partial s^2} \right] + m(\eta + 2\omega \xi - \omega^2 \eta) &= 0 \end{aligned} \right\} \quad (23)$$

Assuming that the bending stiffnesses EI_1 and EI_2 are randomly distributed in the axial direction, that γ is a small constant, and that $\kappa(s)$ and $\nu(s)$ are uncertain, the bending stiffness is expressed as follows:

$$EI_1 = \overline{EI} (1 + \gamma \kappa(s)), \quad EI_2 = \overline{EI} (1 + \gamma \nu(s)) \quad (24)$$

where $\overline{EI}$ is an expected value of EI . Eq.(23) is differential equations having random coefficients. Applying the perturbation method proposed by Hara to those equations, the variances of the mode are evaluated. The solutions of eq.(23) are assumed as follows:

$$\xi = H(s) \cos \lambda t, \quad \eta = H(s) \sin \lambda t \quad (25)$$

Substituting eq.(25) into eq.(23), then we get

$$\left. \begin{aligned} (L_0 + \gamma L_1) H(s) - J(\lambda^2 H(s) + 2\omega \lambda H(s) + \omega^2 H(s)) &= 0 \\ (L_0 + \gamma L_2) H(s) - J(\lambda^2 H(s) + 2\omega \lambda H(s) + \omega^2 H(s)) &= 0 \end{aligned} \right\} \quad (26)$$

where

$$\left. \begin{aligned} L_0 &= \frac{\partial^4}{\partial s^4}, \quad L_1 = \kappa(s) \frac{\partial^4}{\partial s^4} + \frac{\partial \kappa}{\partial s} \frac{\partial^3}{\partial s^3} + \frac{\partial^2 \kappa}{\partial s^2} \frac{\partial^2}{\partial s^2} \\ L_2 &= \nu(s) \frac{\partial^4}{\partial s^4} + \frac{\partial \nu}{\partial s} \frac{\partial^3}{\partial s^3} + \frac{\partial^2 \nu}{\partial s^2} \frac{\partial^2}{\partial s^2}, \quad J = \frac{m}{EI} \end{aligned} \right\} \quad (27)$$

The modal function and the eigenvalue in the symmetric system are $H_k^{(0)}$ and $\lambda_k^{(0)}$ respectively. It is assumed that γL_1 and γL_2 in eq.(26) are perturbed terms and $H(s)$ is developed as follows:

$$H(s) = \sum C_i H_i^{(0)}(s) \quad (28)$$

where C_i is a constant. Substituting eq.(28) into eq.(26), multiplying them by j th mode, and integrating from 0 to L (the length of the shaft), we get the following expressions;

$$\left. \begin{aligned} \sum_i C_i \gamma (j L_1 i) + C_j J (\lambda_j^{(0)2} - \lambda^2) + 2\omega J C_j (\lambda_j^{(0)} - \lambda) + \gamma C_j (j L_1 j) &= 0 \\ \sum_i C_i \gamma (j L_2 i) + C_j J (\lambda_j^{(0)2} - \lambda^2) + 2\omega J C_j (\lambda_j^{(0)} - \lambda) + \gamma C_j (j L_2 j) &= 0 \end{aligned} \right\} \quad (29)$$

where

$$(j L_1, i i) = \int_0^L H_j^{(0)} L_1 i H_i^{(0)} ds \quad (30)$$

C_j and λ perturb by γ . That is,

$$\left. \begin{aligned} C_j &= C_j^{(0)} + \gamma C_j^{(1)} + \gamma^2 C_j^{(2)} + \dots \\ \lambda &= \lambda^{(0)} + \gamma \lambda^{(1)} + \gamma^2 \lambda^{(2)} + \dots \end{aligned} \right\} \quad (31)$$

Substituting eq.(31) into eq.(29), we arrange the equation in the order of γ . Putting γ as zero, we get an approximate value of the zero order. Neglecting the terms over the second order of γ , the first approximate value is obtained. The same procedure is repeated to the third order of γ . The k th mode function is expressed as follows in the second order of γ .

$$\begin{aligned} H_k &= H_k^{(0)} + \gamma \sum_{i \neq k} \frac{(i L_1 k)}{J (\lambda_k^{(0)} - \lambda_i^{(0)}) (\lambda_k^{(0)} + \lambda_i^{(0)} + 2\omega)} H_i^{(0)} \\ &+ \gamma \sum_{i \neq k} \frac{-(i L_1 k) (k L_1 i)}{J (\lambda_k^{(0)} - \lambda_i^{(0)})^2 (\lambda_k^{(0)} + \lambda_i^{(0)} + 2\omega)^2} H_i^{(0)} \\ &+ \gamma \sum_{j \neq k} \frac{(i L_1 j) (j L_1 k)}{J^2 (\lambda_k^{(0)} - \lambda_j^{(0)}) (\lambda_k^{(0)} + \lambda_j^{(0)} + 2\omega) (\lambda_i^{(0)} - \lambda_j^{(0)}) (\lambda_i^{(0)} + \lambda_j^{(0)} + 2\omega)} H_i^{(0)} \end{aligned} \quad (32)$$

The covariance between k th and l th modes is given by its definition as follows.

$$\text{cov}(H_k, H_l) = E[(H_k - E(H_k)) (H_l - E(H_l))] = E(H_k H_l) - E(H_k) E(H_l) \quad (33)$$

$E(H_k)$ and $E(H_k, H_l)$ have the following expressions in the first order of γ .

$$E(H_k) = H_k^{(0)}, \quad E(H_k, H_l) = H_k^{(0)} H_l^{(0)} + \gamma^2 \sum_{i \neq k} \sum_{j \neq l} E(C_i^{(1)} C_j^{(1)} H_i^{(0)} H_j^{(0)}) \quad (34)$$

Therefore the variance of the mode becomes eq.(35).

$$\begin{aligned} \sigma_{H_k}^2 &= \text{cov}(H_k, H_k) \\ &= \gamma^2 \sum_{i \neq k} \sum_{j \neq k} \frac{J^2 (\lambda_k^{(0)} - \lambda_i^{(0)}) (\lambda_k^{(0)} - \lambda_j^{(0)}) (\lambda_i^{(0)} + \lambda_j^{(0)} + 2\omega) (\lambda_k^{(0)} + \lambda_j^{(0)} + 2\omega)}{H_i^{(0)} H_j^{(0)}} \end{aligned} \quad (35)$$

where

$$\begin{aligned} (ij L_1^2 k k) &= \int_0^L \int_0^L H_i^{(0)}(s) H_j^{(0)}(r) \left[E(\kappa(s) \kappa(r)) \frac{\partial^8}{\partial s^4 \partial r^4} + \frac{\partial}{\partial r} E(\kappa(s) \kappa(r)) \frac{\partial^7}{\partial s^4 \partial r^3} \right. \\ &+ \frac{\partial^2}{\partial r^2} E(\kappa(s) \kappa(r)) \frac{\partial^5}{\partial s^4 \partial r^2} + \frac{\partial}{\partial s} E(\kappa(s) \kappa(r)) \frac{\partial^7}{\partial s^3 \partial r^4} + \frac{\partial^2}{\partial s \partial r} E(\kappa(s) \kappa(r)) \frac{\partial^6}{\partial s^3 \partial r^3} \\ &+ \frac{\partial^3}{\partial s \partial r^2} E(\kappa(s) \kappa(r)) \frac{\partial^5}{\partial s^3 \partial r^2} + \frac{\partial^2}{\partial s^2} E(\kappa(s) \kappa(r)) \frac{\partial^5}{\partial s^2 \partial r^4} + \frac{\partial^3}{\partial s^2 \partial r} E(\kappa(s) \kappa(r)) \frac{\partial^5}{\partial s^2 \partial r^3} \\ &\left. + \frac{\partial^4}{\partial s^2 \partial r^2} E(\kappa(s) \kappa(r)) \frac{\partial^4}{\partial s^2 \partial r^2} \right] H_k^{(0)}(s) H_k^{(0)}(r) ds dr \end{aligned} \quad (36)$$

under the assumption of $E(\gamma) = 0$, $E(\nu) = 0$, $E(\nu k) = 0$

3-3 EXAMPLE OF NUMERICAL CALCULATION AND DISCUSSION

The numerical data and the constant values of the continuous system are shown in Tables 1 and 2. The variances of the amplitude and the mode are shown in Figs. 1-4. Fig. 1 shows the coefficient of variance of the amplitude caused by the variance of the eccentricity of the center of gravity. The coefficient of variance becomes maximum before and after the eigenfrequency. It is easy to understand this fact, that is, the tendency of the asymmetry of stiffness appears because of the error of the bending stiffness. Figs. 2 and 3 show the variation of the amplitude caused by the term of the gravity force. As the variations of both $|U|$ and $|V|$ become maximum near half of the eigenfrequency, an increase of the amplitude can be expected. The increase of the amplitude at this point is also caused by the character of the asymmetric rotor, so we have to pay attention more to the increase of the amplitude caused by the gravity force than to that of the eccentric force. The standard deviation of the mode obtained by the perturbation method is shown in Fig 4. In this calculation, it

Fig. 1 Coefficient of variation of $|R|$

Fig. 2 Standard deviation of $|U|$

Fig. 3 Coefficient of variation of $|V|$

is assumed that the auto correlation function of the uncertain elements κ and ν in the axial direction is δ function, and κ and ν are statistically independent. In eq.(35) it is necessary to calculate the infinite series, but it is verified by the numerical calculation that the summation to 40th term is enough. These figures show that the variances of the shape of the mode are quite different from those of the symmetric rotor, and there are such inclination that the variances become maximum at the node of the mode and becomes minimum at the maximum amplitude.

Fig. 4 Standard deviation of vibrational mode

Table 1 Details of simple rotor

m	80 kg
k_1	3 100 kg/cm
k_2	3 100 kg/cm
ω_n	194.9 rad/s
σ_{k_1}/k_1	0.05
σ_{k_2}/k_2	0.05

Table 2 Details of continuous rotor

L	850 mm
d	75 mm
ω_n	131 rad/s
$\sigma_{E11}/E11$	0.05
$\sigma_{E12}/E12$	0.05
τ	0.1
σ_σ	0.1
σ_ϵ	0.1

4. EVALUATION OF THE UNSTABLE PROBABILITY OF THE ROTOR SYSTEM

In this section the unstable probability caused by the errors of asymmetry of the bending stiffness is described using the distribution of the stable condition of the system obtained by maximum likelihood method.

Furthermore, to investigate the accuracy of the analytical results, a simulation is performed by Monte Carlo method and the results of the simulation are compared with those of analysis.

4-1 EVALUATION OF UNSTABLE PROBABILITY OF ROTOR SYSTEM BY MAXIMUM LIKELIHOOD METHOD

In eq.(3) obtained in section 2, ζ and μ are the nondimensional values of the damping ratio of the system and the asymmetric ratio of the bending stiffness of the shaft respectively. And the stability of the system is decided by the relation between those two values.

Putting

$$\zeta^2 - \zeta^4/4 = X, \mu^2 = Y$$

then X and Y vary by the errors of the bending stiffness of the shaft k_1 and k_2 . Therefore, assuming that k_1 and k_2 are random variables the expected value and the standard deviation are obtained by maximum likelihood method.

The expected values and the standard deviations of these two values, that is, X and Y are expected as follows;

$$X = (\bar{X}, \sigma_X), Y = (\bar{Y}, \sigma_Y) \dots\dots\dots(37)$$

Then, the risk P will be obtained as the following expression.

$$P = \frac{1}{\sqrt{2\pi}\sigma_z} \int_{-\infty}^0 \exp\left\{-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma_z}\right)^2\right\} dz \dots\dots\dots(38)$$

where $\bar{z} = \bar{Y} - \bar{X}$, $\sigma_z = \sqrt{\sigma_X^2 + \sigma_Y^2}$

4-2 THE SIMULATION BY MONTE CARLO METHOD

The risk P obtained by in previous section is simulated by Monte Carlo method. It is verified that the characters of every two random numbers obtained by 2000 trials are enough for use.

4-3 EXAMPLES OF THE NUMERICAL CALCULATION AND DISCUSSION

The risks for a simply supported rotor are shown in Figs.5-8. The numerical data of the system are also shown in Table 1. In the calculation the expected value and the coefficient of variance of the bending stiffness k_1 of the shaft are equal to those of k_2 , and they have a negative correlation coefficient. The number of trials of the simulation is 2000. In Figs.5-8 the probabilities of instability are obtained for different mean values of the external damping ratio. The parameter shown in those figures is a correlation coefficient, (a) and (b) show the probability of instability derived from the analysis and the result of the simulation, respectively. As for the influence of the external damping on the system stability, it is found that the larger the damping ratio becomes, the more the appearance of the unstable vibration

is restrained, and that the stronger the correlation becomes the higher the probability of instability becomes. These facts are easily understood because a stronger correction means an asymmetry. By the way in the evaluation, although the coefficient of variance of the bending stiffness of the shaft increases, the probability of instability converges near 0.75 over some value of the coefficient of variance of the bending stiffness. This comes from the fact that probability density was assumed to be normally distributed. The results derived from the evaluation are in good agreement with those obtained from the simulation by Monte Carlo method qualitatively. The variation of the probability of instability caused by the mean value of the external damping ratio

(a) Expected value from theory

(b) Expected value from simulation

Fig.5 Probability of instability

(a) Expected value from theory

(b) Expected value from simulation

Fig.6 Probability of instability

(a) Expected value from theory

(b) Expected value from simulation

Fig. 7 Probability of instability

(a) Expected value from theory

(b) Expected value from simulation

Fig. 8 Probability of instability

and the coefficient of variance of the bending stiffness is shown in Fig. 9. The straight lines show that the unstable vibration occurs with a probability shown in this figure. Moreover, it seems that the unstable vibration occurs more often than with the probability shown in this figure at the right area of the line and less often than with the probability at the left side of it. It is possible to get the quantity of the damping including the bearing by the small bearing approximation, so the allowable value of the error can be obtained from Fig. 9 at the designing or producing stage.

The evaluation of the probability of instability and the results of simulation for the continuous system are shown in Figs. 10 and 11. The probability of instability is evaluated for every mode of this

(a) $\rho = 0.0$ (b) $\rho = -0.5$ (c) $\rho = -1.0$

Fig. 9 Probability of instability due for external damping

(a) Expected value from theory

(b) Expected value from simulation

Fig. 10 Probability of instability for the first mode of the continuous system

system, but just the results of the first and the second modes are shown. As in the boundary condition the continuous system is assumed to be simply supported, it is expected that the analytical results

for the first mode will coincide with those of the simple rotor system. Comparing Fig.7 with Fig.10, this expectation is comes true. Furthermore, compared with the simulation, all tendencies relative to the simple rotor hold good. With regard to the probability of instability in every mode, it is found that unstable vibration occurs with the same probability in every mode. Therefore it seems to be enough to evaluate the probability of instability for a system with one degree of freedom under the assumption of the same boundary conditions in order to evaluate the risk for a rotor system with multi degrees of freedom.

(a) Expected value from theory

(b) Expected value from simulation

Fig.11 Probability of instability for the second mode of the continuous system

5. CONCLUSIONS

The response of the amplitude and the statistics of the stability of the system caused by an error of the bending stiffness of the shaft in two directions were analyzed.

The variation of the amplitude of the rotor becomes maximum at three points, that is, points before and after the eigenfrequency of the symmetrical rotor system, and a point at the half speed of it, so an increase of the amplitude is expectable at those points. The variation of the mode becomes maximum at the mode of the symmetric rotor and becomes minimum at the maximum amplitude, so the amplitude may happen to be large because of a error even at the point of small amplitude in the symmetric rotor.

As for the stability of the rotor system, the probability of instability was evaluated by using the distributions of the errors of the bending stiffness and the external dampings. The smaller the damping ratio becomes, and the bigger the risk becomes. Comparing these probabilities with the results of the simulation by

Monte Carlo method, these are in perfect agreement qualitatively, and in good agreement quantitatively. The results of the evaluation of the probability of instability in every mode have the same values quantitatively.

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