Fluctuation of Oil Film Pressure with Eccentricity and Waviness* (Case of a Light Load and Infinitesimal Elastic Deformation)

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Using a cylinder without vibration and adopting a light load in order to neglect the elastic deformation, the fluctuation of an oil film pressure caused by the eccentricity and the waviness of the cylinder was measured by an instrument manufactured for trial. Then the measured fluctuation of the pressure was compared with the value of the theoretical calculation.

As the result, it was found that the oil film pressure fluctuates periodically with the eccentricity and the waviness, and the degree of fluctuation varies according to the situation in the pressure region. Moreover the calculated fluctuation wave form agreed with the wave form of the experimental results. In regard to the distributions of the maximum values and the minimum values of the fluctuation also, the calculated results agreed with the experimental results.

Key Words: Lubrication Theory, Compressible Flow, Lubrication, Oil Film Pressure, Eccentricity, Waviness

1. Introduction

The problem relating to the rolling-sliding contacts is one of the important subjects in the studies of machine elements and many such studies have been done. Especially analysis of the shape and the pressure distribution of the oil film, which exists between the surfaces of the rolling-sliding contact is a basic subject.

As regards studies of the oil film pressure distribution, there are Martin's lubricating theory⁽¹⁾, Dowson's computation of EHL⁽²⁾, etc.. But most of them treated the subject of the oil film pressure which is generated by the contact of two cylinders without eccentricity and waviness, so there isn't any variation of the oil film pressure and the shape of the oil film with the passing of time. In case of machine elements applied generally, it is more practical, however, to consider cylinders with eccentricity and waviness than to consider a cylinder which is produced with high precision. There is Peppler's study⁽³⁾, for example, concerning the subject done from this viewpoint, but more detailed investigations are needed.

So in this study, we manufactured a measuring instrument for trial, and measured the fluctuation of the oil film pressure with the eccentricity and the waviness of a cylinder. And we got a calculated value of the oil film pressure by substituting the measured value of the eccentricity and the waviness into equations, then we compared the calculated value with the measured value.

2. Notations

- $2a$: Length of the major axis of the ellipse
- $2b$: Length of the minor axis of the ellipse
- h : Oil film thickness
- h_c : Constant
- h_0 : Height of a circle (broken line in Fig.4) from the plate for fixing a pressure sensor
- h_W : Height from static balancing position to action point of w_p
- h^* : Integration constant
- I : Moment of inertia of the test cylinder and arm for fixing it
- k : Spring constant
- K : Volume modulus
- l : Length from supporting pin to spring
- l_a : Length from supporting pin to measuring instrument for oil film pressure
- l_0 : Length of the fine hole
- L_0 : Length of surface of plate for fixing a pressure sensor
- p : Oil film pressure
- p_a : Natural circular frequency
- p_0 : Pressure of the oil pool
- r : Radius of a circle (broken line in Fig.4)
- t : Time
- T : Period
- u_1 : Velocity
- V : Volume of the oil pool
- w_p : Load caused by oil film pressure
- x : Coordinate measured in the direction at right angles to the axis of the test cylinder
- γ : Eccentricity and waviness
- μ : Viscosity

3. Apparatus and test procedure

3.1 Apparatus

A schematic diagram of the apparatus is shown in Fig.1. A test cylinder (2 in Fig.1) is supported by two plunger blocks (8 and 9 in Fig.1) on the arm (3 in Fig.1) for fixing the cylinder, and a ball joint

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(6 in Fig.1) transmits the motion to the test cylinder.

The dimension of the test cylinder is shown in Fig.2. The measuring instrument for the oil film pressure (1 in Fig.1), which is set below the test cylinder, with a semiconductor pressure sensor (8 in Fig.3a) measures the pressure of an oil film which exists between the rolling test cylinder and the surface of the measuring instrument. And a displacement transducer (15 in Fig.1) is set over the arm (3 in Fig.1), then the vertical displacement is measured when the test cylinder rolls.

A weight (5 in Fig.1) fixed at the end of the arm is employed for loading. The arm can turn with the support as a center. And the test cylinder contacts with the surface of the measuring instrument under the weight (5 in Fig.1) and the empty weight of a test cylinder and of the arm. The load can vary with a changing weight, and a spring (4 in Fig.1) is employed to adjust the load. A lubricating system is such that a mineral oil is spurted by a nozzle toward the contact point at a rate of 0.25L/min. As regards the pressure sensor, its allowable pressure is 1 MPa (10.2 kgf/cm²).

A detail drawing of the measuring instrument is shown in Fig.3a. As to the structure of the plate for fixing a pressure sensor (1 in Fig.3a) contacting with a test cylinder, a hole having 3 mm diameter was bored through the plate, and it was stuffed with a pin. One side of the pin was shaved off a little in order that the maximum clearance would become 40 μ m, therefore a fine hole whose shape was a part of circle was formed as in Niemann's experiment⁽⁴⁾ (Fig.3b). After the hole was stuffed with the pin, the surface of the plate (1 in Fig.3a) was ground and then polished by a buffing machine. The bottom of the hole was tapped, and then the pressure sensor was screwed into the plate.

Oil is dropped in the fine hole, and then the pressure is transmitted to the pressure sensor through the fine hole when

Fig.1 Schematic diagram of the apparatus

Fig.2 Dimensions of a test cylinder

a pressure is generated at the surface of the plate.

As regards the measuring instrument, the plate is fixed on a supporting plate (3 in Fig.3a) with bolts and nuts. Then both the plate and the supporting plate can move about 10 mm in a straight line along guides (4 in Fig.3a) on the bed (5 in Fig.3a). After the shifting of the supporting plate by screw (7 in Fig.3a) it is fixed on the bed with bolts. And the movement of the supporting plate is measured with a dial gauge (14 in Fig.1). The ball joint is joined to a gear (10 in Fig.1) which has a small hole (5 mm in diameter). When the test cylinder rolls at a certain angle of rolling, the photo-transistor (11 in Fig.1) receives the light of a signal lamp (12 in Fig.1) through this small hole, and an electric signal is generated. The oil film pressure in the optional angle of rolling can be determined by the measurement of this electric signal (the synchronizing signal) and of the oil film pressure.

3.2 Test procedure

To begin with, the eccentricity and the waviness were measured with an electric micrometer, the sensor of which was in contact with the test cylinder in Fig.1. For measuring the eccentricity and the waviness, the test cylinder was revolved by 5 degrees.

After this measurement the test cylinder was rolled at a certain angular

Fig.3a Detail drawing of the measuring instrument

Fig.3b Structure of a plate for fixing a pressure sensor

velocity. Then the oil film pressure, the displacement of the test cylinder and the synchronizing signal were recorded. Next, the supporting plate was moved a little, and then the length of the movement, the oil film pressure, the displacement of the test cylinder and the synchronizing signal were measured. The supporting plate was moved a little again, then the same measurement was done. This process was repeated, so the distribution and the fluctuation of the oil film pressure could be determined.

Adopted test conditions are shown in Table 1.

4. Calculation

4.1 Analysis of the fluctuation of the oil film pressure with the eccentricity and the waviness

The eccentricity, the waviness, and the vibration of the test cylinder caused by itself are considered as the factors of the fluctuation of the oil film pressure, so theoretical equations are derived taking account of these matters.

As shown in Fig.1, the arm is suspended by a spring which is fixed at the free end of the arm, and it can turn round the supporting pin. Substituting a simpler model for this vibration system, then neglecting the friction and the centrifugal force caused by the eccentricity, we get the following equation.

$$\ddot{h}_W + p_a^2 h_W = \frac{l_a^2}{I} w_p \quad (1)$$

The natural circular frequency is calculated by the following equation.

$$p_a = \sqrt{\frac{k l^2}{I}} \quad (2)$$

In equation(1), w_p which denotes a load caused by the oil film pressure is a function of time, and in steady state it is considered a periodic function having a period T which denotes the period of one rotation of the test cylinder. Introducing Fourier series, we get the following equations.

$$w_p = \frac{w_{10}}{2} + \sum_{n=1}^{\infty} \left[w_{1n} \cos\left(\frac{2n\pi}{T}t\right) + w_{2n} \sin\left(\frac{2n\pi}{T}t\right) \right] \quad (3)$$

$$\left. \begin{aligned} w_{1n} &= \frac{2}{T} \int_{-T/2}^{T/2} w_p \cos\left(\frac{2n\pi}{T}t\right) dt \\ (n &= 0, 1, 2, \dots) \\ w_{2n} &= \frac{2}{T} \int_{-T/2}^{T/2} w_p \sin\left(\frac{2n\pi}{T}t\right) dt \\ (n &= 1, 2, 3, \dots) \end{aligned} \right\} \quad (4)$$

As w_p is the load caused by the oil film pressure, the following equation is obtained.

$$w_p = L_0 \int_{-\infty}^{\infty} p dx \quad (5)$$

In Fig.4 we show a sectional view of the test cylinder which rolls at a circumferential velocity u_1 and which contacts with the surface of the plate. In Fig.4, γ denotes the eccentricity and the waviness. Supposing the value of γ is constant in the direction of the axis of the test cylinder, the Reynolds equation becomes:

$$\frac{\partial}{\partial x} \left[\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right] = 12 \frac{\partial h}{\partial t} + 6 u_1 \frac{\partial h}{\partial x} \quad (6)$$

From Fig.4 the oil film thickness becomes:

$$h = h_0 + \frac{x^2}{2r} - \gamma \quad (7)$$

Now the relation between h_0 in equation(7) and h_W in equation(1) becomes:

$$h_0 = h_W + \text{constant} \quad (8)$$

Substituting equations(3) and (8) into (1), the following equation is obtained.

$$h_0 = h_c + \sum_{n=1}^{\infty} \left[\frac{l_a^2 w_{1n}}{(p_a^2 - w_n^2) I} \cos w_n t + \frac{l_a^2 w_{2n}}{(p_a^2 - w_n^2) I} \sin w_n t \right] \quad (9)$$

$$w_n = 2n\pi/T \quad (10)$$

In equation(7), γ is considered a function of $(x - u_1 t)$ in the pressure region⁽⁵⁾, so the following equation is obtained.

$$\frac{\partial \gamma}{\partial t} = -u_1 \frac{\partial \gamma}{\partial x} \quad (11)$$

Fig.4 Sectional view of a test cylinder rolling with a circumferential velocity u_1

Table 1 Test conditions

Material of a test cylinder	S43C
Material of the measuring instrument	S45C
Lubricating oil	Turbine oil 32
Moment of inertia of a part for fixing a cylinder	7.4 kg·m ²
Spring constant	2500 N/m
Angular velocity	28.6 rad/s

Substituting equation(11) into (6), the following equation is obtained.

$$\frac{\partial}{\partial x} \left[\frac{(h_0 + x^2/2r - \gamma)^3}{\mu} \frac{\partial p}{\partial x} \right] = 12 \frac{\partial h_0}{\partial t} + 6 u_1 \frac{\partial}{\partial x} \left[h_0 + \frac{x^2}{2r} + \gamma \right] \quad (12)$$

The pressure p is determined by solving the equations(3), (5), (9) and (12) simultaneously. When the moment of inertia I is large enough, the axis of the test cylinder is considered to move little. So the equation(9) is approximated by the following equation.

$$h_0 \div h_c \quad (13)$$

Then analysis becomes easy.

Substituting the equation(13) into (12) and integrating it, the following equation is obtained.

$$\frac{\partial p}{\partial x} = 6 \mu u_1 \frac{h_c + x^2/2r + \gamma - h^*}{(h_c + x^2/2r - \gamma)^3} \quad (14)$$

In this paper we calculated the pressure p by the equation(14) and compared it with the experimental value. Namely we measured the value of γ , which was substituted into the equation(14), and then the numerical calculation was done by the digital computer. It should be added that the following boundary conditions were employed.
 $p = 0$ for $x = -\infty$, $x = 0$.

4.2 Correction of oil film pressure

The measurement of the oil film pressure adopted in this study was performed through a fine hole shown in Fig.3b. If the oil is incompressible, the measured value is identical with the oil film pressure. In case, however, the compressibility of the oil cannot be neglected, the oil flows through the fine hole. So an error is generated by viscosity between the measured value and the real value of the oil film pressure. Therefore the following calculation is adopted for correction.

Supposing a sectional view of the fine hole, which is a part of a circle, is elliptic and then neglecting the inertia and the weight of the oil, we find the following equation:

$$\frac{\Delta V}{\Delta t} = \frac{\pi (p - p_0) a^3 b^3}{4 \mu l_0 (a^2 + b^2)} \quad (15)$$

where ΔV denotes the volume of the oil which flows for a short time Δt into the oil pool. Denoting the volume of the oil pool by V and denoting the volume modulus by K , the increase Δp , which is caused by the oil flow into the oil pool, in the pressure p_0 can be written as:

$$\Delta p_0 = K \frac{\Delta V}{V} \quad (16)$$

Considering the limit value of Δt , the following equation is derived from the equations(15) and (16).

$$\frac{dp_0}{dt} = \frac{\pi K a^3 b^3}{4 \mu l_0 V (a^2 + b^2)} (p - p_0) \quad (17)$$

The equation(17) signifies the relation between the oil film pressure and the pressure of the oil pool which is identical with the measured value.

5. Results

In Fig.5 we show an example of the pressure fluctuation with the passing of time at a point where the maximum value of the oil film pressure was measured. Adopting the test cylinder with a smooth surface, the eccentricity and the waviness of which are insignificant, the experimental result is shown in Fig.5a. And the axes of ordinate and abscissa represent the pressure and time respectively. In this case the pressure is changing cyclically with a periodic time which is equivalent to one revolution of the test cylinder. And the pressure changing curve is similar to a sine curve. In Fig.5b we show the experimental result given under the following condition, that is, the surface of the test cylinder was scratched by emery paper in parallel with the axis, therefore the waviness and the roughness became large. From these figures the pressure fluctuation curve is seen to be composed of two waves when the surface of the test cylinder has been scratched by emery paper. One is a wave which is equivalent to the pressure changing curve given under the condition that the test cylinder with a smooth surface was adopted. The other is a wave, the shape of which is intricate and the periodic time of which is short. So some influence of the waviness and roughness on the pressure fluctuation is expected.

In Fig.7 we experimentally show the magnitude of the pressure fluctuation, which varies according to a situation in the pressure region. In this case the experimental conditions are that the weight

Fig.5a Measured value of oil film pressure (a cylinder with a smooth surface)

Fig.5b Measured value of oil film pressure (a cylinder with a rough surface)

is 32.4 N (3.3 kgf), and the temperature of the oil is 26 °C. The eccentricity and the waviness of the test cylinder A which was adopted in the experiment mentioned above are shown in Fig.6. And the axis of abscissa represents the angle of rolling. As seen from Fig.7, the magnitude of the pressure changing with the passage of time becomes maximum at a point where the pressure becomes maximum, and it becomes small with an increase in the distance from this point. And the negative pressure is also generated at the outlet in the pressure region.

Making the waviness of various sizes by scratching the test cylinders in parallel with the axis by emery paper, we experimented next. The eccentricity and the waviness of the test cylinder B are shown in Fig.8 and those of the test cylinder C in Fig.9.

Fig.6 Eccentricity and waviness

Fig.7 Pressure distribution

Fig.8 Eccentricity and waviness

The axis of abscissa represents the angle of rolling, and the origin of the axis represents a point where a synchronizing signal is generated by the photo-transistor. The ordinate at the origin represents the value of the eccentricity and the waviness at a contact point of the test cylinder with the surface of the measuring instrument when the synchronizing signal is generated. When the test cylinder rolls in the experiments, the value of the eccentricity and the waviness at a contact point is given by the ordinate determined by the angle of rolling which varies from 0° to 360° accord-

Fig.9 Eccentricity and waviness

Fig.10 Measured value of oil film pressure (Cylinder B)

Fig.11 Measured value of oil film pressure (Cylinder C)

ing to the rotation of the cylinder. The measured result of the pressure changing, at the point where a peak appears on the pressure distribution curve, in case of the test cylinder B (cf. Fig.8) is shown in Fig.10 with the axis of abscissa which represents time. The experimental conditions are that the weight is 32.4 N (3.3 kgf), and the temperature of the oil is 19 °C. In the same figure we show also the displacement of the arm, which is less than 1 μm , so we consider that the supposition adopted in calculation can be accepted. Likewise the measured result in case of the test cylinder C (cf. Fig.9) is shown in Fig.11. The experimental conditions are that the weight is 30.4 N (3.1 kgf), and the temperature of the oil is 25 °C. As seen from Fig.10 and Fig.11, the pressure changes with a periodic time that is one revolution of the test cylinder, though each wave form repeated differs a little.

From the experimental result mentioned above, we can show the magnitude of the pressure changing at each point in the pressure region in Fig.12 (cylinder B) and Fig.13 (cylinder C). As seen from these figures, in which the full lines in parallel with the axis of ordinate represent the magnitude of the pressure changing at each point, the pressure fluctuation becomes maximum at a point where the pressure

Fig.12 Distribution of oil film pressures

Fig.13 Distribution of oil film pressures

becomes maximum, and it becomes small with an increasing distance from this point.

Adopting the eccentricity and the waviness of the test cylinders B and C (cf. Fig.8 and 9), we calculated the oil film pressure and the pressure in the oil pool by equations (14) and (17). The results are shown in Figs.12 ~ 18.

In Fig.14 we show the changing oil film pressure and the changing pressure of the oil pool at a point where the oil film pressure becomes maximum, by means of the peak-valley values in the change that were calculated by substituting various values of h_c into the equation (14). In the same figure the full lines and the broken lines represent the oil film pressure p and the pressure in the oil pool p_0 respectively. And the experimental result is also shown to compare with the calculated value in the same figure. Comparing the experimental result with the calculated pressure in the oil pool, we can infer that:

$$h_c \div 28 \mu\text{m}$$

Likewise the relation between h_c and the oil film pressure p or the pressure in the

Fig.14 Relation between h_c and oil film pressure (Cylinder B)

Fig.15 Relation between h_c and oil film pressure (Cylinder C)

oil pool p_0 is shown with the experimental result in Fig.15. In this case we infer that:

$$h_c \doteq 23 \mu\text{m}$$

Then in Fig.16 (test cylinder B) and in Fig.17 (test cylinder C) we show each fluctuation with the passage of time of the oil film pressure p and the pressure in the oil pool p_0 calculated by these presumed values of h_c . In the same figure the full line and the broken line represent the oil film pressure and the pressure in the oil pool respectively. And each fluctuation described in these figures is generated at a point where the oil film pressure becomes maximum in the pressure region. Comparing the experimental result in Fig.10 with the calculated pressure in the oil pool p_0 in Fig.16, they have the same wave form and the same point of the synchronizing signal. Likewise the same conclusion is drawn from Fig.11 and Fig.17. Moreover as shown in Fig.12 from these calculations about the test cylinder B we obtain also the distribution curves of the oil film pressure p at the time when the pressure becomes maximum or minimum, and the distribution curves which represent the maximum value or minimum value of the

pressure in the oil pool at each point in the pressure region. The full line and the broken line represent the oil film pressure and the pressure in the oil pool respectively. Likewise in Fig.13 we show the distribution curves obtained from the calculated result with the test cylinder C. In these figures some errors are recognized between the calculated value and the experimental value of the pressure in the oil pool. And it is considered that the errors come from substitution of an ellipse in place of a part of circle as the shape of the fine hole in the calculation.

Next the influence of the eccentricity and the waviness on the oil film pressure or the load is determined by calculation in Fig.18. The axis of abscissa represents the peak-valley values of the eccentricity and the waviness, and the axis of ordinate represents the pressure or the load. The result in Fig.18, which is calculated by adopting the eccentricity and the waviness of the test cylinders A ~ C, represents the maximum values or the minimum values of the load and of the pressure. This result is obtained under the condition that:

$$h_c = 25 \mu\text{m}$$

It can be said that the maximum value and the minimum value of the oil film pressure or of the load become major and minor respectively in proportion to the enlargement of the eccentricity and the waviness. So the change of the oil film pressure or the load increases in proportion to the enlargement of the eccentricity and the waviness.

6. Conclusions

- (1) The oil film pressure changes with a periodic time that is one revolution of the test cylinder.
- (2) There is a difference between the changing of the oil film pressure and that of the pressure in the oil pool, for fine changes in the oil film pressure are not reflected in the measurement.

Fig.16 Calculation of fluctuating oil film pressure (Cylinder B)

Fig.17 Calculation of fluctuating oil film pressure (Cylinder C)

Fig.18 Influence of the eccentricity and the waviness on the oil film pressure or the load

(3) The magnitude of the pressure fluctuation becomes maximum at a point where the pressure becomes maximum, and it becomes small with an increase in the distance from this point.

(4) The calculated value obtained by substituting measured values of the eccentricity and the waviness into the theoretical equations agrees with the experimental value roughly.

(5) The maximum value and the minimum value of the oil film pressure or of the load become major and minor respectively in proportion to the enlargement of the eccentricity and the waviness.

These conclusions mentioned above are given under the restrictions that the load

is small, and the axis of the test cylinder hardly moves.

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