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X-ray Measurement of Residual Stresses in a Plate with Both Surface Diffractions under Single Surface Removal or Both Surface Removal*

By Osamu DOI** and Takashi IMURA***

The former X-ray methods of residual stress distribution measurement in a plate is to measure the surface stress by X-ray on one surface, removed surface or nonremoved one, after successive thin layer removal from one surface.

The authors propose in this paper new methods to obtain the residual stress distribution in a plate from the surface stresses measured by X-ray on both surfaces during the single surface removal or the both surface removal and show an example of the new methods applied to a quenched and tempered alloy steel plate of SKS 5.

Keywords: Strength, X-ray Diffraction, Residual Stress Measurement

1. Introduction

The theory of residual stress measurement of a plate by X-ray diffraction method has been proposed.⁽¹⁾⁽²⁾ In this paper, the authors derived two kinds of new X-ray methods to measure residual stress distribution in a plate.

One is a method to measure the surface stresses by X-ray on both surfaces after removing thin layers from one surface of a plate and to calculate residual stresses from the removed surface by use of both surface stresses (Both Surface Subtraction Method, Both Surface Addition Method). This method has no differential term in the calculating formula and produces a smaller calculating error.

The other is a method to measure the surface stresses by X-ray on both surfaces after removing equal thin layers from both surfaces of a plate and then to calculate the residual stresses by use of both surface stresses (Both Surface Removal Method). This method can calculate the residual stress distributions from both surfaces in a plate and has no differential term in the formula.

2. Single Surface Removal- Both Surface X-Ray Method

After thin layer removal of thickness z from one surface in a plate, surface stresses $\sigma_x(z)$ and $\bar{\sigma}_x(z)$ are measured by X-ray on both surfaces as shown in Fig.1. The relations of residual stresses $\sigma_{yx}(z)$, measured stresses by X-ray on the removed surface $\sigma_{yx}(z)$, measured stresses by X-ray

Fig.1 Coordinates of a plate
(Single surface removal)

on the nonremoved surface $\bar{\sigma}_x(z)$, residual stresses on the nonremoved surface $\sigma_{yx}(h)$, normal stress changes by removal $\sigma_{nx}(z)$ and bending stress changes by removal $\sigma_{bx}(z)$ are expressed as follows.

$$\sigma_{yx}(z) = \sigma_{yx}(z) + \sigma_{nx}(z) + \sigma_{bx}(z) \quad \dots\dots\dots(1)$$

$$\bar{\sigma}_x(z) = \sigma_{yx}(h) + \sigma_{nx}(z) - \sigma_{bx}(z) \quad \dots\dots\dots(2)$$

2.1 Both Surface Addition Method From eqs.(1), (2)

$$\sigma_{yx}(z) + \bar{\sigma}_x(z) - \sigma_{yx}(h) = \sigma_{nx}(z) + 2\sigma_{bx}(z) \quad \dots\dots\dots(3)$$

For unit width

$$\sigma_{nx}(z) = \frac{1}{(h-z)} \int_0^z \sigma_{yx}(\zeta) d\zeta \quad \dots\dots\dots(4)$$

$$\sigma_{yx}(z) + \bar{\sigma}_x(z) - \sigma_{yx}(h) = \bar{\sigma}_x(z) \quad \dots\dots\dots(5)$$

From eqs.(3), (4), (5)

$$\sigma_{yx}(z) + \frac{2}{(h-z)} \int_0^z \sigma_{yx}(\zeta) d\zeta = \bar{\sigma}_x(z) \quad \dots\dots\dots(6)$$

The solution of eq.(6) is

$$\sigma_{yx}(z) = \bar{\sigma}_x(z) - 2(h-z) \int_0^z \frac{\bar{\sigma}_x(\zeta)}{(h-\zeta)^2} d\zeta \quad \dots\dots\dots(7)$$

2.2 Both Surface Subtraction Method From eqs.(1), (2)

$$\sigma_{yx}(z) - \bar{\sigma}_x(z) + \sigma_{yx}(h) = \sigma_{nx}(z) + 2\sigma_{bx}(z) \quad \dots\dots\dots(8)$$

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** Professor, Faculty of Engineering, Hokkaido University, Nishi-8 Kita-13 Kita-ku Sapporo.

*** Instructor, Faculty of Science and Technology, Meijo University, 69-49 Yagotourayama Tenppaku-ku Nagoya.

For unit width

$$\sigma_{yz}(z) = \frac{6}{(h-z)^2} \int_0^z \left(\frac{h+z}{2} - \zeta \right) \sigma_x(\zeta) d\zeta \quad (9)$$

$$\sigma_x(z) - \bar{\sigma}_y(z) + \sigma_y(h) = \bar{\sigma}_y(z) \quad (10)$$

From eqs.(8),(9),(10)

$$\sigma_x(z) + \frac{12}{(h-z)^2} \int_0^z \left(\frac{h+z}{2} - \zeta \right) \sigma_x(\zeta) d\zeta = \bar{\sigma}_y(z) \quad (11)$$

The solution of eq.(11) is

$$\sigma_x(z) = \bar{\sigma}_y(z) + 6(h-z) \int_0^z \frac{\bar{\sigma}_y(\zeta)}{(h-\zeta)^2} d\zeta - 12(h-z)^2 \int_0^z \frac{\bar{\sigma}_y(\zeta)}{(h-\zeta)^3} d\zeta \quad (12)$$

3. Both Surface Removal-Both Surface X-Ray Method

The residual stresses are calculated from surface stresses measured by X-ray on both surfaces removing thin layers with equal thickness from both surfaces of a plate as shown in Fig.2. The relations of residual stresses $\sigma_x(z)$, $\sigma_x(h-z)$ measured stresses by X-ray $\sigma_y(z)$, $\sigma_y(h-z)$ on the surface (1) and (2), normal stress changes $\sigma_{nz}(z)$, $\sigma_{nz}(h-z)$ on the surfaces (1) and (2), bending stress changes $\sigma_{bz}(z)$, $\sigma_{bz}(h-z)$ on the surfaces (1) and (2) are found as follows.

$$\sigma_x(z) = \sigma_y(z) + \sigma_{nz}(z) + \sigma_{bz}(z) + \sigma_{nz}(h-z) - \sigma_{bz}(h-z) \quad (13)$$

$$\sigma_x(h-z) = \sigma_y(h-z) + \sigma_{nz}(z) - \sigma_{bz}(z) + \sigma_{nz}(h-z) + \sigma_{bz}(h-z) \quad (14)$$

For unit width

$$\sigma_{nz}(z) = \frac{1}{(h-2z)^2} \int_0^z \sigma_x(\zeta) d\zeta \quad (15)$$

$$\sigma_{bz}(z) = \frac{6}{(h-2z)^2} \int_0^z \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta \quad (16)$$

$$\sigma_{nz}(h-z) = \frac{1}{(h-2z)^2} \int_{h-z}^h \sigma_x(\zeta) d\zeta \quad (17)$$

$$\sigma_{bz}(h-z) = \frac{6}{(h-2z)^2} \int_{h-z}^h \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta \quad (18)$$

From eqs.(13)~(18)

$$\sigma_x(z) = \sigma_y(z) + \frac{1}{(h-2z)^2} \left\{ \int_0^z \sigma_x(\zeta) d\zeta + \int_{h-z}^h \sigma_x(\zeta) d\zeta \right\} + \frac{6}{(h-2z)^2} \left\{ \int_0^z \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta - \int_{h-z}^h \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta \right\} \quad (19)$$

$$\sigma_x(h-z) = \sigma_y(h-z) + \frac{1}{(h-2z)^2} \left\{ \int_0^z \sigma_x(\zeta) d\zeta + \int_{h-z}^h \sigma_x(\zeta) d\zeta \right\} - \frac{6}{(h-2z)^2} \left\{ \int_0^z \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta - \int_{h-z}^h \left(\frac{h}{2} - \zeta \right) \sigma_x(\zeta) d\zeta \right\} \quad (20)$$

From eqs.(19),(20)

$$\sigma_x(z) + \sigma_x(h-z) = \sigma_y(z) + \sigma_y(h-z) + \frac{2}{(h-2z)^2} \left\{ \int_0^z \sigma_x(\zeta) d\zeta + \int_{h-z}^h \sigma_x(\zeta) d\zeta \right\} \quad (21)$$

The solution of the integral equation (21) is

$$\sigma_x(z) + \sigma_x(h-z) = \sigma_y(z) + \sigma_y(h-z) - 2 \int_0^z \frac{\sigma_y(\zeta) + \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta \quad (22)$$

From eqs.(19),(20)

$$\sigma_x(z) - \sigma_x(h-z) = \sigma_y(z) - \sigma_y(h-z) + \frac{12}{(h-2z)^2} \left\{ \int_0^z \left(\frac{h}{2} - \zeta \right) \sigma_y(\zeta) d\zeta - \int_{h-z}^h \left(\frac{h}{2} - \zeta \right) \sigma_y(\zeta) d\zeta \right\} \quad (23)$$

The solution of the integral equation (23) is

$$\sigma_x(z) - \sigma_x(h-z) = \sigma_y(z) - \sigma_y(h-z) - 6(h-2z) \int_0^z \frac{\sigma_y(\zeta) - \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta \quad (24)$$

From eqs.(22) and (24)

$$\sigma_x(z) = \sigma_y(z) - \int_0^z \frac{\sigma_y(\zeta) + \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta - 3(h-2z) \int_0^z \frac{\sigma_y(\zeta) - \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta \quad (25)$$

$$\sigma_x(h-z) = \sigma_y(h-z) - \int_0^z \frac{\sigma_y(\zeta) + \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta + 3(h-2z) \int_0^z \frac{\sigma_y(\zeta) - \sigma_y(h-\zeta)}{(h-2\zeta)^2} d\zeta \quad (26)$$

Fig.2 Coordinates of a plate (Both surface removal)

4. Examples of application

As an application of this method, residual stresses of a tempered alloy steel plate of SKS 5 (102 mm width, 1.07 mm thickness) are measured. The test piece with a length 195 mm and a width 19 mm is cut away from a center of the test material.

Thin layers are removed by electropolishing. The conditions and the apparatus of electropolishing are shown in Table 1 and Fig.3. X-ray stress measurement is done by Geiger Counter method under the conditions shown in Table 2.

The measured stresses by X-ray with single surface removal are shown in Fig.4 a) and Fig.4 b). The residual stress distributions in the plate can be calculated by the author's new method and former one⁽¹⁾ from the measured values. The residual stress distributions obtained by two different methods are in good agreement. Fig.5 shows residual stress distributions.

The measured stresses by X-ray with both surface removal are shown in Fig.6. The residual stress distributions calculated from Both Surface Removal-Both Surface X-Ray Method are shown in Fig.7. The residual stress distributions in Fig.5 and Fig. 7 agree closely.

Fig.3 Experimental apparatus

Chemical components	% (Wt)	HCl 3 H ₂ O 97
Temperature	°C	20 ~ 22
Current density	A/cm ²	0.54

Table 1. Conditions of electropolishing

Characteristic X-ray	Cr-K α
Tube voltage	25 KV
Tube current	10 mA
Time constant	8 s
Slit divergent angle	1.02°
Irradiated area	4 x 2 mm ²
Determination of peak point	Half value width method

Table 2. Conditions of X-ray stress measurement

a) x-direction

Fig.4 Measured stress by X-ray (Single surface removal)

b) y-direction

Fig.4 Measured stress by X-ray
(Single surface removal) Fig.6 Measured stress by X-ray
(Both surface removal) Fig.5 Residual stress distributions
(Single surface removal) Fig.7 Residual stress distributions
(Both surface removal)

4. Conclusions

(1) The authors proposed two new X-ray methods to measure two principal residual stresses in a plate.

(2) The formula of the Single Surface Removal-Both Surface X-Ray Method has no differential term and produces a smaller calculating error.

(3) Both Surface Removal-Both Surface X-Ray Method can calculate the residual stress distributions from both surfaces and the formula has no differential term.

(4) The authors confirmed experimentally that the results obtained by the new methods agreed with those by the former method.

References

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