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Flows in Diffusers Interfered with Wake behind a Circular Cylinder*

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Effects of the wake behind a cylindrical stay upon internal flows and performances of rectangular cross-sectional diffusers were examined.

When a stay of a moderate diameter is set at an appropriate position in the passage, the flow separation is controlled effectively. This seems to be mainly attributable to the blockage effect of the stay, the displacement effect of the wake flow and the increment of the turbulent energy. As the result, the performance of the diffuser with a fairly large divergent angle is improved.

The flow pattern and pressure recovery are also discussed theoretically in this paper.

Key Words: Boundary Layer, Wake, Separation Control, Internal Flow, Interfered Flow, Diffuser Performance

1. Introduction

Performances and internal flows in various diffusers have been studied and the desirable shape of the diffuser has been decided⁽¹⁾⁻⁽⁷⁾. The effects of nonuniform inlet velocity profiles upon the flow in diffusers and their performances have also been investigated⁽⁸⁾⁻⁽¹⁰⁾. It has been pointed out, as a special case, that the performance may be improved by inserting a suitable obstacle into the separated region of a suddenly enlarging channel⁽¹¹⁾⁽¹²⁾.

From a structural standpoint, it seems necessary for a large-sized diffuser to be reinforced with stays which are installed in the channel. The stays may affect the flow pattern and the performance, but their effects have not been investigated yet. To know these effects, in this paper, the internal flow disturbed by the wake flow and the diffuser performance were examined in rectangular cross-sectional diffusers with a circular cylinder set parallel to the side wall near the inlet region. This study has been continued from the previous paper which dealt with the interference between the boundary layer flow and the wake flow without pressure gradient⁽¹³⁾.

Symbols

u : interfered velocity (see Fig.12).
 u_b : velocity in the boundary layer unaffected by the wake flow (see Fig.12).
 u_w : velocity in the wake unaffected by the wall (see Fig.12).
 u_m : velocity at the wake center (see Fig.12).

u_i : imaginary velocity of the isolated wake at the position corresponding to the wall surface (see Fig.12).
 u_0 : main flow velocity at the inlet.
 $\bar{u}_0$: mean velocity at the inlet.
 u_{max} : maximum velocity.
 u' : turbulent velocity fluctuation.
 u_τ : frictional velocity ($=\sqrt{\tau_0/\rho}$).
 y_0 : y -coordinate of the center of a circular cylinder (see Fig.12).
 y_c : y -coordinate of the wake center (see Fig.12).
 ξ_0 : ξ -coordinate of the center of a circular cylinder (see Fig.12).
 d : diameter of a circular cylinder.
 b : half width of the wake.
 ϕ : half divergent angle of the diffuser.
 W_0 : half width of the inlet passage.
 p : static pressure.
 p_0 : static pressure at the inlet.
 C_p : pressure recovery coefficient ($= (p-p_0)/\frac{1}{2}\rho u_0^2$).
 δ : thickness of the boundary layer (see Fig.12).
 δ^* : displacement thickness of the boundary layer.
 θ : momentum thickness of the boundary layer.
 H : shape factor ($=\delta^*/\theta$).
 τ_0 : shearing stress at the wall.

2. Experimental apparatus and procedure

Diffusers were arranged just behind the nozzle of a wind tunnel. As illustrated in Fig.1, the channel walls have sharp leading edges and flexible throats. To avoid the decrement of the inlet pressure, a reasonable perforate plate is installed at the outlet of the channel.

Air flow was applied here. The divergent angles 2ϕ were 2.8°, 5.6°, 8.4°, 10°, 11.2° and 14° for Diffuser A, and were 20° and 40° for Diffuser B. Dimensions of these diffusers are given in Fig.1. Circular cylinders of diameters $d/W_0 = 0 \sim 2$ were tested and their setting position ξ_0/W_0 was varied between -3.6 and 13.2. Where, W_0 is a half

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width of the inlet channel, and $W_0=25\text{mm}$ for Diffuser A (600mm high) and $W_0=15\text{mm}$ for Diffuser B (240mm high). The co-ordinates (ξ, η) are shown in Fig.1.

The velocity u and the turbulent velocity fluctuation u' were measured with a Pitot tube and a hot-wire anemometer, respectively. The pressure recovery coefficient was evaluated from the pressure at the side wall. To know the effects of the wake flow upon the boundary layer flow in detail, a spectral analysis was also carried out by using a frequency analyzer with a constant-band filtering circuit ($\alpha=4.88\text{ Hz}$).

Through all the experiments, the velocity of the main flow was kept constant at the inlet standard cross-section, that is $u_0=30\text{m/s}$ at $\xi=-125\text{mm}$ for Diffuser A and $u_0=40\text{m/s}$ at $\xi=-70\text{mm}$ for Diffuser B. A two-dimensional flow was confirmed on these sections. On the downstream cross-section at $\xi/W_0=18$ of Diffuser A, the two-dimensional flow was checked between two planes $\pm 150\text{ mm}$ from the middle plane of the passage for various experimental conditions. Accordingly, the flows were measured on the middle plane for Diffuser A. For Diffuser B with a large divergent angle, the flow was measured on many planes, because the two-dimensional flow seems difficult to kept.

3. Discussions of experimental results

3.1 Internal flows

Figure 2 shows the velocity distributions for $2\phi=10^\circ$. Where, u_0 denotes the mean velocity at the inlet and the full lines are theoretical results mentioned later. For a diffuser without a circular cylinder, the flow separates from one side wall at $\xi/W_0 \approx 8$ and inclines toward the other side wall gradually. When the cylinder is installed, however, the separation is controlled successfully and the velocity distributions show symmetrical features anywhere. The separation control may be induced by the decrement of the boundary layer thickness on the wall near the cylinder and the displacement of the wake flow behind the cylinder. The velocity defect of the wake decreases markedly in the downstream and a fairly uniform velocity distribution is observed.

Even though the separation occurs widely in the large divergent channel ($2\phi=20^\circ$), the separated flow is reattached to both walls of the downstream straight channel and a fully developed boundary layer is observed (see Fig.3(a)). As realized in Fig.3(b), however, a suitable cylinder installed in such a diffuser seems to be also useful for controlling the separation. As the result, the velocity distribution becomes roughly uniform on the outlet section of the diffuser and the thickness of the boundary layer decreases considerably. Where, $u_{\max}$ is the maximum velocity on the cross-section.

Figure 4 presents the distribution of the turbulent energy ($E_t=(\sqrt{u'^2})^2$). On the section near the cylinder ($\xi/W_0=5.88$), the energy increases particularly in the wake region and decreases in the boundary layer region except for the region in the immediate neighborhood of the wall in comparison

	al	bl	cl	dl
Diffuser A	450	1000	1680	65
Diffuser B	170	475	2600	60

Fig.1 Experimental apparatus

(a) Case without circular cylinder

(b) Case with circular cylinder

Fig.2 Velocity distributions ($2\phi=10^\circ$)

with the flow without the cylinder. The latter phenomenon seems to occur owing to a steep velocity gradient near the wall caused by the blockage effect of the cylinder. In the downstream, the high energy of the wake spreads over the whole flow region

and the turbulent energy in the boundary layer increases conspicuously unlike the flow without the wake. The increment of the energy is effective to prevent the flow separation.

The characteristics of the turbulent velocity fluctuation for the flow with a wake differ from the one without a wake. Figure 5 indicates the distribution of the energy density spectra $E_f(f) = \langle u'^2 \rangle / \alpha(f)$ (f : frequency, α : constant-band filtering value) (14). In the neighborhood of the edge of the boundary layer ($\eta/W_0=1$), as shown in Fig. 5 (a) which demonstrates the distributions on the cross-section near the cylinder, $E_f(f)$ for the wake flow is considerably large at 0.4 kHz corresponding to the frequency of Karman's vortex sheet but small in the other frequency range unlike the case without the wake flow. On the other hand, $E_f(f)$ with the wake flow increases over the whole frequency range near the side wall ($\eta/W_0=1.5$). In the downstream, $E_f(f)$ with the wake flow increases obviously irrespective of the frequency range and the position, as shown in Fig. 5(b).

When the wake flow interferes with the boundary layer flow, the distribution of the frequency spectra $F_u(k)$ is also different from that without an interference as shown in Fig. 6. Where, $F_u(k) = E_f(f) / (\sqrt{u'^2})^2 u / 2\pi$, $k = 2\pi f / u$ (14)-(16). The relation $F_u(k) \propto k^{-5/3}$ signifies the conveyance of turbulent eddies and $F_u(k) \propto k^{-7}$ signifies the dissipation of the eddies. Near the outer edge of the boundary layer ($\eta/W_0=1$), the region shown by the relation of $F_u(k) \propto k^{-7}$ spreads widely in the flow without the wake, but the relation $F_u(k) \propto k^{-5/3}$ and Karman's vortex sheet appear clearly and the frequency k decreases when the wake flow is imposed. In the inner region of the boundary layer ($\eta/W_0=1.5$), the wake flow does not affect $F_u(k)$ distribution. Furthermore, the frequency of Karman's vortex sheet is about 370 Hz which is the same as the value observed in the flow without pressure gradient, but the sheet can hardly be recognized in the region of $\xi/W_0 > 13$.

3.2 Characteristics of boundary layer

The characteristics of the boundary layer are given in Fig. 7. Here, the displacement thickness $\delta^* = \int (1 - u/u_{\max}) dy$, the

(a) $\xi/W_0=5.88$

Fig. 4 Turbulent energy distributions

(b) $\xi/W_0=8.66$

Fig. 5 Energy density spectrum

(a) Case without circular cylinder (b) Case with circular cylinder

Fig. 3 Equi-velocity lines on the section in the downstream parallel channel ($2\Phi=20^\circ$, $\xi_0/W_0=160$)

momentum thickness $\theta = \int u/u_{\max}(1-u/u_{\max})dy$ and the shape factor $H = \delta^*/\theta$. Each one is the average of the values obtained on both walls. As for the flow without the wake, the displacement thickness increases markedly in the divergent channel and the flow separates from the wall, but as for the flow with the wake, the values δ and θ decreases once by the blockage effect of the cylinder at $\xi/W_0 \approx 4.7$ and increases gradually, but H is kept nearly constant. The existence of a circular cylinder is effective to suppress remarkably the development of the displacement thickness and to control the flow separation.

3.3 Diffuser performance

Figure 8 indicates the relation between the pressure recovery coefficient and the setting position of the cylinder. Where, $c_p = (p - p_0)/(1/2 \rho u_0^2)$ and AR denotes the ratio of the cross-sectional areas. In the case of a diffuser without a cylinder, the pressure scarcely recovers after the separation point (see Fig.8(a)) and C_p decreases once near the throat of a large divergent angle owing to the local separation (see Fig.8(b)). When the diffuser has a downstream parallel channel, the pressure recovers slightly as far as the reattachment point of the separation. When a cylinder is installed in the diffuser, however, the pressure almost recovers in the divergent channel and the C_p values become higher than those in the case of no cylinder. When the cylinder is set extremely near the throat, the inlet passage becomes too narrow and the diffuser performance is lowered (see Fig.8(b), $\xi_0/W_0 = 8$).

(a) $2\phi = 10^\circ$

(b) $2\phi = 20^\circ$

Fig.6 Frequency spectrum

Fig.7 Characteristics of the boundary layer

Fig.8 Effects of the setting position of the cylinder upon the pressure recovery coefficient

The diameter of the cylinder also affects the pressure recovery. Figure 9 shows the effects of the diameter and the setting position upon the pressure recovery coefficient at $AR=3$. The pressure recovery coefficient decreases as ξ_0/W_0 decreases because of the unfavourable blockage of the passage by the cylinder, and this tendency is remarkable as the cylinder diameter increases. When the flow does not separate in the diffuser without the cylinder ($2\phi=5.6^\circ$), the recovery is not improved on account of an additional loss caused by the wake, regardless of the diameter and the setting position of the cylinder.

The effect of the divergent angle upon the pressure recovery is also examined. As shown in Fig.10, the pressure recovery coefficient is improved by setting a moderate cylinder in the range of $2\phi > 7^\circ \sim 8^\circ$ where the flow separation occurs in the diffuser when the cylinder is not installed.

As discussed above, it is clear that the moderate cylinder installed in the suitable position of the diffuser is useful for improving the pressure recovery when the divergent angle is large. Then, the optimum arrangement (diameter and position) is presented in Fig.11, as an example. The figure gives C_p values obtained at $\xi/W_0=160$ where the separated flow is reattached to both walls. The optimum arrangement is determined probably from this figure.

4. Theoretical discussions

4.1 Analytical method

A moderate cylinder prevents the flow separation and makes the flow pattern symmetrical in the diffuser, except in the one with an excessively divergent angle (see Fig.2). Then, the analysis was done by the same procedure presented in the previous paper⁽¹³⁾, under the condition that the boundary layer grew along the wall surface. The co-ordinates and the symbols are given in Fig.12. Moreover, the following equations are described by use of non-dimensional parameters based on W_0 and u_0 , which are indicated by the capital letters.

An interfered flow behind the cylinder is dealt with (see Fig.12). Assuming that the wake flow is not influenced by the wall in the region of $Y > \Delta$, the equation of motion and equation of continuity integrated from $Y=0$ to $Y=Y_c$ (wake center) are written as

$$\frac{d}{dX} \left(\int_0^{\Delta} U^2 dY + \int_{\Delta}^{Y_c} U_w^2 dY \right) = -\frac{1}{2} Y_c \frac{dP}{dX} - U_c^2 \quad (1)$$

$$\frac{d}{dX} \left(\int_0^{\Delta} U dY + \int_{\Delta}^{Y_c} U_w dY \right) = 0 \quad (2)$$

Where, $P=p/(1/2)\rho u_0^2$, $U_c^2=\tau_0/\rho u_0^2$. As a symmetrical flow without a separation is assumed, the position of the wake center located in the center line of the diffuser (ξ -axis in Fig.1) and is given by $Y_c=Y_0+X\tan\phi$.

When the velocity defect of the boundary layer is provided by the relation of U_1-U_0 in the region of $Y < \Delta$ the superposed velocity of the interfered flow may be simply expressed by

$$U=U_w-(U_1-U_0) \quad (3)$$

Fig.9 Effects of the cylinder diameter and the setting position upon the pressure recovery coefficient

Fig.10 Effects of the divergent angle upon the pressure recovery coefficient

Where, U_w is the mean velocity in the wake unaffected by the wall, U_1 is the imaginary velocity of an isolated wake in the position corresponding to the wall surface ($U_1=U_w|_{Y=0}$) and U_0 is the mean velocity in the boundary layer unaffected by the wake flow. The superposed velocity satisfied the condition $U|_{Y=0}=0$.

The same velocity profile of the isolated wake flow as the one suggested by H.Schlichting⁽¹⁸⁾ is applied conveniently in the (X, Y) plane, as it has been verified that the similarity law for U_w profile is valid in the flow field with a slight pressure gradient⁽¹⁷⁾. That is $U_w=U_m[1-F\{1-((Y_c-Y)/B)^{3/2}\}]/(1-F)$, where U_m denotes the velocity at the wake center. Thus, the imaginary velocity

U_1 is expressed by $U_1 = U_m [1 - F(1 - (Y_c/B)^{3/2})^2] / (1 - F)$ for $Y < \Delta$ and $U_1 = U_m / (1 - F)$ for $Y_c > B$. Where, the function $F = 0.98(c_D D \cos \phi / X)^{1/2}$ and the half width of the wake $B = 0.567(c_D D X / \cos \phi)^{1/2}$. The function F and the half width B are expressed in consideration of the effect of the half divergent angle ϕ of the diffuser, and the wake flow is assumed to develop along the center line of the diffuser. The drag coefficient C_D is estimated by using the velocity of the main flow u_d in the setting position of the cylinder.

The velocity in the boundary layer undisturbed by the wake flow given by the wall law $U_s = U_\tau (\ln(U_\tau Y / N) + A) / \kappa$, where, $A = 0.41$, $\kappa = 2.12^{(19)}$ and $N = \nu / W_0 u_0$.

On the other hand, as the relation of $U_b = U_1$ must be obtained at $Y = \Delta$ in the equation (3), the following equation is introduced.

$$U_m [1 - F(1 - (Y_c/B)^{3/2})^2] / (1 - F) = U_\tau (\ln(U_\tau \Delta / N) + A) / \kappa \quad \dots\dots\dots (4)$$

By substituting the equation (3) into the equations (1), (2) and (4), unknown variables are Δ , U_τ , U_m and P . These variables may be calculated according to the Runge-Kutta method, by using one of the following supplemental equations.

(a) Case I : Flow with main flow The following Bernoulli's equation is applied to the main flow.

$$\frac{dP}{dX} + 2 \frac{U_m}{1-F} \frac{d}{dX} \left(\frac{U_m}{1-F} \right) = 0 \quad \dots\dots\dots (5)$$

(b) Case II : Flow without main flow and $\Delta < Y_c$. The frictional velocity U_τ is assumed from the shearing stress presented by H. Blasius⁽²⁰⁾, as follows. The velocity U_1 , here, is used instead of the main flow velocity.

$$U_\tau = (0.0225 U_1^{7/4} (N/\Delta)^{1/4})^{1/2} \quad \dots\dots\dots (6)$$

(c) Case III : Flow without main flow and $\Delta = Y_c$. As the boundary layer has been fully developed,

$$\Delta = Y_c \quad \dots\dots\dots (7)$$

The following numerical analysis was also performed. The upstream flow condition of the cylinder was estimated by the turbulent boundary layer analysis due to E. Truckenbrodt's method⁽²²⁾ by combining the shearing stress given by Ludwig-Tillman⁽²¹⁾. Near the cylinder, the displacement thickness of the cylinder surface was also estimated by Karman-Pohlhausen's method⁽²³⁾ as a laminar boundary layer because $u_d d / \nu \approx 10^4$ in these experiments. The blockage of the passage by the cylinder was modified by this displacement thickness.

4.2 Comparisons with experimental results

The calculated velocity distributions and characteristics of the boundary layer are compared with the experimental ones in Figs. 2 and 7, respectively. The analytical results coincide comparatively well with the experiments when the flow does not separate. A comparison of the pressure recovery coefficient is made in Figs. 8 and 10. Though the additional loss caused by the wake flow is not considered in the analysis, the calculated results can be predicted fairly well,

Fig. 11 Optimum diameter of the circular cylinder and its setting position

Fig. 12 Co-ordinates and symbols
(in the case of reasonable divergent angle)

except for the small divergent angle of the diffuser ($2\phi < 8^\circ$) in which the loss caused by the wake flow considerably affects the pressure recovery. On the other hand, when the divergent angle or the sectional area is excessively large, the analytical results do not coincide with the experiments because the assumed wall law does not hold.

5. Conclusions

The internal flows and performances of rectangular cross-sectional diffusers with cylindrical stays were examined experimentally and theoretically. The main conclusions are as follows.

(1) The flow separation is controlled effectively by installing a moderate circular cylinder parallel to the side wall near the inlet of the diffuser. The separation control mainly may be attributable to three reasons. (a) The cylinder is useful for decreasing once the thickness of the boundary layer on the walls near the setting position of the cylinder. (b) The growth of the boundary layer is suppressed owing to the velocity defect of the wake flow behind the cylinder. (c) The higher turbulent energy produced by the wake flow diffuses into the boundary layer.

(2) By the separation control, the pressure recovery is improved in a diffuser with a large divergent angle.

(3) The flow pattern and pressure recovery are estimated roughly by applying an analytical method using the boundary layer flow superimposed by the wake flow without a pressure gradient.

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