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On Mechanism of Induced Plastic Anisotropy of Polycrystalline Metals

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The experimental results concerning the plastic behaviour of metals along the strain path with a corner have demonstrated some common and fundamental features of the induced plastic anisotropy of polycrystalline metals, for example, the fading memory, the saturation property of pre-strain effect and the delay effect. It is shown that the mechanism of the induced plastic anisotropy is closely related with the internal stress due to the interaction among grains of deformed polycrystalline metals and the common features are related closely with the internal stress.

Computations using a simple mathematical model of polycrystal proposed in relation to the idea of internal stress show a very good qualitative agreement with the experimental results obtained from some typical strain paths by using a thin-walled tubular specimen of brass.

(key Words: Plasticity, Plastic Anisotropy, Polycrystalline Metals, Stress-Strain Relation)

1. Introduction

Plastic behaviour of polycrystalline metals depends on the deformation history. The history effect has been investigated successfully in the strain-controlled torsion-tension experiments where strain paths of a "corner" type have been employed.⁽¹⁾⁻⁽⁶⁾ Figure 1 shows a typical bi-linear strain path with a corner in the two-dimensional vector space of strain $(e_{11}, 2e_{12}/\sqrt{3})$, where e_{12} and e_{11} denote the shear and axial deviatoric strain components, respectively. Such strain paths are obtained by applying a combined load of an axial force and a torque to a thin-walled tubular specimen. In Fig. 1, the segments with arrows indicate the deviatoric stress vector $\bar{s} = (3s_{11}/2, \sqrt{3}s_{12})$ at the corresponding points on the strain path, where s_{12} and s_{11} denote the shear and axial deviatoric stress components, respectively.

The measured stress response along the strain path with a corner is characterized by the direction and magnitude of stress vector $\bar{s}$. The observed difference θ between the directions of stress vector $\bar{s}$ and strain increment vector $d\bar{e} = (de_{11}, 2de_{12}/\sqrt{3})$ called the delay angle, and the observed magnitude of stress vector $|\bar{s}| = \{(3s_{11}/2)^2 + (\sqrt{3}s_{12})^2\}^{1/2}$ are shown in Figs. 2a and 3a, respectively, in relation to the length of strain path after the corner.

As shown in these figures, the stress response exhibits some significant features common in polycrystalline (cubic) metals. In the present paper, the observed common features are listed, discussed and interpreted in terms of the crystal plasticity. For this purpose, idealized single crystal and polycrystal models are proposed referring to Kröner's idea⁽⁷⁾. As the result, it is concluded that the main mechanism of the induced plastic anisotropy should be related closely with internal stress due to the interaction among crystal grains in the polycrystal.

A computational analysis of the induced plastic anisotropy is carried out using a proposed model of single crystal grain. The simplification is justified by the experimental observation that the mild steel (B.C.C.) and the brass (F.C.C.) showed qualitatively the same induced anisotropy behaviours despite the difference in crystallographic structure.⁽¹⁾⁻⁽⁴⁾ In fact, according to the results of computation, the difference in the above-mentioned structures does not affect the qualitative trend but only the grade of the anisotropy. Moreover, in the computation on stress response in the strain-controlled experiments, Lin's approximation⁽⁸⁾ for the internal

Fig. 1 Bi-linear strain path

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stress is used to avoid the iterative procedure incorporated with Kröner's model. The computational time can be saved by these simplifications without loss of any important feature of the original Kröner's model.

The results of computation using such a simplified model agree qualitatively well with the experimental results for a group of strain paths with a pointed corner as well as one with a rounded corner shown in Fig. 5.

2. Experimental Results

First of all, attention will be paid to some essential features of the induced anisotropy appearing in the experimental results described in detail in the previous papers⁽¹⁾⁻⁽⁶⁾. Here, the effect of the third invariant of deviatoric stress has been eliminated from the experimental results by using the coefficient R introduced previously^(3,21), since the discussion will be centered on the effect of the induced anisotropy.

(a) Experimental results

(b) Calculated results

Fig. 2 Delay angle after corner along bi-linear strain path

Figure 2a shows the measured delay angle for a group of bi-linear strain paths of the type shown in Fig. 1.⁽⁴⁾ In this figure, L is the length of strain path defined by

$$L = \int [(de_{11})^2 + (2de_{12}/\sqrt{3})^2]^{1/2}$$

and L_0 the value of L (pre-strain) at the corner point P_0 . These experiments were performed by applying combined loads of an axial force and a torque to thin-walled tubular specimens of polycrystalline brass BsBM2 under the strain-controlled condition.

As found from Fig. 2a, for every bi-linear strain path, the delay angle attains the maximum value at the corner and decreases tending to zero with an increase of deformation along the second straight branch. This means that the material "remembers" just temporarily the anisotropy induced by the foregoing plastic deformation along the first branch. In other words, the material exhibits so called "Fading Memory".

(a) Experimental results

(b) Calculated results

Fig. 3 Magnitude of stress vector after corner along bi-linear strain path

Generally, the curve of delay angle should depend on the pre-strain L_0 . For larger values of the pre-strain L_0 than a certain value, however, the curve may be regarded practically as independent of the value of L_0 , i.e., the plastic anisotropy induced by the first branch is to be saturated. This trend was observed in the experiments on mild steel⁽¹⁾ more clearly than those on brass.

Figure 3a shows a change in the magnitude of stress vector $|\mathbf{s}|$ after the corner, where $|\mathbf{s}_0|$ is the value of $|\mathbf{s}|$ at the corner point P_0 . As found from the figure, the magnitude $|\mathbf{s}|$ shows some amount of a temporary softening just after the corner. In agreement with the saturation property mentioned above, the temporary softening is practically independent of the pre-strain L_0 larger than the value mentioned previously⁽³⁾.

In Fig. 3a, a trend in the fading memory is also apparent. After some increase of deformation along the second straight branch, every curve becomes parallel with the curve for $\theta_c = 0^\circ$ (the dashed line). The curve for $\theta_c = 180^\circ$ may be regarded as parallel with the curve for $\theta_c = 0^\circ$ approximately for $L - L_0 \geq 1.5\%$ (This part of the curve is not seen in Fig. 3a). This means that the shape of the curve showing the relation between the increment of stress magnitude $d|\mathbf{s}|$ and that of the strain dL becomes, after a certain deformation, insensitive to the existence of a corner in the strain path. In this sense, the material exhibits the fading memory in the stress-strain diagrams, too. Same features mentioned above were observed in the experiments on mild steel^(1,2).

3. Model of Polycrystalline Metal

Numerous models on the mechanical behaviour of polycrystalline metals described in terms of the model of single crystal have been studied by many authors (for example, recently, Weng, G.J. and Phillips, A.⁽⁹⁾, Gotoh, M.⁽¹⁰⁾, Abe, T.⁽¹¹⁾, Takahashi, H.⁽¹²⁾, for a survey, see, e.g., Kocks, U. F.⁽¹³⁾, Abe, T.⁽¹⁴⁾).

On the other hand, a simple version of such models is used here to interpret the physical process underlying the essential features of induced plastic anisotropy listed in the previous section.

3.1 Idealized Single Crystal and Its Deformation Mechanism

The slip in an idealized model of single crystal is assumed to be crystallographic but homogeneous. Furthermore, the discrete structure of active slip planes in the actual crystal is not considered here. In the idealized model, a group of slips which have the same orientation tensor β is called a slip system in a grain.

$$\beta = (\bar{n} \otimes \bar{n} + \bar{s} \otimes \bar{s}) / 2, \dots\dots\dots (1)$$

where $\bar{n}$, $\bar{s}$ and $\otimes$ denote a unit normal vector to the slip plane, a unit vector in the slip direction and a diadic product, respectively.

The slip system starts to yield when

the shear stress resolved in the slip direction on the slip plane reaches a critical value (Schmid Law). The critical resolved shear stress in the active slip system generally increases with the slip deformation of the system. This effect is known as work-hardening. Moreover, there are several slip systems in a single crystal which interact with each other causing a mutual work-hardening by changing the critical value of the resolved shear stress. In order to incorporate the effect of mutual work-hardening into the model, we assume a linear form of Taylor's hardening law^(15,16): the critical resolved shear stress in any slip system (both active and latent) in the single crystal changes proportionally to the algebraic sum of plastic shears of all slip systems in a grain. The yield surface of the crystal thus undergoes an isotropic expansion. However, there was confirmed to exist the Bauschinger effect in slip system⁽¹⁷⁾: the slip system ($-\beta$, for example) whose sense is exactly opposite to that of active one (β) may harden less than the latter one (β). Therefore, we modify Taylor's hardening law such that the critical resolved shear stress of the opposite slip system ($-\beta$) is proportionally reduced with an increase of shear strain of the active slip system (β).

3.2 Idealized Polycrystal and Its Deformation Mechanism

A polycrystal in an ideal sense contains a great many single crystal grains in the same orientation, each of which has a different group of the surrounding grains. Since there may be no correlation between the orientations of neighbouring grains, all these surrounding grains taken together may be a large random set and are assumed to be as isotropic as the whole polycrystal is assumed to be. An external stress applied to this model of polycrystal may generally cause a different plastic deformation in each grain from that in the matrix, which may cause an internal stress. Then, the stresses outside and inside of the grain should be different from each other. For a spherical grain with the orientation α in the isotropic matrix, Kröner⁽⁷⁾ established the following equation based on Eshelby's calculation⁽¹⁸⁾.

$$\mathbf{s}^*(\alpha) = \mathbf{s} - \lambda G [\mathbf{e}_p^*(\alpha) - \mathbf{e}_p], \dots\dots\dots (2)$$

where G is the shear modulus,

$$\lambda = 2(7 - 5\nu) / [15(1 - 2\nu)]$$

(ν is Poisson's ratio), $\mathbf{s}^*(\alpha)$ and $\mathbf{e}_p^*(\alpha)$ are deviatoric stress and plastic strain deviators in the single crystal grain with the orientation α . The tensor $\mathbf{s}$ and $\mathbf{e}_p$ are the macroscopic stress and plastic strain deviators in the polycrystal, which are identified with the averages of $\mathbf{s}^*(\alpha)$ and $\mathbf{e}_p^*(\alpha)$ over all orientations α , respectively. The formula (2) is a key expression for understanding the mechanism of induced plastic anisotropy in polycrystalline metals. Regardless of any quantitative detail, this formula gives us an explanation for the experimental features discussed in Sec. 2.

For example, the internal stress

$$\lambda G[e_P^*(\alpha) - e_P]$$

can not grow so high. In fact, the difference between $e_P^*(\alpha)$ and e_P larger than a few percent may cause a fracture of polycrystal. Moreover, before the fracture happens, the increase of internal stress may make other slip systems active and so-called multi-slip state appears in the grain. One mechanism of multi-slip due to the internal stress has been explained by Kocks⁽¹³⁾. The multi-slip increases the degree of freedom for the deformation of grain. Remember that the internal stress may not occur in a polycrystal whose all grains are isotropic and have an identical critical value of resolved shear stress. Now, it can be understood that the multi-slip limits the increase of internal stress. This limitation of internal stress explains the saturation of pre-strain effect in the induced plastic anisotropy.

The fading memory may be also related to the occurrence of multi-slip due to the internal stress. For example, let us consider the plastic behaviour along the bilinear strain path shown in Fig. 1. After a certain increase of deformation along the first branch of the strain path, the multi-slip may occur in almost of all grains in the polycrystal. From a geometrical point of view, this situation may be interpreted such that the microscopic stress $s^*(\alpha)$ in the grain reaches any of the corners of the yield surface (Fig. 4) of the grain and stays there. Just after the corner, the situation is disturbed and $s^*(\alpha)$ moves to another corner of the yield surface after a certain increase of deformation. The macroscopic stress state by averaging such $s^*(\alpha)$ over all the grains may be similar to that before the corner of strain path⁽¹⁹⁾. This mechanism explains the fading memory phenomenon observed in the experiments.

The plastic anisotropy caused by the internal stress causes also a difference (delay angle) between the directions of stress vector $\bar{s}$ and strain incremental vector $d\bar{e}$.

4. Computation

4.1 Lin's Approximation for Internal Stress

In Lin's model of polycrystal, each grain is assumed to have the same (total) strain equal to the macroscopic (averaged) strain, i.e., $e = e^*(\alpha)$. From this assumption, by using the decompositions of deviatoric strain $e^*(\alpha) = e_E^*(\alpha) + e_P^*(\alpha)$, $e = e_E + e_P$ and Hooke's law for the deviatoric components $s^*(\alpha) = 2Ge_E^*(\alpha)$, $s = 2Ge_E$, the following expression of internal stress in Lin's model can be obtained,

$$s^*(\alpha) = s - 2G[e_P^*(\alpha) - e_P]. \quad (3)$$

Comparing Eqs. (2) with (3), we see that, while the coefficient λ in Kröner's model depends on the material through Poisson's ratio, Lin's model sets the coefficient λ equal to 2 regardless of the material. The principal difference between these two models lies in the validity of the compat-

ibility and equilibrium conditions across the grain boundaries in the polycrystal. Kröner's model satisfies, through Eq. (2), both of these conditions. On the other hand, Lin's approximation satisfies the compatibility condition trivially by assuming the strains to be uniform (i.e., $e = e^*(\alpha)$), but violates the equilibrium condition (except for the case in the material with Poisson's ratio $\nu = 0.8$, i.e., $\lambda = 2$).

However, the comparison between Eqs. (2) and (3) indicates that Lin's model may still keep the essential features of the internal stress which seems to be the main mechanism of induced anisotropy. Moreover, Lin's model has a special advantage in the calculation for the strain-controlled path of the type shown in Fig. 1. For the prescribed macroscopic strain path, we can get macroscopic stress response in the following straight forward procedure, without any iterative calculation which must be used in the case of Kröner's model.

An increment of microscopic plastic strain $de_P^*(\alpha)$ in a single crystal with orientation α (the value of α determines the orientation factor $\beta^{(k)}$ of the slip system in the grain) is given by the following expression (the symbol α is omitted for simplicity hereafter).

$$de_P^* = \sum_k d\gamma^{(k)} \beta^{(k)}, \quad (4)$$

where $d\gamma^{(k)}$ is an increment of shear strain in the k -th slip system represented by the orientation factor $\beta^{(k)}$ (Eq. (1)).

According to the Schmid law, the k -th slip system starts to yield when

$$\text{tr}(s^* \beta^{(k)}) = \tau_0, \quad (5)$$

where τ_0 is the initial value of critical resolved shear stress. In the system, we have $d\gamma^{(k)} > 0$

$$\text{when } \text{tr}(ds^* \beta^{(k)}) = d\tau^{(k)} \quad (6a)$$

$$\text{and } \text{tr}(s^* \beta^{(k)}) = \tau^{(k)}, \quad (6b)$$

$$\text{and } d\gamma^{(k)} = 0$$

$$\text{when } \text{tr}(ds^* \beta^{(k)}) < d\tau^{(k)} \quad (7a)$$

$$\text{and } \text{tr}(s^* \beta^{(k)}) = \tau^{(k)} \quad (7b)$$

$$\text{or } \text{tr}(s^* \beta^{(k)}) < \tau^{(k)}, \quad (7c)$$

where $\tau^{(k)}$ is a current value of critical resolved shear stress in the k -th slip system ($\beta^{(k)}$).

From Eqs. (4) and (6), by using the relation $de_E^* = de^* - de_P^*$, $s^* = 2Ge_E^*$ and Lin's assumption $de^* = de$, the following system of equations can be obtained for N active slip systems in the grain.

$$2G[\text{tr}(de \beta^{(k)}) - \sum d\gamma^{(m)} \text{tr}(\beta^{(m)} \beta^{(k)})] = d\tau^{(k)}, \\ (m, k = 1, 2, \dots, N). \quad (8)$$

For the prescribed strain increment de , the shear strain $d\gamma^{(k)}$ is found from Eq. (8), and then de_P^* can be obtained by using Eq. (4). Now the stress ds^* is derived from Hooke's law, and the macroscopic stress ds is the average of ds^* over all orientations.

4.2 Simplification of Model for Two-dimensional Strain Path

In an actual single crystal, the number of slip systems and their orientations depend on the crystallographic structure. However, the essential features of induced anisotropy observed in the experiments do not seem to be so sensitive to the exact crystallographic structure. For example, mild steel (B.C.C.) and brass (F.C.C.) show qualitatively the same induced anisotropy response, irrespective of the difference between crystallographic structure⁽¹⁾⁻⁽⁴⁾. Therefore, in order to avoid an excess complexity caused by considering in detail the structure, both single crystal and polycrystal models are simplified in the following manner.

Every single crystal in our model is assumed to have a single slip plane X-Y with the normal directed to Z-axis, and three pairs of effective slip directions intersecting with each other at an angle 60° are considered in that plane X-Y, where the local coordinate system XYZ is of the Descartes type. When a combined load composed of two shear stress components τ_{zx}^* and τ_{zy}^* is applied to a single crystal model, the corresponding yield surface is represented by a hexagon whose each side (labeled i) corresponds to the critical resolved shear stress ($\sqrt{3}\tau^{(i)}$) of the i-th slip system ($i = 1, 2, \dots, 6$) in the deviatoric stress space ($\sqrt{3}\tau_{zx}^*$, $\sqrt{3}\tau_{zy}^*$). Fig. 4 shows schematically an initial yield surface (dashed line) and a current one (solid) of the single crystal model. According to the notation shown in Fig. 4, the hardening rule discussed in Sec. 3 is expressed in the following forms, for the case that the slip systems 1 and 3 are active, for example.

$$d\tau^{(1)} = d\tau^{(3)} = d\tau^{(5)} = d\tau^{(6)} = K(d\gamma^{(1)} + d\gamma^{(3)}), \quad \dots \dots \dots (9a)$$

$$d\tau^{(2)} = K(d\gamma^{(1)} + d\gamma^{(3)}) - K_0 d\gamma^{(1)}, \quad \dots \dots \dots (9b)$$

$$d\tau^{(4)} = K(d\gamma^{(1)} + d\gamma^{(3)}) - K_0 d\gamma^{(3)}, \quad \dots \dots \dots (9c)$$

where K and K_0 are material constants.

In order to get the stress response of a polycrystal along the prescribed two-dimensional strain path shown in Fig. 1, the polycrystal model is assumed to consist of 60 grains whose slip planes lie in the plane x-y of the reference coordinate system xyz (the Descartes type) and whose slip directions are uniformly distributed in the plane x-y. When the strain path of two shear strain components e_{xz} and e_{yz} are applied to this aggregate, two shear stress components τ_{xz}^* , τ_{yz}^* and two shear strain components e_{xz}^* , e_{yz}^* appear in each grain. As found from this deformation mechanism, we can describe a two-dimensional macroscopic stress response of τ_{xz} and τ_{yz} (τ_{xz} and τ_{yz} are the averaged value of τ_{xz}^* and τ_{yz}^* over all grains, respectively). In this case, the deviatoric stress vector, strain vector, the corresponding invariants and the delay angle are defined as follows: vector

$$\begin{aligned} \vec{s} &= (\sqrt{3}\tau_{xz}, \sqrt{3}\tau_{yz}), \\ \vec{e} &= (2e_{xz}/\sqrt{3}, 2e_{yz}/\sqrt{3}), \end{aligned} \quad \dots \dots \dots (10)$$

magnitude

$$\left. \begin{aligned} |\vec{s}| &= (3\tau_{xz}^2 + 3\tau_{yz}^2)^{1/2}, \\ |\vec{e}| &= (4e_{xz}^2/3 + 4e_{yz}^2/3)^{1/2} \end{aligned} \right\} \quad \dots \dots \dots (11)$$

length of strain path

$$L = \int [4(de_{xz})^2/3 + 4(de_{yz})^2/3]^{1/2} \dots \dots \dots (12)$$

delay angle

$$\theta = \arccos[\vec{s} \cdot \vec{e} / (|\vec{s}| |\vec{e}|)], \quad \dots \dots \dots (13)$$

The equivalency between the two-dimensional deviatoric strain spaces ($2e_{xz}/\sqrt{3}$, $2e_{yz}/\sqrt{3}$) used in the present computation and (e_{11} , $2e_{12}/\sqrt{3}$) used in the experiments is based on the special postulate of isotropy proposed by Ilyushin⁽²⁰⁾ discussed in the previous paper⁽²¹⁾.

As found from the above discussion, the model of a polycrystal is fully specified by the elastic constant G and three material constants of the plastic behaviour: the initial value of critical resolved shear stress τ_0 , the coefficient K in the linearized Taylor's work-hardening law and the coefficient K_0 of proportional reduction of τ^∞ related to the Bauschinger effect. The constants G , τ_0 and K can be estimated from a stress-strain relation obtained by a uniaxial tension test and the constant K_0 can be estimated from an amount of permanent softening⁽¹⁷⁾ related to the Bauschinger effect obtained from an experiment along the strain path with corner angle $\theta_c = 180^\circ$. For the polycrystalline brass used in the corresponding experiments, these material constants were found as $\tau_0 = 7.93 \times 10$ MPa, $G = 2.94 \times 10^4$ MPa and $K = K_0 = 4.90 \times 10^{-2}$ MPa, respectively. As mentioned already, the single crystal model used here has three pairs of slip systems. The number of slip systems was selected by a trial using the material constants estimated according to the procedure mentioned above. In the trial, two and four pairs of slip systems as well as three pairs were tested by computation. The model with two pairs of slip systems has a square type of yield surface and leads to a stronger plastic anisotropy while the one with four pairs of slip systems has an octahedral type of yield surface and leads to a weaker anisotropy than the experimental observations.

5. Comparison between Computational and Experimental Results

By using Lin's approximation for internal stress, the simplified model of polycrystalline metal and the material con-

Fig. 4 Yield surface of single crystal model

stants estimated in the last section, we obtained the computational results shown in Figs. 2b and 3b for strain paths with a corner in the deviatoric strain space ($2e_{xz}/\sqrt{3}$, $2e_{zy}/\sqrt{3}$). Fig. 2b shows a change of delay angle after the corner. As found from the figure, the computational results agree well qualitatively with the experimental ones shown in Fig. 2a except for the results of $\theta_c = 150^\circ$. The discrepancy in the latter case comes mainly from a simplified assumption of the linearized stress-strain relation in every slip system [Eq. (9)]. Fig. 3b shows the stress-strain diagrams obtained by the computation. As found from the figure, the computational results are in a good qualitative agreement with the experimental ones shown in Fig. 3a.

In order to justify again the proposed simple model, the model was applied to another typical example of deformation history. The history is represented by a strain path composed of two straight

Fig. 5 Strain path with a circular corner

(a) Experimental results

(b) Calculated results

Fig. 6 Delay angle along strain path with circular corner

branches with a quarter-circle in two-dimensional vector space of strain (Fig. 5). The experiments⁽²²⁾ were performed also by applying combined loads of axial force and torque to a thin-walled tubular specimen under strain control. The material was the same kind of brass used in the experiments along the strain path with a pointed corner mentioned above, and the radii of circular parts of the strain paths were selected as 0.045, 0.09, 0.19 and 0.45%, and the pre-strain L_0 was fixed at 1.5%. Figures 6a and 7a show the experimental results and Figs. 6b and 7b show the corresponding computational results. In these figures, $R = 0.0\%$ means an orthogonal bilinear strain path with a pointed corner. Figures 6a and 6b show changes of delay angle after the point P_0 . In these figures, the peak of each curve corresponds to the value at the point P_1 in each strain path. Figures 7a and 7b show the stress-strain diagrams after the point P_0 . It is found from Figs. 6a~7b that the computational results obtained from the proposed simple polycrystalline model are in an excellent qualitative agreement with the corresponding experimental results for the strain path with a curved part.

6. Conclusion

Fundamental properties of an induced

(a) Experimental results

(b) Calculated results

Fig. 7 Magnitude of stress vector along strain path with circular corner

plastic anisotropy occurring in the polycrystalline metals, namely the fading memory, the saturation property of pre-strain effect and the delay effect were interpreted by means of idealized single crystal and polycrystal models based on the crystal plasticity. Moreover, the computational results obtained by using the two-dimensional simple models of single crystal and polycrystal proposed here were ascertained to reproduce qualitatively well the complex change in the delay angle and the magnitude of stress vector along the strain paths with a pointed corner as well as the ones with a rounded corner in the two-dimensional vector space of strain. The proposed model were as follows :

1) The simple model of single crystal has a slip plane with three pairs of slip directions (six slip systems). Taylor's work hardening law of slip system was modified referring to the Bauschinger effect. In this single crystal model, there are four material constants which can be determined by two simple tests : uniaxial tension and tension-compression tests.

2) The idea proposed by Kröner on the internal stress was applied in order to consider the interaction between grains. In the computation, Lin's approximation was used for estimating the internal stress arising in the deformation along the strain controlled strain path without the use of iterative procedure which is inevitable in Kröner's model.

The interpretation of the common features by using the idealized models and the qualitative agreement of the computational results obtained by using the simplified models proposed here with the corresponding experimental results indicate that the mechanism of induced plastic anisotropy is governed mainly by the internal stress due to the interaction among grains in the polycrystalline aggregate.

In conclusion, the proposed simple models keeping the essential features of internal stress seems to be very effective for the qualitative prediction of plastic behaviour of polycrystalline metals along the complex deformation history.

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