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Fluid Flow and Heat Transfer around Two Circular Cylinders*

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Succeeding to the former report on two circular cylinders with different diameters, the flow behavior around two cylinders with the same diameters and the local mass transfer around the downstream one have been made clear experimentally in the range of $Re=(1.5\sim 8)\times 10^4$ and the clearance $x^*=(0\sim 5)d$. The main results are summarized as follows. (1) Flow in a closed vortex region formed between two cylinders can be classified. (2) The maximum Nu (Sh) up to the jump clearance closely depends on the velocity gradient at the reattachment point. (3) Overall Nusselt number is proportional to $Re^{0.3}$ and shows an increase of 20-30% as compared with that of a single cylinder in spite of the change of the flow pattern due to the clearance.

Heat Transfer Fluid Behavior Two-Circular Cylinders Jump Phenomena

1. Introduction

In various arrangements of tube bank, there are a number of studies⁽¹⁾ on the flow characteristics (for example, fluctuating forces, shedding vortex et al.) of a downstream cylinder in the wake of an upstream one. On the other hand, for the tube bank used widely as a heat exchanger, the pressure loss and overall heat-transfer coefficient have been examined.⁽²⁾ However, in spite of the importance of the evaluation and analysis of the heat-transfer characteristics on the tube bank, the local heat transfer coefficient around the downstream cylinder in the wake of the upstream cylinder has been little clarified.⁽³⁾

In order to simulate the flow and heat transfer characteristics around the tube banks arranged in lattice, many studies^(4,5) on two circular cylinders in line arrangement and recently on three cylinders with constant pitch have been reported. Kostic⁽⁶⁾ has not made mention of the turbulence behavior in the flow field. On the other hand, Aiba⁽⁵⁾ stated that the strength of vortical layer stretched from the upstream cylinder is the most important factor for the heat transfer coefficient at the reattachment point of the downstream cylinder. However, his definition of the vortex strength was not clear and also he did not make clear the distinction between the velocity fluctuation with Karman vortex and free-stream turbulence. Furthermore, as these experi-

ments were made under the condition of uniform heat flux, an experiment under the iso-thermal wall is desired.

In the former report⁽⁶⁾, in which the diameter of the downstream cylinder is larger than that of the upstream cylinder, it was confirmed that the formation process of the wake near the downstream cylinder and the unsteady and three-dimensional fluid flow affect strongly the heat transfer mechanism in the separated region. Further, in the paper, the relation between fluid flow behavior and heat transfer mechanism around two cylinders with the same diameter is demonstrated especially paying attention to the unsteadiness mainly caused by a vortical flow. Namely, the flow observation beneath the cylinder surface has been made by using the smoke-wire technique and measurement of turbulence by hot-wire anemometer. And the local heat-transfer coefficient has been measured by analogy to mass transfer (naphthalene sublimation method) which corresponds to the condition of uniform wall temperature.

2. Experimental Apparatus and Method

The test section of the wind tunnel was 300x400 mm in cross section and the vertical walls of the test section had been taken away for the purpose of reducing the blockage effect. Cylinder diameter was $d_1=d_2=20^\circ$ or 38° mm, and the clearance (x^*) between two cylinders was $(0\sim 5)d$. Free-stream turbulence intensity was about 0.2% in the experimental range of $Re=(2\sim 8)\times 10^4$. The measurements and arrangements of the local pressure, mass transfer, mean and fluctuating velocities by means of hot-wire anemometer, and flow visualization technique were exactly the same as those of the former report⁽⁶⁾. Fig.1 and Table 1 show the coordinates and nomenclature, respectively.

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Table 1 Nomenclature

C_D	: Pressure Drag Coefficient $= \int_0^\pi C_p \cos(\theta) d\theta$
C_p	: Pressure Coefficient $= (p - p_\infty) / [1/2(\rho U_\infty^2)]$
d	: Cylinder Diameter
D	: Diffusion Coefficient
f	: Vortex Shedding Frequency
h, h_p	: Local Heat and Mass Transfer
k	: Heat Conductivity
p	: Static Pressure
P	: Static Pressure of Free-Stream
Nu	: Local Nusselt Number $= hd/k$
Re	: Reynolds Number $= U_\infty d / \nu$
S	: Strouhal Number $= fd / U_\infty$
Sh	: Local Sherwood Number $= h_p d / D$
Tu	: Turbulence Intensity $= \sqrt{\bar{u}^2} / U_\infty$
u	: Local Velocity
$\hat{u}$	: Velocity at Outer Edge of Laminar Boundary Layer
U_∞	: Free-Stream Velocity
$\sqrt{\bar{u}^2}$	: rms Value of Fluctuating Velocity of u
x^*	: Clearance between Two Cylinders (see also in Fig.1)
y^*	: Normal Distance from the Cylinder Surface (see also in Fig.1)
θ	: Angle from the Front Stagnation Point (see also Fig.1)
θ_m	: Angle of C_{pmax}
θ_s	: Separation Point
ν	: Kinematic Viscosity
ρ	: Density
Suffix	
1	: Upstream Cylinder
2	: Downstream Cylinder
max	: Maximum Value
—	: Mean Value

Fig.1 Coordinate System

3. Characteristics of Flow Behavior

It is well known that the flow pattern around two cylinders changes depending on the arrangement, and a remarkable change of the flow pattern occurs in line arrangement.⁽⁸⁾ Therefore, in order to confirm the relation between the local heat transfer and the flow pattern, the knowledge of a flow near the surface of the circular cylinder will be required.

Fig.2 Pressure Drag Coefficients

3.1 Surface-pressure Distributions and Pressure-drag Coefficients

The variations of C_{D1} (upstream) and C_{D2} (downstream) with the clearance (x^*/d) are shown in Fig.2. C_{D1} and C_{D2} change abruptly at $x^*/d=2.7$. This critical clearance is almost the same as that of the previous reports^(8,9). At a clearance smaller than the critical clearance (called before jump), a thrust force acts on the downstream cylinders ($C_{D2} < 0$). This is due to a negative pressure at the front area of the cylinder, as shown by C_p distribution in Fig.3(b). And also the stagnation pressure is nearly equal to the base pressure of the upstream cylinder (the value at $\theta=180^\circ$ in Fig.3(a)). This result means that a closed vortex is formed in the region between the two circular cylinders.

3.2 Flow Behavior of the Closed Vortex Region

As shown in Fig.2, the thrust force- ($-C_{D2}$) decreases with an increase in x^*/d until zero at $x^*/d=1.5$ and in the range of $x^*/d \geq 1.5$, C_{D2} slightly increases again. However, C_{D2} shows a step change at $x^*/d=2.7$. Biemann's result shown in the same figure indicates the same trend. This suggests that the flow in closed vortex region changes at $x^*/d=1.5$. In this experimental range of Re , Bloor⁽¹⁰⁾ showed that the vortex formation length (L) of a single circular cylinder is nearly equal to $1.5d$ (corresponds to $x^*/d=1.0$). Therefore, as the downstream cylinder is within the vortex formation region of the upstream cylinder in the range of $x^*/d < 1$, the vortex concentration of separating shear layer from the upstream cylinder is prevented and consequently, both shear layers do not interfere with each other. Finally, a quasi-stationary vortex is formed between the cylinders (the flow in closed vortex region is mixing weakly). On the other hand, the behavior of the downstream cylinder for the upstream cylinder is the same as that of the splitter plate set in the wake of a cylinder. As a result, even if x^*/d becomes larger, the separated shear layer from the upstream cylinder is stretched downstream (stretching effect) and the vortex shedding does not occur. However, Strouhal number does not change critically at $x^*/d=2.7$ as will be discussed later

Fig.3 Pressure Distributions around the Circular Cylinders ($Re=5 \times 10^4$)

(Fig.6). Therefore, the shear layer at some clearance in the range of $x^*/d=1\sim 2.7$, may be more unstable. Considering the change of C_{p2} and the singularity of C_p profile at $x^*/d=1.5$, it is suggested that the characteristics of the shear layer will change near the clearance of $x^*/d=1.5$. As an experiment to show the flow behavior in closed vortex region changes at $x^*/d=1.5$, an out-put of a hot-wire with the higher frequency component cut off was recorded, said hot-wire begin set at the front stagnation ($y^*=0.5$ mm from the cylinder surface) of the downstream cylinder. Fig.4 shows the typical results for $x^*/d=0.5, 2.0$ and 3.5 .

In the case of $x^*/d=0.5$, the fluctuation with high frequency cannot be detected while at $x^*/d=2.0$ a periodical fluctuation (twice the Strouhal frequency of the downstream cylinder) appears intermittently. This fact shows that a swing of the shear layer occurs and both shear layers begin to interfere with each other across the center line because of a discontinuous vortex concentration. In the case of $x^*/d=3.5$, an alternate flow with twice the frequency of Strouhal number of a single cylinder occurs due to the influence of Karman vortex shed from the upstream cylinder.

Next, from the C_p profile (see Fig.3(b)) of the downstream cylinder, we shall discuss the change of flow behaviour in closed vortex region. At $x^*/d=1.0$, the position of the minimum value of C_p moves toward the downstream side from the front stagnation point (point A in this figure) and the pressure recovery occurs toward the front stagnation point. This fact suggests that the flow from the reattachment point to the front stagnation point separates near the point A and is entrained into the shear layer from the upstream cylinder. On the other hand, at $x^*/d=1.5$, the position of the minimum value of pressure moves forward to the front stagnation point. Then, the flow along the cylinder surface forms a narrow separating region beneath the front stagnation point and joins at the center line. This joined fluid flows reversely toward the rear region of the upstream cylinder. These phenomena are confirmed by the flow visualization and also can be understood from the fact that the base pressure of the upstream cylinder is higher than the front stagnation pressure of the down-

Fig.4 Signals of the Fluctuation Velocity at the Front Stagnation Point ($\theta=0^\circ$) of the Downstream Cylinder

stream cylinder (at $x^*/d=1.0$, both pressures have nearly the same value.). In the case of $x^*/d=2$, the position of the minimum pressure makes a backward movement from the front stagnation point. This comes from the fact that the flow toward the rear position of the upstream cylinder is prevented because of an intermittent rolling up of the shear layer.

3.3 Reattachment Point

Kostic⁽⁴⁾ reported that the position of C_{pmax} moved toward the front stagnation point with an increase in x^*/d , and the jump phenomena occurred shortly. As shown in Fig.5, however, the reattachment point moves forward in the case of $x^*/d \leq 1.5$, and that point moves backward in the range of $x^*/d > 1.5$ because of the stretching effect of shear layer (discussed in the former section). Therefore, the experimental measurement by Kostic is questionable. In this figure, the variation of the position of the maximum heat(mass)-transfer coefficient near the reattachment point with x^*/d is also shown.

3.4 Strouhal Number

Fig.6 shows the variation of the vortex shedding frequency from the cylinder with x^*/d , while S_1 and S_2 are the Strouhal numbers measured at downstream of the upstream cylinder and the downstream one, respectively. This result is in good agreement with the results by Okajima⁽¹²⁾ ($Re=1.7 \times 10^5$). However, S does not show such a critical change as C_p does. From as predictable from the fact that an intermittent rolling-up of shear layer occurs near at $x^*/d=2$, it seems that the flow pattern becomes unstable in the range of $x^*/d=2.3 \sim 3.0$ where the downstream cylinder does not completely break away

Fig.5 Variations of the Position of C_{pmax} and Sh_{max}

Fig.6 Variations of Strouhal Number with the Clearances

from the vortex formation region of the upstream cylinder. Namely, because the vortex rolling of the upstream cylinder occurs intermittently before and behind the downstream cylinder and these phenomena last, two kinds of special frequency components are observed. In this paper, unstable flow pattern means that a vortex shedding from upstream cylinder sometimes occurs between the two cylinders and sometimes downstream of the downstream one.

3.5 Flow Structure near Separation Point

Considering the flow behavior near the cylinder surface, it is important to determine the separation point. However, it is difficult to evaluate the separation point of a flow with a free-stream turbulence because an appropriate technique is not established. Okajima⁽¹²⁾ stated that oil flow pattern of the downstream cylinder at $x^*/d=2$ and $Re=2.5 \times 10^5$ did not

Fig.7 Separation Point of the Downstream Cylinder

Fig.8 Observation of the Surface Flow by Means of the Smoke-wire Method

show clearly a trace of oil film, being covered by the wake of the upstream cylinder. However, at the subcritical Re range of this experiment ($Re=5 \times 10^4$) the separation line can be recognized very clearly. Fig.7 shows the variation of both the separation point and the point of the minimum Sh with x^*/d . At the clearance before jump phenomena, the separation point is about $\theta_s=115^\circ$ (except at $x^*/d=0.03$) and agrees with the Sh_{min} point. That is, the flow along the downstream cylinder after the reattachment point separates at $\theta_s=115^\circ \sim 120^\circ$ farther downstream than that of a single cylinder ($\theta_s=78^\circ$). Though Kostic regarded this point as the transition point (and turbulent separation point is $\theta_s=145^\circ \sim 150^\circ$), his conclusion is questionable.

Next, after jump, the separation point comes at about $\theta_s=100^\circ$ by the oil-film technique, but the position of the minimum Sh is about $\theta=85^\circ$. Both positions do not coincide with each other and are different from those before jump. Therefore, in order to make clear the near-surface flow just before and behind the separation point, flow visualizations have been conducted by the smoke-wire method at $x^*/d=3.5$. Figs.8(a) and (b) are streak-line photographs at $\theta=90^\circ$ in the case of $Re=2.5 \times 10^4$. Smoke-wire ($d=70 \mu m$) is set at $y^*=0.16$ mm from the cylinder surface and upper part (A) and lower part (B) in this figure are photographs taken at the same time in the normal direction and the tangential direction to the cylinder surface wall, respectively. Fig.(a) shows the attached flow along the wall surface and (b) shows the intermittent reverse flow (arrow mark in this figure shows the ejection of the fluid bulk). Consequently, on time average, it is considered that the flow separates at the downstream near $\theta=90^\circ$, that is, $\theta_s=95^\circ \sim 105^\circ$. At $\theta=120^\circ$ as shown in Fig.(c), an unsteady three-dimensional fluid flow can be observed. And judging from the velocity distribution around the cylinder at special instant discussed in Fig.12, the separation point on time average does not move backward after $\theta=\pm 120^\circ$ and therefore the conclusion by Kostic is doubtful, in which the transition is supposed to occur at $\theta=95^\circ$ (and the turbulent separation point is $\theta=145^\circ \sim 150^\circ$).

4. Heat Transfer Characteristics

4.1 General Characteristics of Local Sherwood Number

Figs.9(a) and (b) show the local Sh profiles at $Re=const.$ ($Re=5 \times 10^4$) at several clearances before and after jump phenomena. At first, on the local Sh profile before jump (Fig.(a)), the maximum value of local sherwood number is obtained at the forward portion by $5 \sim 10^\circ$ upstream from the position of C_{pmax} as shown in Fig.7. The variation of Sh_{max} near the reattachment point with x^*/d corresponds

to the fluid behavior in the closed vortex region mentioned above. As shown in Fig.10, Sh_{max} decreases abruptly with an increase of the clearance until $x^*/d=1.5$, but in the range of $x^*/d=1.5\sqrt{2.3}$ it remains constant, and decreases discontinuously at $x^*/d=2.5$. And then, these values of Sh_{max} are remarkably higher than those at the front stagnation point of the single cylinder in the flow with low free-stream turbulence. Moreover, the local Sh decreases with the development of a boundary layer toward the downstream from the reattachment point and then becomes a minimum Sh .

In the separated region, Sh profile has a second maximum value and Sh at the rear stagnation point is smaller than that of the single cylinder. However, it is noteworthy that the value of Sh in the

Fig.9 Distributions of the Local Sherwood Numbers ($Re=5 \times 10^4$)

Fig.10 Variations of Sh_{max} with the Clearances

separated region is smaller than that at the front region of the cylinder. And Sh decreases monotonously toward $\theta=0^\circ$, but at $x^*/d=1.5$, the minimum Sh occurs near $\theta=0^\circ$ and its value is higher than that at other x^*/d . This corresponds to the flow behavior discussed in section 3.2. The Sh profile at $x^*/d=2.5$ is similar to that after the jump phenomena and this seems to be caused by an increase of the intermittent ratio of the vortex rolling in the downstream of the upstream cylinder as discussed before.

On the other hand, considering the local Sh profile after jump phenomena (Fig.9), the maximum of Sh comes at the front stagnation point ($\theta=0^\circ$) and the front part of the downstream cylinder is affected by the alternate free stream under the influence of Karman vortex shed from the upstream cylinder. In order to recognize the comparatively regular and large fluctuating fluid motion along the front region of the cylinder wall, an simultaneous measurements of the two hot-wire signals (set at $y^*=0.5$ mm) have been done. Then, the typical example in the case of $x^*/d=3.5$ is shown in Fig.11. At $\theta=\pm 30^\circ$ in Fig.(a), both signals are in inverse phase to each other and it is suggested that the flow in the boundary layer fluctuates with the Karman vortex

Fig.11 Typical Traces of the Sampling Hot-wire Out Put

frequency. At $\theta = -50^\circ$ in Fig.(b), a correspondence between the maximum output at $\theta = +30^\circ$ and the minimum output at $\theta = -30^\circ$ as shown in Fig.(a) can not be seen and amplitude modulation occurs. The minimum output at $\theta = -30^\circ$ seems to be the stagnation point as described in Fig.12. Therefore, this fact suggests that the stagnation point is not formed corresponding to the Karman vortex shedding. An intermittent spike can be seen at $\theta = -120^\circ$ (Fig.(c)) and the output signals of the hot-wire fluctuate with low level and high frequency, and the position of $\theta = 120^\circ$ seems to belong to the separated region. Taking the signals of the hot-wire fixed at $\theta = +30^\circ$ as a monitor and selecting the peripheral out put under the condition where the hot-wire signals at $\theta = -30^\circ$ become maximum, the peripheral flow field at some special instant can be calculated as shown in fig.12. The flow forms the stagnation point at $\theta = -30^\circ$, and accelerates counter-clockwise such as the single cylinder (dotted line in this figure) does. But its acceleration region is narrow and the flow separates at $\theta = 105^\circ$. Moreover, in the clockwise direction ($+\theta$ side), the velocity becomes maximum at $\theta = +60^\circ$ and at its downstream the flow seems to have a "phase difference", which suggests the movements of the separation point and the related stagnation point. In either case, the separation point does not move backward to $\theta = \pm 120^\circ$ and beyond. On the contrary, when the stagnation point is formed at $\theta = +30^\circ$, the velocity profile becomes a reversal of this figure. Corresponding to the flow field mentioned above, the local Sh profile does not make a large decrease near the separation point such as the single cylinder. The reason why the Sh profile in the separated region does not increase so monotonously as the single cylinder, is in an outstanding recovery of the back pressure as shown in Cp profile of Fig.3(b). After all, the local Sh after the jump clearance is almost constant in the peripheral direction. And the local Sh tends to decrease over the entire circumference with x^*/d , but the change of its decrease in this experimental range of $x^*/d = 3 \sim 5$ is not so large.

4.2 Arrangement of Numax

Fig.13(a) and (b) show the dependency of Numax on Re as a parameter of x^*/d . Depending on the flow behavior in the closed vortex regions discussed above, its

Fig.12 Velocity Distribution around the Circular Cylinder at Special Instant

dependency on Re at $x^*/d \leq 1.5$ becomes different from that at $x^*/d = 1.5 \sim 2.3$, that is, for the former $\text{Numax} \propto \text{Re}^{0.5}$ and for the latter, $\text{Numax} \propto \text{Re}^{2/3}$, respectively. According to the results by Kostić⁽⁴⁾ as shown in this figure, Numax remains constant in spite of the clearance in the range of $x^*/d = 0.6 \sim 1.7$. However, Aiba⁽⁵⁾ reported that for the middle one of three cylinders in line arrangement, Numax depended on the cylinder clearance. Therefore, the experimental measurement by Kostić seems to be questionable. But, Aiba's conclusion is also doubtful, in which the variation of the reattachment heat transfer coefficient with x^*/d is said to depend on the vortex strength in closed vortex region (Cp defined by $\text{Cpmax} = \text{Cpf}$: where Cpf means the Cp value at the front stagnation point). In the range of $x^*/d = 0.03 \sim 1.5$ where the Numax depends clearly on x^*/d , if the heat transfer coefficient at the reattachment point is assumed to depend on the velocity gradient around the stagnation point such as the front stagnation point in the laminar boundary layer, the following relation is obtained,

$$h_{\max} d / k = 0.940 \sqrt{\frac{d}{U_{\infty}} \left(\frac{d\bar{u}}{d\bar{x}} \right)_{\theta=\theta_m}} / 3.58 \text{Re}^{0.5} \quad (Pr = 0.71)$$

where the value in the root at the right

(a)

(b)

Fig.13 Variations of Numax with Re and the Clearances

hand means the ratio of the non-dimensional velocity gradient at the reattachment point to that of the single cylinder in the uniform flow, and $\hat{x}$ means the distance along the cylinder surface from the point of C_{pmax} toward the front stagnation point. Now, approximating the measured C_p in the region facing the closed vortex region to the fifth order and odd polynomial series, the velocity gradients at the reattachment point, $(d\hat{u}/d\hat{x})_{\theta=\theta_m}$, will be given. Fig.14 shows the relation between its velocity gradient and $Numax$ at $Re=5 \times 10^4$. It is clearly shown in the figure that a large variation of $Numax$ with x^*/d is closely correlated by the velocity gradient near the reattachment point. Introducing Re^* as follows ;

$$Re^* = Re \frac{d}{U_\infty} \left(\frac{d\hat{u}}{d\hat{x}} \right)_{\theta=\theta_m} / 3.58 \quad \dots\dots\dots (1)$$

As shown in Fig.15, $Numax$ can be approximated by the following correlation

$$Numax = 1.67 Re^{*0.5} \quad (0.03 \leq x^*/d \leq 1.5) \quad \dots\dots\dots (2)$$

$Numax$ is larger by about 80% than the theoretical value of the single cylinder in laminar boundary layer. Next, in the range of $x^*/d=1.5 \sim 2.3$ before the jump clearance and in the range of $x^*/d=2.7 \sim 4$ after the jump clearance, $Numax$ is not

Fig.14 $Numax$ and Velocity Gradients at the Reattachment Point

Fig.15 Arrangement of $Numax$ ($0.03 \leq x^*/d \leq 1.5$)

influenced by the clearance and the empirical relations are obtained as follows,

$$Numax = 0.208 Re^{*2/3} \quad (1.5 < x^*/d \leq 2.3) \quad \dots\dots (3)$$

$$Numax = 0.181 Re^{*2/3} \quad (2.3 < x^*/d \leq 4) \quad \dots\dots (4)$$

4.3 $Numax$ and Fluid Flow Behavior beneath the Cylinder Wall

As above mentioned in the case of two circular cylinders in line arrangement, it is a characteristic feature that the heat-transfer coefficient is very high at the reattachment stagnation region around the downstream cylinder. Recently, Kasagi⁽¹⁴⁾ pointed out that there is some possibility of heat-transfer augmentation resulting from the existence of a large-scale eddy in the reattachment flow of free-shear layer. On the other hand, on the augmentation of the heat-transfer coefficient at the stagnation point by the free-stream turbulence, Kestin⁽¹⁵⁾ proposed a vorticity amplification theory. Pertaining to this point, the fluid-flow structure has been observed at the reattached stagnation region around the downstream cylinder. Fig.16 shows photographs of an instantaneous streak-line produced by the smoke-wire method at $Re=2.5 \times 10^4$. Fig.(a) shows a branching flow at $x^*/d=1.0$ and an ejection of fluid bulk larger than the boundary layer thickness. Fig.(b) shows a flow toward downstream from the reattach-

Fig.16 Fluid Flow Behavior at the Reattachment Region of the Downstream Cylinder

ment point (main stream direction) at $x^*/d=0.03$ and it is observed how the flow becomes three-dimensional and unsteady with a fairly good regularity in the axis direction. Figs.(c) and (d) show how the flow at $x^*/d=0.5$ and 1.0 moves toward the upstream from the reattachment point (reverse flow) and that the ejection of fluid bulk with regularity can be seen. These flow structures may be similar to the longitudinal vortex structure as Yokobori pointed out.

4.4 Overall Nusselt Number ($\bar{Nu}$)

Fig.17 shows the dependency of $\bar{Nu}$ on Re for the typical clearance. In spite of the clearance, that is, before and after the jump clearance, $\bar{Nu} \propto Re^{2/3}$ and the influence of the clearance on Nu is not so large except at narrower clearance of $x^*/d=0.03$. $\bar{Nu}$ shows an increase of 20~30% over that of the single cylinder ($Tu=0.2\%$). Then, at constant Re ($Re=5 \times 10^4$), the variation of $\bar{Nu}$ with clearance x^*/d is shown in Fig.18, where the abscissa is normalized such as $\bar{Nu}/Re^{2/3}$. The value of $\bar{Nu}/Re^{2/3}$ before jump clearance is somewhat higher than that after jump, but its difference is not so large. In practice, regardless of the cylinder clearance, overall Nusselt number ($\bar{Nu}$) can be correlated as follows ;

$$N_u = 0.155 Re^{2/3} \quad (5)$$

Fig.17 Variations of Overall Nusselt Number with Reynolds Number

Fig.18 Variations of the Overall Nu with the Clearances

where the results of Kostic⁽²⁾ in the range of lower Re , as shown in the same figure, have higher values than those of the present experiment

5. Conclusions

At the subcritical Re region ($Re=1.5 \times 10^4 \sim 8.0 \times 10^4$), measurements have been conducted on both the fluid flow around two circular cylinders in line arrangement and the heat(mass) transfer around the downstream cylinder. The results can be summarized as follows;

(1) The flow patterns are classified into three types in the closed vortex region between two cylinders. (i) $0.08 \leq x^*/d \leq 1.5$; no influence between up and down shear layers of the upstream cylinder, and formation of a quasi-stationary vortex (ii) $1.5 < x^*/d \leq 2.3$; intermittent rolling up of the vortex behind the upstream cylinder (iii) $2.3 < x^*/d \leq 2.7$; maintenance of the intermittent vortex rolling and no formation of quasi-stationary vortex.

(2) The positions of C_{pmax} around the downstream cylinder move toward the front stagnation point ($\theta=0^\circ$) at $x^*/d \leq 1.5$, and retreat downstream slightly owing to the stretching effect of shear layer, and remain constant at $1.5 < x^*/d < 2.7$. Sh_{max} is moved forward by $5 \sim 10^\circ$ of the position of C_{pmax} .

(3) Separation point comes at $\theta_s=115^\circ$ before the jump clearance and at $\theta_s=100^\circ$ after the jump. It is not acceptable that Kostic regarded these points as the transition point. The minimum of the local Sh profile occurs at the separation point before the jump clearance. But, after the jump clearance, it moves forward from the separation point and a clear decrease of Sh cannot be seen because of the influence of the fluctuation of the boundary layer.

(4) At a clearance smaller than the jump clearance, that is, $0.03 \leq x^*/d \leq 1.5$, the remarkable change of $Numax$ at the reattachment region with x^*/d depends strongly on the velocity gradient at the reattachment point. Relating to the mechanism of heat-transfer augmentation, a turbulent vortex similar to the longitudinal vortex can be observed. Empirical correlations of $Numax$ are given at $0.03 \leq x^*/d \leq 1.5$, $1.5 < x^*/d \leq 2.3$, and $2.3 < x^*/d \leq 4$, respectively.

(5) At the clearance before the jump clearance (especially in the range of $x^*/d \leq 1.5$ where a quasi-stationary vortex is formed), the local Sherwood number in the front portion of the downstream cylinder has a remarkably higher value than that in the separated region. On the other hand, at the clearance larger than the jump clearance, the change of the peripheral Sh profile is not so large.

(6) The overall Nusselt number around the downstream cylinder is proportional to $Re^{2/3}$, and is affected slightly by x^*/d in spite of the flow pattern change. And its value is higher by 20~30% than that of the single cylinder.

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