Behavior of Magnetic Dynamic Absorber*

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In order to reduce the amplitude of a vibrating system excited by an external force, a magnetic dynamic absorber can be utilized. The system with the magnetic dynamic absorber has non-linear characteristics. Generally for the non-linear system there exist two steady solutions which are locally stable in a certain interval of the excited frequency. Which of the steady solutions is realized in the system can be determined by the help of a certain disturbance in the initial conditions. By using a phase plane this subject above mentioned, has been investigated by Ch. Hayashi and A. Tondl.

In this paper, concerning the system with a damped magnetic dynamic absorber excited by an external periodic force, the regions and probabilities of occurrence of disturbances in initial conditions which realize steady solutions of the system were calculated and solved by Tondl's method by means of the digital computer.

Key words : Theory of vibration, Magnetic Dynamic Absorber

1. Introduction

A vibrating system with a damped magnetic dynamic absorber has non-linear characteristics as described in the previous paper. When the non-linear system is excited by an external periodic force, the response curves have the characteristics either of a hard spring or a soft one. There exist two or more steady solutions which are stable locally at a certain excited frequency. When the system is being vibrated with a certain value of frequency, which of the two steady solutions above mentioned can be realized depends on the initial conditions of the system. These ideas and methods are discussed by Ch. Hayashi and A. Tondl. The former wrote about the system with one degree of freedom and the latter about the system with two degrees of freedom. In these investigations, a locally steady solution which can be achieved by the help of certain disturbances in the initial conditions is obtained by an analogue computer, and the regions of the initial conditions realized by each steady solution are drawn in the phase plane (Hayashi plane), and the probability of occurrence of a disturbance in the initial conditions which leads to one given steady solution was solved.

In this paper, when a system with a damped magnetic dynamic absorber is excited by an external periodic force, the regions corresponding to the steady states are calculated in the phase plane of the initial conditions by the above method with the help of digital computers. The probabilities and others are solved as well. The behaviors of the system are analyzed in the resonant domain.

2. Equation of Motion

Figure 1 shows a model of the dynamic system which is vibrating horizontally. ① is the principal system, whose mass is \( M \). ② is the magnetic dynamic absorber. ③ and ④ are side magnets fixed on the magnetic dynamic absorber. ⑥ is the absorber mass which contains permanent magnets and moves right and left along the guide axis between both magnets, whose mass is \( m \). ⑦ is a viscous damper working between ② and ⑥. \( c \) is the coefficient of viscosity. \( P \) is the amplitude of external periodic force. \( \omega \) is the frequency of external periodic force. Magnetic force working between a side magnet and an absorber magnet is in inverse proportion to the square of linear function of the distance between them. \( B_1 \) and \( B_2 \) are magnetic factors of

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the force working between a side magnet and an absorber magnet \( (i=1, 2) \). \( L_s \) is the distance between both side magnets \( \mathcal{Q} \) and \( \mathcal{Q} \). \( L_s \) is the distance between the absorber mass and each side magnet in equilibrium \( (i=1, 2) \). \( x_i \) is the displacement of principal mass. \( x_p \) is the distance of absorber mass. \( x_f \) is a relative displacement \( (x_p = x_s - x_f) \).

Equations of motion are obtained in the following forms.

\[
M \frac{dx_1}{dt} + k_1x_1 = f(x_s) + P \sin \omega t
\]
\[
m \frac{dx_2}{dt} = -f(x_f)
\]

(1)

where,

\[
f(x_f) = \frac{d^2x_2}{dt^2} + \frac{1}{1 + \beta_1 \alpha_1^2} \left[ 1 - \frac{1}{1 + \beta_2 \alpha_2^2} \right] - \frac{1}{1 + \beta_2 \alpha_2^2}
\]

and the dimensionless quantities are defined as follows.

\[
X_1 = x_1/L_s, \quad X_2 = x_2/L_s, \quad A_x = P/k, \quad \alpha_1 = \sqrt{k/M}
\]
\[
\alpha = \frac{m}{M}, \quad \tau = \omega / \sqrt{kM}, \quad \tau = \omega / \alpha
\]
\[
K_1 = \beta_1 / (1 + \beta_1), \quad K_2 = \beta_2 / (1 + \beta_2)
\]

The dimensionless equations of horizontal motion of symmetrical system are as follows.

\[
\frac{d^2X_1}{dt^2} + X_1 = g(X_1) + A_x \sin \omega t
\]
\[
\alpha_1 \frac{d^2X_2}{dt^2} = -g(X_2)
\]

(2)

where,

\[
g(X_2) = 2\gamma \frac{dX_2}{dt} + K_1 \left[ 1 - \frac{1}{1 + \beta_2 \alpha_2^2} \right] - K_2 \left[ 1 - \frac{1}{1 + \beta_2 \alpha_2^2} \right]
\]

(3)

3. The probability of occurrence of disturbances in the initial conditions

In a vibrating system, the state of motion is decided by the displacement and the velocity of the system at arbitrary time. So the state of the system with two degrees of freedom at a time is shown as a point in the four-dimensional space (Hayashi's space). The steady state vibrations are realized by a closed trajectory with a certain period \( T, T = 2\pi / \omega \) in this space. In the resonant vibration of a nonlinear system with two degrees of freedom, two locally stable steady states can exist at a certain frequency, and then there are two closed trajectories. The larger amplitude response curves are termed the resonant solution curve, and the smaller ones are termed the non-resonant solution curve. At time \( \tau = 0 \), a point on the closed trajectories corresponding to the steady state is defined as a singular point at time \( \tau = 0 \). The state point starting from the initial condition on each closed trajectory moves on its own closed trajectory. If a disturbance in the initial condition is given to a system in the steady state, the state of the system will lead to either one trajectory or the other according to the disturbances in the initial conditions. So there exists a region of disturbances in the initial conditions in which the state of the system arrives at a closed trajectory in the phase space, the other region corresponding to the other closed trajectory as well. The phase space for the state of the system with one degree of freedom is a plane. The circle of disturbance in the initial condition is defined as a circle with a radius \( R \) centered at a singular point at an initial time \( \tau = 0 \). When the trajectories with starting points on the disturbance circle arrive at a steady solutions closed trajectory which corresponds to a singular point, the region of starting points is denoted by the length \( L \) of the arc of the circle of disturbances.

\[ P(R) = \frac{L}{2\pi R} \]

(4)

\( P(R) \) is defined as the probability with a steady state solution given by the singular point which is reached from any point on a circle of disturbance in the initial conditions and the circle has radius \( R \) centered at the same singular point. Actually, when \( k \) points out of finite value \( n \) points on a circle arrive at a steady solution, \( P(R) \) is given in the following form.

**Table 1 Coordinate of singular points**

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( X )</th>
<th>( Y )</th>
<th>( \pi/2 )</th>
<th>( \pi/4 )</th>
<th>( \pi/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>resonant solution</td>
<td>( X_1 )</td>
<td>0.278</td>
<td>0.416</td>
<td>0.278</td>
<td>-0.416</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>0.004</td>
<td>0.776</td>
<td>0.004</td>
<td>-0.776</td>
<td></td>
</tr>
<tr>
<td>( X_{1/4} )</td>
<td>0.436</td>
<td>0.270</td>
<td>-0.436</td>
<td>-0.270</td>
<td></td>
</tr>
<tr>
<td>( X_{1/2} )</td>
<td>0.754</td>
<td>0.504</td>
<td>-0.754</td>
<td>-0.504</td>
<td></td>
</tr>
<tr>
<td>non-resonant solution</td>
<td>( X_1 )</td>
<td>-0.045</td>
<td>0.014</td>
<td>0.045</td>
<td>0.014</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>-0.068</td>
<td>-0.169</td>
<td>0.098</td>
<td>0.169</td>
<td></td>
</tr>
<tr>
<td>( X_{1/4} )</td>
<td>0.013</td>
<td>0.014</td>
<td>-0.013</td>
<td>-0.044</td>
<td></td>
</tr>
<tr>
<td>( X_{1/2} )</td>
<td>-0.164</td>
<td>0.093</td>
<td>0.164</td>
<td>0.093</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 2 Characteristic curves of the amplitude of the system**

(a) Principal system (b) Absorber system

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\[ P(R) = \frac{k}{n} \quad (5) \]

The probability \( P(R) \) can be obtained at arbitrary time not only at time \( t = 0 \) but for a period, so that the average value of a period is defined by \( \bar{P}(R) \).

\[
P_{\bar{t}} = \frac{1}{R} \left[ \int_{0}^{R} n(x) \, dx \right] = \left[ \frac{1}{2}(P_0 + P_1) + \sum_{k=2}^{n} P_k \right] / n
\]

\[ P_{\bar{t}} = (2/R) \left[ \int_{0}^{R} x p(x) \, dx \right] = P_0 / n \quad + \frac{\sum_{k=2}^{n} K P_k}{n^2} \quad (6) \]

\( P_{\bar{t}} \) is the equal probability of the occurrence of disturbances on any circle of disturbances. \( P_{\bar{t}} \) is the equal probability of the occurrence of an arbitrary starting point inside the circle of disturbances.

In a system with two degrees of free-

Fig. 3 Domains of the disturbances in the initial conditions (resonant solution = 1)
dom, there exist four dimensional phase spaces. In this case, the above probabilities by means of principal circles of the spheres are investigated by Tondl. In the system with two degrees of freedom, there exist six possible combinations of principal circles. In a certain principal circle, two of initial conditions correspond to the values of a disturbance point which is on the circle with radius $R$ centered at the singular points belonging to the principal circle at a defined time, and the other two values of the initial conditions keep those of the singular point at the same defined time. The probability of occurrence in the system is obtained by the same methods with the system with one degree of freedom, and all values in the six principal circles are averaged.

4. Application to the motion of magnetic dynamic absorber

On the horizontal vibration in which

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Fig. 3 Domains of the disturbances in the initial conditions (resonant solution - 2)
the magnetic forces of both side magnets are symmetric, the non-linear term $g(x)$ in the equation of motion can be approximated to the following form.

$$g(x) = 2\pi d x^2 + C_{g1} x + C_n x$$

(7)

where

$$C_{g1} = 4K_1P_1, \quad C_n = 8K_1P_1^3$$

Fig. 2 shows the response curves for $\lambda^2$ in $\alpha = 0.116, \gamma = 0.013, C_{g1} = 0.083, C_n = 0.336, A_1 = 0.018$. As this is a system with two degrees of freedom, it has four dimensional spaces and six possible combinations of principal circles. Table 1 shows singular points in each 1/4 period for a frequency $\lambda = 0.891 (\lambda^2 = 0.8)$.

4-1 The domains of disturbances in the initial conditions centered at singular point corresponding to resonant solution

Figure 3 shows the regions of disturbances in which the system that has the initial conditions at any point of the disturbance circle with radius $R$ centered at a singular point will lead to the steady state corresponding to the singular point. They could be obtained by using Lunge–Kutta–Gill's (automatically indenting) method for initial conditions.
which are corresponding to 72 points on the respective 15 circles. Fig.3(a)-(d) shows the domains of attraction (regions of initial conditions) for the initial times \( t = 0, 1/(\pi/\lambda), \pi/\lambda \) and \( 3/2(\pi/\lambda) \). The hatched areas are the regions of initial points (conditions) leading to curves of the non-resonant solution. Fig.4 shows the probabilities \( P(r) \) for regions leading to curves of the resonant solution on various principal circles of the spheres of disturbances. Fig.5 shows the average probabilities \( P, P_G, \) and \( P_{CG} \) on all principal circles of the spheres.

4-2 The domains of disturbances in initial conditions centered at singular points corresponding to non-resonant solutions.

Figure 6 shows the regions of initial conditions centered at singular points for non-resonant solution. The hatched areas are the regions of initial condi-

Fig.6 Domains of the disturbances in the initial conditions (non-resonant solution)
tions leading to the non-resonant solution for the initial times \( \tau = 0 \), and \( \tau = \pi / \lambda \). Figure 7 shows corresponding probabilities \( P(R) \) for regions leading to the non-resonant solution on the respective principal circles of disturbances. Fig. 8 shows the curves of functions \( \overline{F} \), \( \overline{F}_{OD} \) and \( \overline{F}_{CD} \) which are the average probabilities on all the principal circles as well as those of (4-1). Fig. 9 presents the 3 dimensional domains of the initial conditions leading to the resonant solution. It shows the figures which are combined by \( X - \lambda t/\Lambda \), \( X - x t/\Lambda \), \( X - \lambda t/\Lambda \) sections in Fig. 3 at time \( \tau = 0 \).

![Fig. 7 Curves of the probability of occurrence of disturbances (non-resonant solution)](image)

![Fig. 6 Average of the probability (non-resonant solution)](image)

![Fig. 9](image)

\[ \text{** Fig. 6** Shows results of calculation at the initial times } \tau = 0 \text{ and } \tau = \pi / 2 \lambda . \text{ In this case the restoring forces of the equation of motion can be symmetrical, therefore the figures at } \tau = \pi / \lambda \text{ and } 3 \pi / 2 \lambda \text{ are respectively in point symmetry to those at } \tau = 0 \text{ and } \tau = \pi / 2 \lambda . \]
5. Conclusions

A vibrating system with a damped magnetic dynamic absorber periodically excited has two locally steady conditions corresponding to one frequency in the resonant interval of the excited frequency because of the nonlinearity of the system. Which of the two conditions is realized depends on initial conditions. Probability of transition of steady solutions achieved by the help of a certain disturbance in the initial conditions was obtained by Tondl's method. The solutions here are for \( \lambda = 0.894 \). If the radius \( R \) centered at one singular point is smaller than 0.6, there exists an initial steady solution corresponding to it. However if \( R \) exceeds 0.6, probabilities in which the steady solution moves to the other increase rapidly. The probabilities on the circle of the singular point in the non-resonant case are larger than those in the resonant one. Though the experiment on the non-linear system is indeed difficult, these methods can be used to clarify the method of the experiment and analyze the experimental results.

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References

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