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Bending Strength of Spur Gear Teeth*

(2nd Report, Estimation of Strength
based on Plastic Strain Amplitude)

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Bending fatigue test of spur gear teeth with 8 various tooth fillet made of normalized 0.45% carbon steel are carried out. It is shown that the maximum elastic stress along the tooth fillet is applicable as the rating stress of the fatigue strength in the case of the same rack.

The Fe-3Si steel etching technique is used to detect plastic flow around the tooth fillet in fatigue test. And the cyclic stress-strain hysteresis loops at the tooth fillet of normalized gear teeth are measured by strain gage of 0.3 mm long. As the results, the linear cumulative damage rule of plastic strain amplitude and the proposed modified full wave method based on the plastic strain amplitude give good estimation for the varying load.

Key Words: Gear, Fatigue, Normalized 0.45% carbon steel, Etching technique, Plastic strain amplitude

1. Introduction

In the previous paper⁽¹⁾, bending fatigue tests of spur gear teeth made of normalized 0.45% carbon steel (JIS S45C) were carried out, where the fatigue strength was defined with the maximum elastic tensile stress along the tooth fillet calculated by the finite element method⁽²⁾, and several cumulative fatigue damage rules were discussed on the basis of the test results. But the fatigue strength at 10^7 cycles under constant amplitude load was shown to be larger than the yield stress of the test specimen, which indicated the necessity for a finer examination of the fatigue strength considering the plastic behavior at the root fillet.

From the past investigations, it is reported⁽³⁾ that the fatigue strength of gear teeth is larger than the yield stress, but there are only a few investigations made so far considering the plastic behavior at the root fillet, except those by Aida, et al⁽⁴⁾, who examined the relation between the slip

bands generated at the root fillet of annealed S15C gear teeth and the number of cycles under constant amplitude load.

In this paper, bending fatigue tests of spur gear teeth with 8 various tooth fillets of rack cutter, addendum modification coefficients and numbers of teeth were carried out with S45C normalized gears, and the fatigue strength and the position of the fatigue crack initiation at the root fillet in tensile side were examined. Furthermore the Fe-3Si steel etching technique was used to observe microscopically the slip bands generated around the root fillet, and plastic region and plastic strain were measured in crack initiation process. As the result, the fatigue strengths under varying load obtained by the previous paper⁽¹⁾ were investigated based on the plastic strain amplitude at the root fillet.

2. Effect of the shape of root fillet
on the fatigue strength of S45C
normalized gear teeth

Table 1 Dimensions of test gears and test conditions

Named gear group		Series A				Series B			
Cutter Dimen- sion	Pressure angle	20°				20°			
	Radius of cutter corner	0.375 m				0.238 m			
	Top clearance	0.25 m				0.157 m			
Named tooth profile		A	A1	A2	A3	B	B1	B2	B3
Dimen- sions of test gear	Module m	4				3			
	Number of teeth Z	18				27			
	Addendum modification coefficient x	0	0.2	0.4	-0.573	0	0.2	0.4	0
	Face width b mm	10				10			
Loading point mm		40.0	40.8	40.0	41.6	40.0	40.8	40.0	64.0
Maximum fillet elastic tensile stress $S/(Pn/b)$ 1/mm		1.01	0.935	0.721	1.55	1.10	1.05	0.838	1.01
Fatigue tester		Servo Pulsator type				Servo Pulsator type			

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2.1 Dimensions of test gears and test rigs .

Dimensions of the test gears divided into two series together with rack cutter are shown in Table 1. Test gears in series A and B were generated by JIS B1701 basic rack cutter and another rack cutter respectively. Tooth profiles A and B are shown in Fig.1. The gear blanks except Tooth profile B3 were prepared from the same rolled bar used in the previous paper⁽¹⁾ and had the same heat treatment condition as in the previous paper. The mean values of Vickers hardness of Tooth profile B3 and another are Hv 187 and Hv 189 respectively, so that the difference of mechanical properties between these gear teeth may be ignored. An electrohydraulic servo-controlled fatigue tester presented in the last paper was used for the plane bending fatigue tests of gear teeth. Also a pulsator type fatigue tester⁽⁵⁾ utilizing a fuel injection pump of diesel engine was constructed and used in some of the tests, in which the applied load was controlled by on-off operation of a relief valve in conjunction with a servo motor controlled by a pulsator controller; the accuracy of load control during test was about 3 %. The frequency of sinusoidal load was set at 40 HZ and 18 HZ for an electrohydraulic servo-controlled tester and a pulsator tester, respectively. Loading points of each tooth are indicated by the position along the center line of tooth as shown in table 1. The rating stress of the fatigue strength is defined with the

Fig.1 Comparison of geometric configuration between Tooth profiles A and B

Fig.2 Fatigue life by constant amplitude load test (Tooth profiles A and B)

maximum elastic tensile stress along the tooth fillet, and the maximum elastic tensile stress related to the loading point and the applied tester for each tooth are also indicated in Table 1.

2.2 Fatigue test results

The fatigue test results of gear tooth profile B under constant amplitude load are shown in Fig.2. The test results of Tooth profile A already described in the previous paper are also shown in the figure for comparison. In the case of Tooth profile B, the P-S-N curves in which the fatigue strength obeys the normal distribution were shown to be in a close agreement with the test results, as is the case with Tooth profile A, and mean μ_N , variance σ_N^2 of the fatigue life are given as follows:

$$\exp(\mu_N) = N_\mu = \left(\frac{7.65 \times 10^6}{S - 555} \right)^{1.17} - 7.42 \times 10^4 \quad (1)$$

$$\sigma_N = 22 \text{ MPa} \quad (S > 555 \text{ MPa})$$

The fatigue strength in the time endurance region of Tooth profile B is about 100 MPa (10 kgf/mm²) larger than that of Tooth profile A. The test results of the other tooth profiles are shown in Fig.3, and separated into series A and B depending on the rack cutter by a single dot broken line as shown

Fig.3 Fatigue life by constant amplitude load test with 8 various tooth profiles

Fig.4 An example of measurement of permanent deflection (Tooth profile B1, 695 MPa)

in the figure, also the addendum modification coefficient and the number of teeth have little effect on the fatigue strength. Moreover the effect of the loading point on the fatigue strength of Tooth profile B is not recognized. The relations among the permanent tooth deflection and number of cycles and the position of crack initiation were investigated. Fig.4 shows the relation between the profile error measured on the compressive side of Tooth profile B1 and the number of cycles at the stress level 695 MPa at which the gear tooth was ruptured in 4.04×10^5 cycles. The tooth does not deflect permanently until 3.84×10^4 cycles as shown in Fig.4(b). A fatigue crack was detected at 4.0×10^5 cycles (figure (c)), in which the crack length was about 0.1 mm. Therefore, the permanent tooth deflection is supposed to depend mainly on the crack opening and the life of the crack propagation process is merely 1~7% of fatigue life, which is similar to the other tooth profiles.

The positions of fatigue crack initiation were observed by a universal projector of 20 magnifications, and the results are shown in Fig.5. The position of crack initiation is indicated by the length l along the root fillet surface in tensile side with the position of Hofer's 30° -tangent section as the origin, in which the length l is normalized by module m . There are some cases where the fatigue cracks initiate at 2 or 3 positions with distance of l/m 0.2, in which

the position of the crack leading to a breakage is defined as the position of crack initiation. Mean l_μ and variance l_σ of position of crack initiation l of each tooth profile are expressed in Fig.5. Black dot marks and triangle marks in the figure correspond to the positions of Niemann's section, Lewis's section (inscribed parabola), respectively. The fine line expresses the stress distribution along the root fillet surface calculated by the finite element method⁽²⁾, and the position of the maximum fillet stress is indicated by circular mark. The fatigue crack initiates generally in the region between Niemann's section and Lewis's section, which lie at the position a little toward the tip from the position of the maximum fillet stress; that is similar to other tooth profiles. Mean position l_μ lies where the stress is 3~6% smaller than the maximum fillet stress. There was no difference between the stress at the mean position of the crack initiation and the maximum fillet stress in rating stress of the fatigue test results.

The position of crack initiation, therefore, does not coincide with that of the maximum fillet stress and lies at the position a little to the tip side. But so long as the same rack cutter is used, the fatigue strength does not depend on the addendum modification coefficient and the number of tooth, so that the maximum fillet elastic stress can be taken as a rating index of the fatigue strength.

As mentioned above, comparing the fatigue strengths of series A and B generated by the various rack cutters, the fatigue strength differ from each other and that of B is about 100 MPa (10 kgf/mm²) larger. Considering the fact that the life of the crack initiation process occupies about 90% and over of the fatigue life, the plastic behavior at the root fillet will be examined in the next discussion. The effects of surface roughness, microscopic projection and work-hardened layer at the root fillet on the fatigue strength were also examined, but these effects were not found to depend on the shape of the rack cutter.

3. Microscopic observations on the plastic behavior at the root fillet with the Fe-3Si steel

It is impossible to observe microscopically the slip line of 0.45% carbon steel, so the Fe-3Si steel etching technique was used, whose yield stress was controlled to be similar to that of 0.45% carbon steel. Table 2 shows the chemical composi-

A : $l_\mu/m = 0.140$, $l_\sigma/m = 0.052$
 A1 : $l_\mu/m = 0.101$, $l_\sigma/m = 0.032$
 A2 : $l_\mu/m = 0.061$, $l_\sigma/m = 0.031$
 A3 : $l_\mu/m = 0.150$, $l_\sigma/m = 0.040$

B : $l_\mu/m = 0.101$, $l_\sigma/m = 0.066$
 B1 : $l_\mu/m = 0.056$, $l_\sigma/m = 0.031$
 B2 : $l_\mu/m = 0.026$, $l_\sigma/m = 0.034$
 B3 : $l_\mu/m = -0.020$, $l_\sigma/m = 0.030$

Fig.5 Relation between the position of fatigue crack initiation and the root fillet stress

Table 2 Chemical compositions of Fe-3Si steel (Wt %)

C	Si	Mn	P	S	PB
0.01	3.02	0.31	0.011	0.012	0.06

Table 3 Mechanical properties of Fe-3Si steel

Heat treatment	Yield stress MPa	Tensile strength MPa	True fracture strength MPa	Elongation %	Reduction in area %	Vickers hardness Hv
780 c 1hr vacuous annealing	431*	607	615	9.7	7.1	160

* stress at plastic strain 0.2%

tions of the Fe-3Si steel applied to test, of which the heat treatment conditions and the mechanical properties are shown in Table 3. The values of elongation and reduction in area are smaller than those of S45C steel, since the Fe-3Si steel generally has a property of brittleness at normal room temperature about 100°C⁽⁶⁾, but the Fe-3Si steel was tested at room temperature as the fatigue tests were carried out in the range of small plastic strains. Etching solution was one developed by Morris⁽⁷⁾, and the process of etching was one followed in the investigations of Hahn⁽⁸⁾ and Ando et al⁽⁹⁾.

3.1 Etching response of Fe-3Si tensile bars plastically strained

Fig.6 shows the etching response of stepped Fe-3Si bars according to JIS 13B plastically strained under static tension load. In the range of plastic strain about 0.5%, slip lines are produced. The more the plastic strain increase, the larger the slip lines grow and come to cross one another. In the range of plastic strain 1%, dislocations are produced so many that the etching

(a) Loaded to 90% of yield stress, $e_p=0\%$

K-M 0.5 mm

(b) $e_p=0.5\%$

(e) $e_p=3\%$

(c) $e_p=0.8\%$

(f) $e_p=7\%$

(d) $e_p=1\%$

(g) $e_p=10\%$

Fig.6 Relation between plastic strain and etching response

response appears dark. Beyond the range of plastic strain 1%, white regions appear partly and all of the crystal grains look white at plastic strain 10% as shown in Fig.6(g), which indicate that the etching response stops due to insufficient carbon in solution for decoration⁽¹⁰⁾. It has been already reported that the etching response under compressive load is the same as that under tensile load⁽⁸⁾. The plastic strain to be mentioned below corresponds to the static tensile strain indicated in Fig.6.

3.2 Relation between the shape of root fillet and the etching response

Fatigue tests were carried out with Tooth profiles A and B, which aimed to examine the effect of the shape of tooth profile on the etching response. Test results under constant amplitude load are shown in Fig.7.

Fig.7 Fatigue life by constant amplitude load test (Fe-3Si steel)

The fatigue lives of Tooth Profiles A and B are denoted by circular marks and black dot marks, respectively. The obtained fatigue strength is larger than the yield stress, and the fatigue strength of Tooth profile B is larger by about 80 MPa than that of Tooth profile A. The same results were obtained in S45C normalized gear teeth. The points marked by $\circ$ and $\bullet$ indicate the number of cycles at which the fatigue test was suspended for the purpose of the observations of the etching response.

Fig.8 shows the etching response of Tooth profile A at the stress level 538 MPa. In loading of one cycle, the etching response reveals the slip lines on both the tension and the compression side at the root fillet and the region between Hofer's section and Lewis's section was plastically strained. The distance between slip lines is so wide that the plastic strain is taken to be 0.5% or less in consideration of the etching response shown in Fig.6. After 10^7 cycles, the distance between slip lines become narrower compared with that of one cycle, which shows the root fillet part to be plastically

Fig.8 Etching response of Tooth profile A (538 MPa)

Fig.9 Etching response of Tooth profile A (797 MPa)

strained about 0.5%. A fatigue crack can not be observed on the tension side. On the compression side a non-propagating crack is observed as indicated in Fig.8(b) by an arrow, and the etching response appears as a white region around the crack, which region will be strained plastically about 10%.

The etching responses of Tooth profiles A and B at the normal load 7840 KN/cm were compared. Fig.9 shows the test results of Tooth profile A at the stress level 797 MPa. In loading of one cycle, the tension side is strained plastically about 0.8% at the most. The plastic region grows in tip direction with the repeated cycles and saturate till 10^4 cycles; and then the plastic behavior becomes steady. An example of the saturated plastic behavior is shown in Fig.10(b), in which slip lines are clustered mostly around Niemann's section. Fig.10(c) shows the etching response of a ruptured tooth. The material peels off at the position of crack initiation partly, so that it may not be always true that the fatigue crack initiates at the neighborhood of Lewis's section and grows by 0.2 mm toward the tip direction at an angle of almost 45° with the root surface and so propagates in the normal tooth direction.

Fig.10 Relation between the number of cycles and the plastic region (plastically strained about 0.5%) (Tooth profile A, 797 MPa)

Fig.11 Comparison of the plastic behavior of Tooth profiles A and B

A portion of the crack appears as a white etching response, which will be strained plastically over 10%. The relations between the number of cycles and the plastic region (plastic strain of about 0.5% and over) are shown in Fig.10. In the case of Tooth profile B, the plastic behavior saturates at 10^4 cycles, which is the same as the case of Tooth A, but the etching response stable region is narrower compared with that of Tooth profile A and is strained plastically about 1%. The relations among the maximum plastic strain and the plastic region strained about 0.5% and the number of cycles are shown in Fig.11 comparing Tooth profiles A and B. Comparing the plastic regions strained about 0.5% of Tooth profiles A and B, Tooth profile A is about 2 times as deep in surface normal direction and about 1.3 times as long in the root fillet surface as Tooth profile B. Tooth profile A is plastically strained about 0.8% at the most, while Tooth profile B is about 1% at the most. From these results, the reason why the fatigue strengths of Tooth profiles A and B are different from each other may lie in the difference of the plastic behavior at root fillet in the fatigue crack initiation process.

4. Rating of the fatigue strength under varying load based on the plastic strain amplitude at the root fillet

In the previous paper, program load tests with the peak value fluctuation obeying the normal distribution were carried out, and it was surmised that the fatigue strength of S45C normalized gear teeth could not always be prescribed by the cumulative damage rule such as Miner's rule with emphasis placed only on cyclic stress. This result may be due to the disregard of the effect of the plastic behavior at the root fillet. In this paper, the fatigue strength under varying load will be investigated afresh based on the cyclic stress-strain hysteresis at the root fillet.

Fig.12 Examples of hysteresis loop

4.1 Relation between the plastic behavior under constant amplitude load and that under program load

Measurements of the cyclic stress-strain hysteresis of Tooth profiles A and B made of S45C normalized steel were performed. The hysteresis loops were observed to saturate at 10^4 cycles, so that the hysteresis loops after 10^4 cycles were measured by strain gages of 0.3 mm long stuck at the position of the maximum elastic tensile stress. The measurements indicate the hysteresis loop to be a symmetrically increasing-decreasing process of load as shown in Fig.12. The stress σ and the plastic strain ϵ_p at the stress level S are defined as shown in Fig.13(a), and the plastic strain amplitude $\Delta\epsilon_p$ is indicated by the plastic strain ϵ_p at $\sigma=S$, in which $\Delta\epsilon_p$ is chosen as a representative parameter for the energy of the hysteresis loop. Fig.13 shows the case of Tooth profile A.

The plastic strain ϵ_p under constant amplitude load is related to the stress σ by

$$\begin{aligned} \text{Tooth profile A: } \epsilon_p &= a\sigma^{3.22} \\ \text{Tooth profile B: } \epsilon_p &= b\sigma^{3.57} \end{aligned} \quad (2)$$

$(20 \times 10^{-6} \leq \epsilon_p \leq 1000 \times 10^{-6})$

where a and b depend on the stress level S . The relations between S and $\Delta\epsilon_p$ are also given by

$$\begin{aligned} \text{Tooth profile A: } S &= 228(\Delta\epsilon_p + 40.0)^{0.181} \\ \text{Tooth profile B: } S &= 146(\Delta\epsilon_p + 80.0)^{0.257} \end{aligned} \quad (3)$$

The hysteresis loops under program load were measured after 2.4×10^4 cycles. The plastic strain amplitude is given by the following equations,

$$\begin{aligned} \text{Tooth profile A: } \Delta\epsilon_{pi} &= a_v S_i^{3.22} \\ \text{Tooth profile B: } \Delta\epsilon_{pi} &= b_v S_i^{3.57} \end{aligned} \quad (4)$$

$(i=1, \dots, 16; S_{16}=S_{\max})$

1 MPa = 0.102 kgf/mm²

(d) stress-plastic strain diagram

Fig.13 Relation between stress and plastic strain (Tooth profile A)

where a_v and b_v depend on the maximum stress level $S_{\max}$ as

$$\begin{aligned} a_v &= ((S_{\max}/228)^{5.52} - 40.0) / S_{\max}^{3.22} \\ b_v &= ((S_{\max}/146)^{3.89} - 80.0) / S_{\max}^{3.57} \end{aligned} \quad (5)$$

(a) coef. var.=1/12

(b) coef. var.=1/9

Fig.14 Fatigue life of Tooth profile A by program load test

(a) coef. var.=1/12

(b) coef. var.=1/9

Fig.15 Fatigue life of Tooth profile B by program load test

The hysteresis loops were measured after 2.4×10^4 cycles, whereby the number of cycles of the maximum stress level $S_{\max}$ was merely 170. But as shown in Fig.13(d), both $\Delta \epsilon_p$ (A) at $S_{\max}$ and that (O) under constant amplitude load are expressed by a single dot broken line as shown in Fig.13(d), so that it is presumed that the hysteresis loops in the steady state are measured even under program load.

In estimating the fatigue life under program load, the linear cumulative damage rule based on the plastic strain amplitude⁽¹¹⁾ was used. Using Eq.(3) and the mean S-N curve, the relation between the mean life N_M and $\Delta \epsilon_p$ was obtained. The estimation of the mean S-N curve under program load was made using Eqs.(4),(5) and the above relation. The estimated curves are shown by a double dot broken line in Figs.14 and 15 for Tooth profiles A and B respectively. In the case of Tooth profile A, the estimated curves are slightly shorter than the test results but approximately agree with the test results above 10^6 cycles. In the case of Tooth profile B, the estimated curves are in good agreement with the test results even in the time endurance region.

The underestimation of the fatigue life of Tooth profile A in the time endurance region indicates that the rating of the plastic behavior in the region around the fatigue crack could not be precisely made by the measurement of a point on the root fillet surface which is strained plastically with a gradient. The estimation of fatigue life of Tooth profile B well agrees with the test results comparing with those of Tooth profile A. The reason may be explained; the plastic region of Tooth profile B is narrow relatively and the effective hysteresis contributing to the fatigue rupture can be measured.

4.2 Examination of the pattern of fillet stress waveform

In the previous paper, the fatigue tests were carried out for Tooth profile A with two patterns of fillet stress waveforms as shown in Fig.16, and the test results are shown in Fig.17. In this paper, these test results will be investigated based on the plastic

(a) Pattern A

(b) Pattern B

Fig.16 Two examples of dynamic tooth fillet stress

strain amplitude. According to the full wave count method⁽²⁾, Pattern A is separated into the first wave with stress level S_{p1} and the second wave with mean stress S_{m2} and stress amplitude $S_2/2$, as shown in Fig.18. Since the second wave is a superposed component, the plastic strain amplitude caused by the stress level S_2 is given by $\Delta\epsilon_{p2}$, as shown in Fig.19. The stress level which is needed to cause the same plastic strain amplitude under constant load is $\bar{S}_2$ (S_2) as shown in Fig.18(a). So the stress amplitude of the second wave should be expressed as $\bar{S}_2/2$. Using the time endurance diagram of the gear teeth which is shown in Fig.17 in the previous paper, the estimation of fatigue life by Miner's rule with a modified full wave count method above mentioned is in good agreement with the test results as shown in Fig.17 by a solid line. The estimation of fatigue life of Pattern B is the same as the one by constant amplitude load.

5. Conclusions

The main results obtained from this investigation are summarized as follows:

(1) A part of the root fillet of gear tooth made of normalized 0.45% carbon steel

Fig.17 Effect of pattern of fillet stress waveform on fatigue life

Fig.18 Count of Pattern A based on cyclic stress-strain hysteresis

Fig.19 Relation between stress and plastic strain under Pattern A

reaches plastic region in fatigue test. But, so long as the same rack cutter is used, the fatigue strength does not depend on the addendum modification coefficient and the number of teeth, so that the maximum fillet elastic tensile stress is applicable as the rating stress of the fatigue strength.

(2) According to the result of a microscopic observation of the plastic behavior at the root fillet with the Fe-3Si steel etching technique, the reason why the fatigue strength differs according to the rack cutter used is in the difference of the plastic behavior in the fatigue crack initiation process.

(3) The energy of the cyclic stress-strain hysteresis at the root fillet was represented by the plastic strain amplitude, and the fatigue strengths under varying load were investigated considering the plastic strain amplitude;

(a) The estimation of life of S45C normalized gear teeth by the linear cumulative damage rule based on the plastic strain amplitude gives good approximation to the test results above 10^6 cycles and is available as the cumulative damage rule.

(b) The modified full wave count method above mentioned is applicable in rating the effect of the pattern of fillet stress waveform on fatigue strength and can be adopted as the count method.

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