621.822.5

Paper No. 211-19

Dynamic Characteristics of Tilting Pad Gas Journal

Bearing Supported by Rotary Spring*

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Experimental and theoretical research works have been conducted to make clear the dynamic characteristics of tilting pad gas journal bearing supported by rotary spring. First, experimental results on the effects of the moment of inertia of the pad and the bearing clearance are compared with theoretical calculations by frequency response method to confirm that the theory is effective to predict the dynamic characteristics of this kind of bearing. Second, the effects of the other design parameters, especially the effect of the stiffness of pad support rotary spring, are investigated theoretically. Finally, theoretically predicted maps of whirl instability threshold, which is the main interest in the bearing design, are presented.

Key Words: Bearing, Tilting Pad Gas Journal Bearing Supported by Rotary Spring, Dynamic Characteristics, Experimental Study, Theoretical Study.

1. Introduction

It had been said that a rotor supported by tilting pad journal bearings, lubricated with oil or gas, was free from whirl instability until Lund(1), McCabe et al.(2) pointed out theoretically that the rotor could become unstable if the moment of inertia of the pad was too great. It seems, however, that heretofore most of the research works on the dynamic characteristics of a tilting pad gas journal bearing have been restricted to theoretical investigations. As far as the author knows, experimental verification of the theory on the whirl instability threshold has not been reported yet.

This paper deals with a special type of tilting pad gas journal bearing patented by Dr. H. Sixsmith where each pad is supported by a rotary spring instead of a conventional pivot. It presents the results of some experiments and a theoretical study of the effect of various parameters on the dynamic characteristics of the bearing, especially the effect of the moment of inertia of the pad and the rotary stiffness of the pad support structure.

2. Nomenclature

$A_{0c}, A_{0s}, \dots, A_{2c}, A_{2s}$: gas film coefficients

D : journal diameter

* Received 7th May, 1981.

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f_x, f_y : gas film forces in x and y directions

$F_x = f_x/paR^2, F_y = f_y/paR^2$

H : dimensionless clearance = 1
+ $\varepsilon \cos(\phi + \varphi)$

I_d : transverse moment of inertia of the shaft about gravity center

I_p : moment of inertia of the pad about the rotating axis

$\bar{I}_p = paI_p(c/R)^5/36\mu^2RR_r^2$

k : rotary stiffness of the pad support spring

$K = ck/paR^2R_r^2$

L : bearing width

l : distance between bearings

m : shaft mass

$M = pm(c/R)^5/72\mu^2R$

$\bar{M}_f$: gas film moment acting on the pad

$\bar{M}_f = M_f/paR^2R_r$

p : gas film pressure

pa : ambient pressure

$P = p/pa$

R : journal radius

R_p : pad curvature radius

R_r : distance between pad rotating center and bearing center

c : ground clearance = $R_p - R$

e : distance between pad curvature center and journal center

e_x, e_y : journal eccentricities in x and y directions

$i = \sqrt{-1}$

t : time

z : distance coordinate in pad width direction

$\Gamma = R_r\tau/C$

$\psi = (PH)^2$

A : compressibility parameter
= $6\mu\omega(R/c)^2/pa$

Ω : whirl frequency ratio = ν/ω

α : pad wrapping angle

α_p : angle between pad leading edge and pad rotating center

β : angle between -y direction and pad rotating center

r : pad inclination angle

- δ : distance between geometrical and gravity center of the shaft
 $\delta^* = \delta / c$
 $\epsilon = e / c$
 $\epsilon_x = e_x / c, \epsilon_y = e_y / c$
 $\zeta = Z / R$
 ω : angular speed of shaft rotation
 ν : angular frequency of shaft whirl
 μ : viscosity
 ϕ, θ : angular coordinate
 φ : angle between pad rotation center and journal eccentricity direction
 $\tau = \nu t$
 Subscripts
 0 : static equilibrium state
 in : initial setting
 1 : perturbed value
 (k) : state of pad k

3. Test bearings, test apparatus and experimental method

Configurations of two test bearings A and B are shown in Fig.1. Each pad is supported on a support structure consisting of two plates brazed to the bearing housing and the pad to form a roof shape. This structure is very stiff against pad displacements in all directions except pitching rotation about the axis where the two plates meet. Each pad is initially tilted slightly by a pre-load spring, but not so much as to touch the shaft. This type of tilting pad bearing has an advantage of less starting friction than the conventional pivoted pad bearing where at least one of the pads is radially preloaded against the shaft, thus causing a high starting friction torque. However, the rotary stiffness of the pad support

structure restricts a free pitching motion of the pad and may cause shaft instability at high rotating speed. One of the main purposes of this work is to make clear the effect of the rotary stiffness of the pad support structure on the whirl instability threshold. The two test bearings have the same diameter and width, but their pad support structures have different configurations and stiffnesses. The rotary stiffnesses of bearings A and B are 2,410 N cm/rad and 1,420 N cm/rad respectively. The effect of the moment of inertia of the pad is investigated by varying the mass attached to the pad.

A schematic diagram of the experimental apparatus is shown in Fig.2. A vertical rotor weighing 494 g is supported by two radial bearings and a pressurized thrust air bearing. As the first bending critical speed of the shaft is 4,330 1/s, the shaft can be regarded as rigid up to 2,500 1/s which is the maximum speed attained by the turbine drive.

Instead of testing two identical upper and lower bearings, a special arrangement of the two bearings was adopted so that the dynamic performance of the upper test bearing could be measured directly by minimizing the effect of the lower bearing on shaft vibration. In the case of a rigid shaft with negligible gyroscopic effect, as is the case with this test apparatus, this measurement can be accomplished by placing the two bearings symmetrically about the gravity center of the shaft and setting the distance between them, ℓ , at $\ell = 2 \sqrt{I_d/m}$. In this system, the vibration of the shaft at the center of the test bearing is equal to the

Fig. 1. Test bearing

Fig. 2. Test apparatus

vibration of the shaft in the equivalent system where the shaft, whose mass is half that of the actual shaft, is supported only by the test bearing and whirls in a translational mode due to the equivalent external force which is equal to the dynamic load acting on the test bearing in the actual system. Because of the difficulty in placing the electrode for vibration pick-up at the center of the bearing, a pair of electrodes are placed symmetrically about the test bearing. The vibration of the pad is measured by an electrode placed in the position shown in Fig.1. Several shafts with slightly different diameters are prepared to investigate the effect of bearing clearance.

4. Theoretical analysis

The theoretical model analyzed here is shown in Fig.3. It is assumed that three pads having the same specifications are

Fig. 3. Analytical model

O : Center of Bearing
Os : Center of Journal
Or : Center of Pad Rotation
Op : Center of Pad Curvature

Fig. 4. Coordinate system

arranged symmetrically around the journal. Additionally, we treat the case of a symmetrically vertical rotor-bearings system which is shown in Fig.3(b). The shaft whirl mode is assumed to be cylindrical.

4.1 Gas film force

Dynamic gas film force of a pad is computed with the frequency response method first employed by Lund to analyze a tilting pad gas bearing. The basic equation of compressible fluid lubrication, assuming an isothermal viscous flow, is given as follows.

$$\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial \tau^2} - \frac{1}{H} \frac{\partial H}{\partial \phi} \frac{\partial \psi}{\partial \phi} - \frac{2}{H} \frac{\partial^2 H}{\partial \phi^2} \psi = \frac{A}{H\sqrt{\psi}} \left(\frac{\partial \psi}{\partial \phi} + 2\Omega \frac{\partial \psi}{\partial \tau} \right) \quad (1)$$

It is assumed that the bearing clearance varies harmonically with time with a small amplitude around the static equilibrium state. Putting ε and ϕ equal to $\varepsilon_0 + \varepsilon_1 e^{i\tau}$ and $\phi_0 + \phi_1 e^{i\tau}$ respectively, H and P in eq.(1) can be expressed as follows.

$$H = 1 + \varepsilon_0 \cos \theta + \varepsilon_1 e^{i\tau} \cos \theta - \varepsilon_0 \phi_1 e^{i\tau} \sin \theta \quad (2)$$

$$P = P_0 + \varepsilon_1 e^{i\tau} P_1 + \varepsilon_0 \phi_1 e^{i\tau} P_2 \quad (3)$$

where $\theta = \phi + \phi_0$. Inserting eqs.(2) and (3) into eq.(1), we can derive equations for P_0 , P_1 and P_2 respectively. These equations are solved by a finite difference method with a boundary condition such that the film pressure on the edge of the pad equals

the ambient pressure. Dimensionless film forces F_x and F_y acting on the shaft in the x and y directions respectively and dimensionless film moment acting on the pad about the pad rotation center, $\bar{M}_f$, are given as:

$$\left. \begin{aligned} F_x &= - \int_{-L/D}^{L/D} \int_{\beta}^{\beta+\alpha} (P-1) \sin(\theta) \\ &\quad - \varphi_0 + \beta) d\theta d\zeta \\ F_y &= \int_{-L/D}^{L/D} \int_{\beta}^{\beta+\alpha} (P-1) \cos(\theta) \\ &\quad - \varphi_0 + \beta) d\theta d\zeta \\ \bar{M}_f &= - \int_{-L/D}^{L/D} \int_{\beta}^{\beta+\alpha} (P-1) \sin(\theta) \\ &\quad - \varphi_0) d\theta d\zeta \end{aligned} \right\} \quad (4)$$

Inserting eq.(4) and $\varepsilon_0 = \Gamma_0$, $\varepsilon_1 = \varepsilon_{x1} \cos \beta + \varepsilon_{y1} \sin \beta + I_1$, $\phi_0 = \pi/2$, $\varepsilon_0 \phi_1 = \varepsilon_{x1} \sin \beta - \varepsilon_{y1} \cos \beta$ into eq.(3), F_x , F_y and $\bar{M}_f$ can be expressed as the sum of the static equilibrium term and the perturbed time dependent term, the latter being a linear function of $\varepsilon_{x1} e^{i\tau}$, $\varepsilon_{y1} e^{i\tau}$ and $I_1 e^{i\tau}$. The journal center at static equilibrium was assumed to be at the bearing center, as we dealt with the vertical shaft only. Summing up the film force generated by the

three pads, film force and moment of the bearing are expressed as:

$$\begin{pmatrix} F_x \\ F_y \\ \bar{M}_f^{(1)} \\ \bar{M}_f^{(2)} \\ \bar{M}_f^{(3)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ A_{0c} \\ A_{0s} \\ A_{0c} \end{pmatrix} + \begin{pmatrix} \frac{3}{2}(A_{1c}-A_{2s}) & -\frac{3}{2}(A_{1s}+A_{2c}) & -A_{1c} & \frac{1}{2}(\sqrt{3}A_{1s}+A_{1c}) & -\frac{1}{2}(\sqrt{3}A_{1s}-A_{1c}) \\ \frac{3}{2}(A_{1s}+A_{2c}) & \frac{3}{2}(A_{1c}-A_{2s}) & -A_{1s} & \frac{1}{2}(A_{1s}-\sqrt{3}A_{1c}) & \frac{1}{2}(A_{1s}+\sqrt{3}A_{1c}) \\ -A_{1c} & A_{2c} & A_{1c} & 0 & 0 \\ \frac{1}{2}(A_{1c}-\sqrt{3}A_{2c}) & -\frac{1}{2}(\sqrt{3}A_{1c}+A_{2c}) & 0 & A_{1c} & 0 \\ \frac{1}{2}(A_{1c}+\sqrt{3}A_{2c}) & \frac{1}{2}(\sqrt{3}A_{1c}-A_{2c}) & 0 & 0 & A_{1c} \end{pmatrix} \begin{pmatrix} \varepsilon_{x1} \\ \varepsilon_{y1} \\ \Gamma_1^{(1)} \\ \Gamma_1^{(2)} \\ \Gamma_1^{(3)} \end{pmatrix} e^{i\tau} \dots (5)$$

where

$$\begin{aligned} A_{0c} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} (P_0-1) \cos \theta d\theta d\xi, & A_{0s} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} (P_0-1) \sin \theta d\theta d\xi \\ A_{1c} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} P_1 \cos \theta d\theta d\xi, & A_{1s} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} P_1 \sin \theta d\theta d\xi \\ A_{2c} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} P_2 \cos \theta d\theta d\xi, & A_{2s} &= \int_{-L/D}^{L/D} \int_{\xi}^{\xi+\alpha} P_2 \sin \theta d\theta d\xi \end{aligned}$$

4.2 Pad inclination angle at static equilibrium

Static equilibrium of the moment about the pad rotation center is expressed as:

$$\Gamma_0 = \Gamma_{in} + \frac{1}{K} A_{0c} \dots (6)$$

From this equation, the pad inclination angle at the static equilibrium state can be computed.

4.3 Unbalance induced whirl

In the symmetrically vertical rigid shaft and bearings system where unbalanced weight exists at the center plane of the shaft as shown in Fig.3(b), equations of motion of the shaft and the pads are expressed as:

$$m \frac{d^2 e_x}{dt^2} = 2f_x + m\bar{\omega}^2 e^{i\omega t}, \quad m \frac{d^2 e_y}{dt^2} = 2f_y + m\bar{\omega}^2 e^{i(\omega t - \pi/2)} \dots (7)$$

$$I_p \frac{d^2 \gamma^{(k)}}{dt^2} + k\gamma^{(k)} = M_f^{(k)} \quad (k=1, 2, 3) \dots (8)$$

By putting eq.(5) into the non-dimensional form of these equations, solutions are obtained as:

$$\varepsilon_{x1} = \frac{M\bar{A}^2(-A_{1c}+K-\bar{I}_p\bar{A}^2)\delta^*}{[(3/2)(A_{2s}-A_{1c}-i(A_{1s}+A_{2c}))-M\bar{A}^2](-A_{1c}+K-\bar{I}_p\bar{A}^2)-(3/2)(A_{1c}+iA_{1s})(A_{1c}+iA_{2c})} \dots (9)$$

$$\Gamma_1^{(1)} = \frac{-M\bar{A}^2(A_{1c}+iA_{2c})\delta^*}{[(3/2)(A_{2s}-A_{1c}-i(A_{1s}+A_{2c}))-M\bar{A}^2](-A_{1c}+K-\bar{I}_p\bar{A}^2)-(3/2)(A_{1c}+iA_{1s})(A_{1c}+iA_{2c})} \dots (10)$$

$$\varepsilon_{y1} = \varepsilon_{x1} e^{-i(\pi/2)} \dots (11)$$

$$\Gamma_1^{(2)} = \Gamma_1^{(1)} e^{-(2/3)\pi i} \dots (12)$$

$$\Gamma_1^{(3)} = \Gamma_1^{(1)} e^{(2/3)\pi i} \dots (13)$$

As the values of gas film coefficients A_{1c} , A_{1s} , A_{2c} , A_{2s} in these equations, we adopt those corresponding to $\bar{\omega} = 1$, because whirl frequency ν equals rotating speed ω in this case.

4.4 Instability threshold

We consider the state where the shaft is forced to whirl in the direction of rotation at a whirl speed of ν by external force $F_{exe} i\nu t$. Whirling amplitudes in this state become

$$\begin{bmatrix} \frac{3}{2}(A_{2s}-A_{1c}-i(A_{1s}+A_{2c}))-M\bar{\omega}^2\bar{A}^2 & \frac{3}{2}(A_{1c}+iA_{1s}) \\ A_{1c}+iA_{2c} & -A_{1c}+K-\bar{I}_p\bar{\omega}^2\bar{A}^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{x1} \\ \Gamma_1^{(1)} \end{Bmatrix} = \begin{Bmatrix} F_{ex} \\ 0 \end{Bmatrix} \dots (14)$$

$$\varepsilon_{y1} = \varepsilon_{x1} e^{-i(\pi/2)} \dots (15)$$

$$\Gamma_1^{(2)} = \Gamma_1^{(1)} e^{-(2/3)\pi i} \dots (16)$$

$$\Gamma_1^{(3)} = \Gamma_1^{(1)} e^{(2/3)\pi i} \dots (17)$$

The condition for obtaining non-trivial solutions to the above equations in the case of zero external force is as follows.

$$W(\bar{\omega}) = \begin{vmatrix} \frac{3}{2}(A_{2s}-A_{1c}-i(A_{1s}+A_{2c}))-M\bar{\omega}^2\bar{A}^2 & \frac{3}{2}(A_{1c}+iA_{1s}) \\ A_{1c}+iA_{2c} & -A_{1c}+K-\bar{I}_p\bar{\omega}^2\bar{A}^2 \end{vmatrix} = 0 \dots (18)$$

This equation gives the condition of sustained whirl. The amplitude of the whirl may diverge or dampen if this condition is not satisfied. To judge if a given system is stable or not against whirl, we can plot a vector diagram of $W(\Omega)$ and employ Nyquist's method.

5. Experimental results and discussions

5.1 Pad inclination angle

Prior to investigating the dynamic characteristics, the inclination angle against the shaft speed was measured and compared with the theoretical results. These results are shown in Fig.5. Instead of the inclination angle, the eccentricity of the curvature center of the pad to the bearing center, CF_0 , is plotted. Although the results show good agreement at low shaft speed, there is a clear tendency for the deviation between the predicted and measured values to increase with an increasing shaft speed. Note that with Bearing A, the measured eccentricity of the pad becomes larger than the predicted value as the shaft speed increases, while in the case of Bearing B the opposite tendency appears. These phenomena can be explained by the thermal expansion of the shaft and the pad. In the case of Bearing B where the pad's axis of rotation lies within the pad, the actual clearance becomes smaller with an increasing shaft speed because of the thermal expansion of the shaft and the pad. However, in the case of Bearing A where the pad's axis of rotation lies in the housing, the pad support structure, whose temperature rise is small and thus causes less thermal expansion than the pad, lifts up the pad due to the latter's thermal expansion, and thereby compensates for the decrease of bearing clearance. The displacement probe took this "lift up" as an increase in the inclination angle. That is the cause of the discrepancy between the experimental and theoretical results.

5.2 Shaft and pad vibration due to the unbalance

Before conducting the experiment, the

Fig. 5. Eccentricity of pad

shaft was dynamically balanced so that the amplitude of the vibration of the shaft and the pad was less than $1 \mu\text{m}$. An Unbalanced mass was attached to the shaft at the place shown in Fig.2. The vibration amplitudes of the shaft at the center of the test bearing and the pad were measured and compared with the theoretical values. The results are shown in Fig. 6. The vibration amplitude of the pad shown here is a peak to peak vibration amplitude of the center of the pad curvature. There are two amplitude peaks, one at low speed and the other at high speed. The low speed peak is a resonance of the shaft mass and gas film spring, while the high speed peak is a resonance of pad inertia and pad support spring plus the gas film. Tendency for the resonance speed to fall with an increasing bearing clearance may be explained by the decrease in gas film stiffness. Qualitative and quantitative agreements between the theoretical and experimental results are good except in the bearing with a relatively small clearance where the effect of thermal expansion is more predominant.

5.3 Onset speed of self-excited whirl

Measured and predicted values of whirl onset speed are shown in Fig.7. It is clear that with a reduced moment of inertia of the pad, stability increases and that there is an optimum clearance where whirl onset speed reaches its maximum value. Here, too, agreement between the experimental and theoretical results is good, taking into

Fig. 6. Unbalance whirl

account the fact that the theoretical calculation does not include the effect of thermal expansion.

From the results stated above, it became clear that the frequency response method was effective in predicting the dynamic characteristics of this type of bearing. Therefore, the effects of the other design parameters, namely pad support stiffness, k , and the initial amount of pad tilting, CF_{in} , on whirl onset speed were theoretically investigated with Bearing A. The results are shown in Fig. 8. From this figure it can be seen that there is an optimum value of k at which whirl onset speed reaches maximum, and that with an increasing CF_{in} , stability may be improved considerably when k is large, though there is little stabilizing effect when k is small. It is worthwhile to add that the lower and the upper limits of k correspond to the cases of a freely pivoted bearing and a 3-lobed bearing respectively. Supporting a pad with a rotary spring prevents the pad from following the shaft movement and is considered to have a negative effect on stability. However, it has a positive effect on stability, maintaining a high tilting angle of the pad against the restoring moment of the gas film. These positive and negative effects are the reason why the stiffness of the pad support has an optimum value.

6. Map of Instability threshold against whirl

As stated above, it is clear that the frequency response method is effective for predicting the dynamic characteristics of this kind of bearing. The maps of stability threshold, which is the main interest in practical design, are produced based on the theory and presented in Figs 9 ~ 12. These maps can be applied to a symmetrically vertical rigid shaft and bearings system which is the theoretical model this analysis is based on. These maps also assume a cylindrical whirl. For a system where the effect of the gyroscopic moment of the shaft is negligible, stability threshold against conical whirl can be predicted from these maps by putting m in the definition of M as $4Id/l$. Dimensionless parameters other than those defined in these maps are as follows.

$$\alpha = 112^\circ, \alpha_p/\alpha = 0.625, L/D = 1.29$$

These maps yield several useful design guides. In the case of small M , increasing K is effective for improving stability, while in the case of large M , the opposite is true. Stability is improved by decreasing I_p , but, in the case of large K , the stabilizing effect is small.

7. Conclusions

Experiments have been conducted on the

effects of various design parameters on the dynamic characteristics of rotary-spring supported tilting pad gas journal bearings. The results have been compared and discussed with the theoretical values obtained using the frequency response method. The main conclusions are as follows.

- (1) Whirl onset speed is decreased when the moment of inertia of the pad is increased.
- (2) There is an optimum bearing clearance at which the whirl onset speed reaches maximum.
- (3) Whirl onset speed and unbalance response characteristics can be effectively predicted by the frequency response method.
- (4) The theoretical simulations conducted to investigate the effect of the rotary stiffness of the pad support structure on the whirl instability threshold make it clear that there is an optimum stiffness value.
- (5) Theoretically obtained instability threshold maps for this kind of bearing are presented.

This research work was conducted at the University of Oxford with guidance and helpful discussions of Dr. H. Sixsmith to whom the author would like to express his

Fig. 7. Whirl onset speed

Fig. 8. Whirl onset speed vs. k , CF_{in}

sincere thanks.

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Fig. 9. Instability threshold map

Fig. 10. Instability threshold map

Fig. 11. Instability threshold map

Fig. 12. Instability threshold map