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Study on Bending Fatigue Strength of Bevel Gears*

(2nd Report, Bending Fatigue Strength of
Straight Bevel Gears of Gleason Type)

By Satoshi ODA**, Takao KOIDE***

and Katsuhiko HIGUCHI****

The bending fatigue strength and the bending fatigue breakage of straight bevel gears of Gleason type were investigated by carrying out bending fatigue tests (Wöhler method) using a bevel gear pulsator of hydraulic type. The bending fatigue limit loads were also determined using the progressive load increase methods proposed by Prot, Henry and others, which consist in testing a few specimens raising the stress in equal increment until finally the specimen breaks, and they were compared with those obtained by Wöhler method.

Furthermore the existing equations of the bending strength for straight bevel gears were compared on the basis of the experimental results.

1. Introduction

In the previous paper⁽¹⁾, the characteristics of tooth profile of straight bevel gears of Gleason type were examined and the root stresses were investigated

experimentally by making use of a static loading apparatus, which was developed by the authors. Furthermore various calculating methods of contact ratio for straight bevel gears were compared on the basis of the measured contact ratios.

In the present paper, the bending fatigue strength and the bending fatigue breakage of straight bevel gears of Gleason type were investigated by carrying out bending fatigue tests (Wöhler method) using a bending fatigue testing machine for bevel gear. The bending fatigue limit loads for bevel gears were also determined using the progressive load increase methods proposed by Prot, Henry and others⁽²⁾, and they were compared with those obtained by Wöhler method.

Furthermore the existing equations of bending strength for straight bevel gears were compared on the basis of the experimental results.

Table 1 Dimensions of test gears

Gear sign	G.A		G.B		
	Pinion G.A ₁	Gear G.A ₂	Pinion G.B ₁	Gear G.B ₂	
Tooth profile	Gleason tooth				
Module	m	4			
Cutter pressure angle α_c	20°				
Number of teeth	z	24	48	18	36
Face width	b	20 mm			
Shaft angle	Σ	90°			
Pitch diameter	d ₀	96 mm	192 mm	72 mm	144 mm
Pitch angle	γ_0	26°34′	63°26′	26°34′	63°26′
Outer cone distance	R	107.33 mm		80.50 mm	
Addendum	h _k	5.38 mm	2.62 mm	5.38 mm	2.62 mm
Dedendum	h _f	3.37 mm	6.13 mm	3.37 mm	6.13 mm
Working method	Generated				

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** Professor, Faculty of Engineering, Tottori University, 4-101 Minami, Koyama-cho, Tottori.

*** Research Assistant, Faculty of Engineering, Tottori University, 4-101 Minami, Koyama-cho, Tottori.

**** Engineer, Kowa Seiko Co., Ltd., 1440 Ikejiri, Sayama-cho, Minamikawachi-gun, Osaka.

2. Experimental methods and apparatus

2.1 Test gears

Straight bevel gears of Gleason type used in this experiment are made of S45C normalized steel (heat treatment condition : 850°C × 1.5h water quenched and 620°C × 1.5h air cooled, hardness : $H_V = 200$). Table 1 shows the dimensions of test gear

pairs G.A and G.B. Figure 1 shows the shapes and dimensions of G.A. The characteristics of tooth profile of these bevel gears are described in 1st report⁽¹⁾ in detail.

2.2 Measurement of root stresses

The root stresses were measured by carrying out a static loading test. The strain gages were pasted for each pinion at the middle and both ends (2 mm toward the middle) of tooth trace on the tensile fillet determined by Hofer's method (see Fig.5 in 1st report⁽¹⁾). A strain gage was also pasted at the middle of tooth trace for gear of G.A, and the root stresses of pinion and gear were measured at the same time. This experiment was carried out under the normal distribution of load along tooth trace, which was examined by smearing red lead on the tooth surface in the case of the pinion torque $T = 2 \text{ kgm}$.

(a) Gear

(b) Pinion

Fig.1 Shapes and dimensions of test gears (G.A)

Fig.2 Static loading and bending fatigue testing machine

Fig.3 Comparison of strain waveforms at root fillet and at shaft

Stress values were obtained by multiplying strain readings by modulus of elasticity $E = 21000 \text{ kg/mm}^2$.

Figure 2 shows the static loading apparatus used in this experiment. A test gear can be loaded at arbitrary meshing positions with a manual oil pump by fixing a supporting gear. The applied load value is detected from the reading of the strain gages pasted at the shaft. A circumferential backlash $c_0 = 150 \mu$, which is equal to the minimum value specified in JGMA⁽³⁾, is selected in this experiment.

2.3 Bending fatigue test

2.3.1 Bending fatigue test under constant amplitude load (Wöhler method)

Figure 2 shows the bending fatigue testing machine for bevel gear used in this experiment. This testing machine is of hydraulic type and consists of a fuel injection pump and its driving apparatus, a pressure controller, a loading apparatus and a gear supporting apparatus. The frequency of load applications is about 510 c/min. The applied load value is detected from the reading of the strain gages pasted at the shaft. The loading apparatus is made available for both the static loading and bending fatigue tests by changing the oil pressure generator.

Bending fatigue tests were carried out with gears meshing at the worst loading position on the basis of the static loading test results. A circumferential backlash $c_0 = 300 \mu$, which is equal to about a half of maximum value ($c_0 = 620 \mu$) specified in JGMA⁽³⁾. Figure 3 shows the strain waveforms measured at the root fillet of bevel gear tooth and at the shaft. It is seen from this figure that both waveforms are equally good and of perfect

one side loading condition. Compared with the running test using a power circulating type gear testing machine, a bending fatigue test using this testing machine has advantages in that no effect of dynamic load is involved in the test results and that the S-N curve can be obtained economically with only one pair of gears.

In order to observe the process leading to a bending fatigue breakage, bending fatigue tests for G.B were carried out under an applied torque $T = 1.3 T_u$ (T_u : bending fatigue limit torque). Specimens were demounted from the testing machine at several stages in the fatigue process and cut at four positions on the tooth trace as shown in Fig.4. Then the crack initiation and propagation at the root fillets of these sections (A~F) were observed by the optical microscope.

2.3.2 Bending fatigue tests by progressive load increase methods (Prot and Henry methods)⁽²⁾

Determination of the fatigue limit calls for fatigue tests on several specimens. This is not practicable, however, in certain cases when only a few samples are available or the testing is restricted by shortage of time or by financial considerations. In such circumstances the use of the progressive load increase methods, which consist in raising the stress in a few specimens in equal increment until finally they break.

In order to examine the validity of the progressive load increase methods for bevel gears, bending fatigue tests by progressive load increase methods were carried out for G.B. A stepwise increasing torque as shown in Fig.5 was applied on the gear tooth in this experiment. In Fig. 5, the breaking torque T_F due to the

Fig.4 Cutting sections of gear tooth (G.B)

Fig.5 Relation between stepwise increasing torque and linearly increasing torque

stepwise increasing torque is converted into T_R , one due to a linearly increasing torque according to the following relation.

$$T_R = T_F - T_{au}/2 + N_u' \beta \quad (1)$$

where T_{au} : unit torque amplitude, N_u' : number of load cycles at last step, β : rate of increase of torque.

When the breaking torque T_R is obtained, the following relations between T_R and β are assumed by Prot and Henry methods.

$$\text{Prot method : } T_R = T_w + K \beta^{1/2} \quad (2)$$

$$\text{Henry method : } T_R = T_w + K \beta^n \quad (3)$$

where T_w : Wöhler fatigue limit torque to be determined, K, n : constants. The constant n in Eq.(3) should be selected such as to linearize the relation between T_R and β^n . The fatigue limit torque is obtained by plotting the relations between T_R and $\beta^{1/2}, \beta^n$.

In this experiment, the unit torque amplitude T_{au} is 3 kgm and the rates β of increase of torque are 4.7×10^{-6} , 2.2×10^{-5} , 5.4×10^{-5} kgm/c. The initial torque T_0 in Fig.5 is selected as $0.8 T_u$ on the basis

(a) G.A ($T = 5.5$ kgm)

(b) G.B ($T = 4$ kgm)

Fig.6 Measured root stresses

of the bending fatigue limit torque T_u obtained by Wöhler method.

It is known that there is an effect of coxing in steel materials when the rate α of increase of stress is less than 10^{-6} kg/mm²/c⁽²⁾⁽⁴⁾. The minimum value of α corresponding to β used in this experiment is found to be 7.7×10^{-6} kg/mm²/c from the static loading test results⁽¹⁾, and it can be considered that there is no effect of coxing.

The bending fatigue testing machine used in this experiment is shown in Fig.2.

3. Experimental results and discussions

3.1 Measured results of root stresses

Figure 6(a) shows the measured root stresses of pinion of G.A for the applied torque $T = 5.5$ kgm and Fig.6(b) shows those of G.B for $T = 4$ kgm. In these figures, the angle of rotation of a pinion is taken as abscissa and the pitch point as the origin, and the stepped line shows the range of single and double tooth pairs contact. It is found from Fig.6(a) and (b) that the root stresses at the toe, heel and middle of the tooth trace become larger in the order mentioned in the whole range of

Fig.7 Measured root stresses at middle of tooth trace for pinion and gear (G.A, $T = 5.5$ kgm)

Fig.8 Point of transition of engagement (G.A)

engagement for both G.A and G.B.

Figure 7 shows the measured root stresses at the middle of tooth trace for pinion and gear of G.A for the applied torque $T = 5.5$ kgm. The maximum values of root stresses of pinion and gear are found to be almost equal, and the bending strength of straight bevel gears of Gleason type might be considered to be well balanced between pinion and gear⁽¹⁾.

In order to determine the worst loading point (outer point of single tooth pair contact) for the bending fatigue tests in which a test pinion and a supporting gear are meshed, the change of the range of single tooth pair contact or double one with an increasing applied torque was examined by carrying out a static loading test under a higher torque. Figure 8 shows this result for G.A with the applied torque as ordinate and the angle

(a) G.A (gear)

(b) G.B (pinion)

Fig.9 Bending fatigue test results

Fig.10 Crack length distribution (G.B)

of rotation of a pinion as abscissa. In this figure, a (b) shows the point of transition from double tooth pairs (single tooth pair) contact to single tooth pair (double tooth pairs) contact. It is seen from this figure that the range of single tooth pair contact becomes narrower and that of double tooth pairs contact wider with an increasing applied torque.

3.2 Bending fatigue test results under constant amplitude load (Wöhler methods)

Figure 9(a) and (b) show the bending fatigue test results for G.A and G.B respectively with the pinion torque as ordinate and the number of load cycles as abscissa. The bending fatigue breakage of G.A occurred at the gear tooth and that of G.B at the pinion tooth. The reason might be as follows. The range of single tooth pair contact of G.A is narrower for a higher torque (see Fig.8). Since in this range the root stress value of the gear becomes larger than that of the pinion, the bending fatigue breakage of the G.A pair occurs at the tooth of gear member. It is seen from Fig.9 that the bending fatigue limit load (circumferential load $P_u = 1149$ kg (torque $T_u = 50$ kgm)) of G.A is larger than that ($P_u = 793$ kg ($T_u = 25$ kgm)) of G.B. This might be because the maximum root stress value of G.A is smaller than that of G.B for the same circumferential load (see Figs.6 and 7).

3.3 Position of crack initiation

A crack was observed at the root fillet on the tensile side at $N = 9 \times 10^5$ for an applied torque $T = 32.5$ kgm ($= 1.3 T_u$), and the crack length distribution is shown in Fig.10. Figure 11 shows the measured root stress distribution in longitudinal direction at the worst loading point. Comparing the results of Figs.10 and 11, it can be considered that the crack initiates at the position of maximum root stress on the tensile side and propagates toward both ends with an increasing number of load cycles.

Figure 12 shows the position of crack initiation and the directions of crack propagation in sections A, B, C (see Fig. 4). The positions of crack initiation lie in the neighborhood of critical section determined by Hofer's method, but the direction of crack propagation differs in each section.

3.4 Bending fatigue test results by progressive load increase method

Figure 13 shows the $T_R - \beta$ diagram for G.B obtained by the bending fatigue

tests by progressive load increase method. The breaking torque T_R shows a tendency to increase with an increasing β . Figure 14 shows the $T_R - \beta^n$ diagram obtained by Prot and Henry methods (Eqs.(2), (3)) using the result of Fig.13.

Fig.11 Root stress distribution in longitudinal direction (G.B)

Fig.12 Position of crack initiation and direction of crack propagation (G.B)

Fig.13 Bending fatigue test results by progressive load increase method (G.B)

Fig.14 $T_R - \beta^n$ diagram (G.B)

Fig.15 Comparison of allowable circumferential loads of bevel gears calculated from various formulas for bending strength

It is seen from Fig.14 that the bending fatigue limit torques estimated by Prot and Henry methods are 27.6 kgm and 26.2 kgm respectively and almost equal to the result (25 kgm) by Wöhler method. It might be considered from this fact that the progressive load increase methods (Prot and Henry methods) are applicable to an estimation of the bending fatigue limit of bevel gears.

3.5 Comparison of various formulas for bending strength of straight bevel gears

Allowable transmitted loads (circumferential loads) for each test gear (G.A, G.B) calculated from various formulas^{(5)~(9)} for bending strength of straight bevel gears and the bending fatigue test results carried out using the bending fatigue testing machine for bevel gears are summarized in Fig.15. Calculation is carried out with the dynamic load factor (the velocity factor) included in each formula taken as unity since no effect of dynamic load is involved in the bending fatigue test results, and with the life factor, reliability factor and so on taken as unity. It is found from Fig.15 that the allowable transmitted loads calculated are much smaller than the experimental results and that there is a considerable difference in allowable transmitted loads depending on calculating formulas.

4. Conclusions

The main results obtained from this investigation are summarized as follows.

(1) The root stresses at the toe, heel and middle of tooth trace become larger in the order mentioned in the whole range of engagement.

(2) A crack initiates at the position of maximum root stress on the tensile side and propagates toward both ends with an increasing number of load cycles.

(3) The positions of crack initiation in each normal section lie in the neighborhood of critical section determined by Hofer's method, but the direction of crack propagation differs in each normal section.

(4) In some cases the bending fatigue strength of gear tooth becomes lower than that of pinion tooth.

(5) The progressive load increase methods (Prot and Henry methods) are applicable to an estimation of the bending fatigue limit of straight bevel gear of Gleason type.

(6) The allowable transmitted loads calculated by various formulas for bending strength of straight bevel gears are much smaller than the experimental results, and there is a large difference in allowable transmitted loads depending on calculating formulas.

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