Study on Bending Fatigue Strength of Helical Gears*
(3rd Report, Mechanism of Bending Fatigue Breakage)

By Satoshi ODA** and Takao KOIDE***

In this paper, the relations between the stress distributions on root fillet and the crack initiation and propagation in helical gear, thin rim spur gear teeth and cantilever plate were examined and the mechanism of bending fatigue breakage of helical gears was investigated. It was found that a crack of helical gear teeth occurs at the position of maximum root stress on the tensile fillet and the directions of crack propagation differ in each normal section.

Furthermore, the existing testing methods of bending strength for helical gear teeth were compared on the basis of the experimental results.

1. Introduction

In order to discuss the bending strength of helical gears under various loading conditions or to raise the bending fatigue strength, it would be necessary to investigate the root stress distributions and the mechanism of bending fatigue breakage, which are basic problems for the bending fatigue strength of gear teeth.

In the case of helical gears, the load distribution on the contact line and root stress distribution are generally nonuniform along the tooth trace because the contact line crosses the tooth surface diagonally at an inclination, and the helical gear tooth has incomplete parts at the tooth ends. Hence the mechanism of bending fatigue breakage of helical gears is considered to be considerably different from the case of spur gears[1].

In the present paper, the relations between the root stress distributions and the crack initiation and propagation in helical gear, thin rim spur gear teeth and cantilever plate were examined by carrying out static loading and bending fatigue tests, and the mechanism of bending fatigue breakage of helical gears was investigated.

Furthermore, the existing testing methods of bending strength for helical gear teeth were compared on the basis of the experimental results.

2. Experimental methods and apparatus

2.1 Test gears

Dimensions of helical gears used in this experiment are shown in Table 1. A test gear is made of S45C normalized steel (hardness: Hv = 230) and is hobbed to

<table>
<thead>
<tr>
<th>Table 1 Dimensions of helical gears</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth profile</td>
</tr>
<tr>
<td>Normal module, mₘ</td>
</tr>
<tr>
<td>Normal pressure angle, αₘ</td>
</tr>
<tr>
<td>Helix angle, β₀</td>
</tr>
<tr>
<td>Number of teeth, z</td>
</tr>
<tr>
<td>Pitch circle diameter, d₀</td>
</tr>
<tr>
<td>Face width, b</td>
</tr>
<tr>
<td>Transverse contact ratio, eₜ</td>
</tr>
<tr>
<td>Overlap ratio, eₛ</td>
</tr>
</tbody>
</table>

Fig. 1 Shape and dimensions of cantilever plate

---

* Received 24th September, 1980.
** Professor, Faculty of Engineering, Tottori University, 4-101 Minami, Koyama-cho, Tottori.
*** Research Assistant, Faculty of Engineering, Tottori University, 4-101 Minami, Koyama-cho, Tottori.
an accuracy of 4th class in JIS. A supporting gear meshing with the test gear is made of SCM415 steel with teeth surfaces ground after casehardening to an accuracy of 1st class in JIS.

In order to clarify the mechanism of bending fatigue breakage of helical gears, static loading and bending fatigue tests were carried out using a cantilever plate whose shape and dimensions are shown in Fig.1. The cantilever plate specimen is made of S15C normalized steel (HV = 136) and finished with an end mill. Moreover, static loading and bending fatigue tests were carried out using a thin rim spur gear whose structure and dimensions are shown in Table 2 and Fig.2.

2.2 Bending fatigue test for helical gear

Figure 3 shows a bending fatigue testing machine for cylindrical gear used in this experiment. This testing machine is of hydraulic type and consists of a fuel injection pump and its driving apparatus, a pressure controller, a pulsating torque generator and gear supporting apparatus. The frequency of load applications is about 500 c/min. The center distance is variable and the meshing positions can be determined by measuring the distance between two specific gear teeth with a reading microscope.

Bending fatigue tests were carried out with gears meshing at the worst loading position which was determined from the static loading test results(12). In order to observe the process leading to a bending fatigue breakage, bending fatigue tests were carried out under a circumferential load P = 1.3 P_U (P_U : bending fatigue limit load). Specimens were demounted from the testing machine at several stages in the fatigue process and cut at six positions as shown in Fig.4. These specimens were polished with fine emery papers and buffed, and etched in Nital (nitric acid 3% + methyl alcohol 97%), and then the crack initiation and propagation at the root fillet on a normal section were observed by an optical microscope.

2.3 Measurement of root stresses and bending fatigue tests for cantilever plate

2.3.1 Static bending test

The root stresses of a cantilever plate were measured by a static bending test under concentrated and uniform loads. Strain gages were pasted at five positions on the tensile fillet determined by Hofer's method. The static bending testing machine used in this experiment was of hydraulic type and the gear supporting apparatus was remodeled for the cantilever plate specimen. A concentrated load was applied with a steel ball Φ 6 at a point separated by 3.7 mm from the free edge at the middle of face width, and a uniform load with a loading bar of tip radius 3 mm at a position separated by 3.7 mm from the free edge. The total applied load P in both cases is 500 kg.

![Diagram of bending fatigue testing machine](image)

![Diagram of cutting sections of helical gear tooth](image)

![Table 2 Dimensions of thin rim spur gear](image)

<table>
<thead>
<tr>
<th>Tooth profile</th>
<th>Standard - Stub tooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module</td>
<td>4.5</td>
</tr>
<tr>
<td>Cutter pressure angle α</td>
<td>20°</td>
</tr>
<tr>
<td>Number of teeth z</td>
<td>72</td>
</tr>
<tr>
<td>Face width b</td>
<td>30 mm</td>
</tr>
<tr>
<td>Rim thickness l_w</td>
<td>1.5 m*</td>
</tr>
<tr>
<td>Web thickness b_w</td>
<td>4 mm</td>
</tr>
<tr>
<td>Finishing method</td>
<td>Hobbed</td>
</tr>
<tr>
<td>Material</td>
<td>835C</td>
</tr>
<tr>
<td>Treatment condition</td>
<td>Normalized (hardness : HV = 186)</td>
</tr>
</tbody>
</table>

* m : Module
2.3.2 Bending fatigue test

Bending fatigue tests were carried out by using a bending fatigue testing machine of hydraulic type, and S-N curves were obtained. In order to observe the process leading to a bending fatigue breakage, the bending fatigue tests were carried out under an applied load $P = 1.1 \text{ Pu}$. Specimens were demounted from the testing machine at several stages in the fatigue process and cut at three positions as shown in Fig. 5. Then the crack initiation and propagation at the root fillet were observed by an optical microscope.

2.4 Measurement of root stresses and bending fatigue tests for thin rim spur gear

Static bending and bending fatigue tests were carried out with a thin rim spur gear whose dimensions and structure are shown in Table 2 and Fig. 2 by using a static bending and bending fatigue testing machine. In order to observe the process leading to a bending fatigue breakage, the bending fatigue tests were carried out under a normal tooth load $P = 1.3 \text{ Pu}$. Specimens were demounted from the testing machine at several stages in the fatigue process and cut at three positions as shown in Fig. 6. Then the crack initiation and propagation at the root fillet were observed by an optical microscope.

3. Experimental results and discussions

3.1 Root stress distribution and position of crack initiation of helical gear teeth

Figure 7 shows the S-N curve for the helical gear and the numbers of load cycles ($a$ to $d$) at which the observation of the fatigue process was carried out. A crack was observed at the root fillet on the tensile side at $N = 5 \times 10^5$ and on both the tensile and compressive sides at $N = 8 \times 10^5$. Figure 8 shows microphotographs of the crack on the tensile side at $N = 5 \times 10^5$. Figure 9 shows the crack length distributions on the tensile side at $N = 5 \times 10^5$, $8 \times 10^5$ and Fig. 10 on the compressive side at $N = 8 \times 10^5$.

Figure 11(a), (b) show the measured root stress distributions in longitudinal direction on tensile and compressive sides of helical gear teeth respectively. It is seen from Fig. 11 that the ratio of

![Position of load application](image)

Fig. 5 Cutting sections of cantilever plate

![Fig. 6 Cutting sections of thin rim spur gear tooth](image)

![Fig. 7 Bending fatigue test results for helical gear](image)

![Fig. 8 Microstructures at root fillet on tensile side of helical gear tooth ($P = 1.3 \text{ Pu}, N = 5 \times 10^5$)](image)
maximum root stress on the compressive side to that on the tensile side is about 1.1, which is smaller than in the case of spur gears\(^{(1)}\). Comparing the root stress distribution at the meshing position \(9\) in Fig.11(a) and the crack length distribution in Fig.9, it can be considered that a crack initiates at the position of maximum root stress, which is distant slightly from the root end, and propagates toward both ends with an increasing number of load cycles. The crack length distribution may be considered to be different from the root stress distribution before the crack initiation.

In the case of spur gears cracks appear in some cases first on the compressive side, and later on the tensile side\(^{(1)}\), while in the case of helical gears they seem to appear first on the tensile side. This might be explained as follows. Figure 12 shows the relation between \(-\sigma_r/\sigma_t\) and \(l/s\) (\(\sigma_r\), \(\sigma_t\): root stresses on the compressive and tensile sides, \(l\): length from loading point to Hofer’s critical section, \(s\): tooth thickness at critical section in the case of spur gears\(^{(1)}\)). In this figure \(\mu\) denotes the frictional coefficient and \(\rho\) the radius of curvature of fillet curve at critical section. It is found from this figure that the ratio \(-\sigma_r/\sigma_t\) decreases as the loading point moves from the tip toward the root. Since whether a crack initiates earlier on the compressive side or on the tensile side depends on the relationship between the ratio \((-\sigma_r/\sigma_t)\) of the stress \(\sigma_t\) on the compressive side to that \(\sigma_r\) on the tensile side and the ratio \((\sigma_{up}/\sigma_{tp})\) of the bending fatigue limit \(\sigma_{tp}\) under pulsating compression to that \(\sigma_{up}\) under pulsating tension, it is found from Fig.12 that the possibility of the crack appearing first on the tensile side becomes higher as the loading point moves farther from the tip toward the root.

In the case of helical gears the loading point differs in each section and extends from the neighborhood of the tip toward the root at the worst loading position, since the contact line crosses the tooth surface diagonally. The position of load application corresponding to the maximum root stress is at a certain distance from

![Fig.9 Crack length distribution on tensile side of helical gear tooth \((P = 1.3 \text{ P}_{\text{U}})\)]

![Fig.10 Crack length distribution on compressive side of helical gear tooth \((P = 1.3 \text{ P}_{\text{U}}, N = 8 \times 10^5)\)]

![Fig.11 Measured root stress distributions in longitudinal direction (helical gear)\(^{11}\)]

\(m_0 = 6, \beta_0 = 20^\circ, z = 36, b = 60 \text{ mm}, P = 1000 \text{ kg}\)
the tooth tip.

Figure 13 shows the positions of crack initiation and the directions of crack propagation for each section shown in Fig. 4. The positions of crack initiation lie in the neighborhood of critical section determined by Hofer's method, but the directions of crack propagation differ in each section. This might be attributed to the fact that the stress conditions of root fillet differ in each section since the positions of load application in each section differ as indicated by arrows.

In the actual running conditions, the root stress distribution along the tooth trace changes with the progress of meshing, but the maximum root stresses occurring in each section are shown by an envelope curve for a group of curves, which represent the root stress distributions at each meshing position. The maximum root stress in this experiment occurs at the meshing position 9 as mentioned above. Provided the crack initiation of gear teeth is dominated by the maximum root stress, the validity of this bending fatigue testing method, in which the gear member of the pair is rigidly locked against rotation, might be confirmed. Hence the bending fatigue limit obtained from this testing method might be considered to be almost equal to that obtained from the running test for the case when the dynamic additional load is negligibly small.

3.2 Calculated and measured results of root stresses of cantilever plate

Figure 14 shows the calculated and measured results of root stresses of a cantilever plate under the concentrated and uniform loads. The calculated results were obtained by applying the moment-image method (5) and the principle of superposition to the moment distribution due to a concentrated load calculated for a cantilever plate with infinite length by Fujita (6).

It is found from this figure that the maximum root stress and stress distribution differ depending on the loading conditions even if the total loads are the same. The measured maximum root stress due to the concentrated load is about 12% larger than that due to the uniform load.

3.3 Bending fatigue test results for cantilever plate

Figure 15 shows the bending fatigue test results for a cantilever plate under the concentrated and uniform loads. The ordinate indicates the applied load in Fig. 15(a) and the true root stress in Fig. 15(b). The bending fatigue limit load $P_d$ under a concentrated load is about 6% smaller in comparison with the case of a uniform load but the bending fatigue limits are almost equal. From Fig. 15(b) it is found that the time strength under a uniform load is smaller than that under a
concentrated load. This might be because the crack propagation under a uniform load is faster than under a concentrated load, since a higher stress occurs over a larger area of the face width as shown in Fig.14.

3.4 Root stress distribution and position of crack initiation of cantilever plate

A crack was observed only at the middle of face width on the tensile side at \( N = 7.3 \times 10^5 \) under a concentrated load \( P = 1.1 \, P_u \), and the length of the crack was about 1.3 mm. Figure 16(a) shows the crack length distribution on the tensile side at \( N = 8.8 \times 10^5 \) under a concentrated load \( P = 1.1 \, P_u \). The crack is longest at the middle of face width. Comparing the crack length distribution in Fig.16(a) and the root stress distribution due to a concentrated load in Fig.14, both distributions are similar to each other. It can be considered from these results that the crack initiates at the position of maximum stress (at middle of face width) and propagates toward both ends with an increasing number of load cycles.

Figure 16(b) shows the crack length distribution on the tensile side at \( N = 5 \times 10^5 \) under a uniform load \( P = 1.1 \, P_u \). The crack length is almost equal at the sections B, C, D in Fig.5 and shorter at both ends. It can be considered from the results shown in Figs.14 and 16(b) that the crack initiates at the position of maximum stress similarly to the case of a concentrated load.

Figure 17(a), (b) show the propagating pattern of a crack under a concentrated and a uniform load respectively.

3.5 Root stress distribution and position of crack initiation of thin rim spur gear teeth

Figure 18 shows the measured root stress distribution of a thin rim spur gear under a normal tooth load \( P = 400 \, \text{kg} \). The stress value at the middle of face width (just above the web) is higher than that at the end. Figure 19 shows the bending fatigue test results. The bending fatigue limit load (normal tooth load) is found to be \( P_u/b = 63.3 \, \text{kg/mm} \). A crack was observed at the root fillet on the tensile side at \( N = 3.8 \times 10^5 \) (a in Fig.19) under a normal tooth load \( P = 1.3 \, P_u \), and the crack length distribution is shown in Fig.20. The crack is longest at the middle of face width and shorter at both ends.
It is found from Figs. 18 and 20 that the crack initiates at the position of maximum stress.

3.6 Testing methods of bending strength for helical gears

From the above discussions it is found that the bending fatigue breakage of the helical gear teeth is dominated by the maximum root stress. The crack initiates first at the position of maximum root stress on the tensile side and propagates toward both ends with an increasing number of load cycles and on the compressive side later. Since the cracks of the helical gear teeth initiate at the position of maximum root stress as mentioned above, the bending fatigue limit can be obtained accurately by the testing method adopted in this experiment, in which the gear member of the pair is rigidly locked against rotation.

Figure 21 shows the relations between the helix angle and the bending fatigue strength ratio \( F_{uh}/F_{us} \) (\( F_{uh}, F_{us} \) : bending fatigue limit loads for helical and spur gears respectively) obtained by various testing methods\(^{(7)}\). The points ● in this figure represent the results of a pulsator test, in which a gear tooth is loaded by a loading bar with a test gear fixed; the chain lines are the running test results, and the points ○ are test results obtained from this experiment. It may be considered from this figure that the bending fatigue testing method adopted in this experiment is more similar in loading condition to the running test than to the pulsator test (loaded by a loading bar).

### Fig.18 Measured root stresses of thin rim spur gear (\( P = 400 \, \text{kg} \))

### Fig.19 Bending fatigue test results for thin rim spur gear

#### 4. Conclusions

The main results obtained from this investigation are summarized as follows.

1. The ratio of maximum root stress on the compressive side to that on the tensile side for helical gears is smaller than for spur gears. A crack initiates at the position of maximum root stress on the tensile side, which is at a certain distance from the tooth end, and propagates toward both ends with an increasing number of load cycles.

2. The crack length distribution in helical gear teeth is different from the root stress distribution before the crack initiation.

3. In the case of helical gear, the positions of crack initiation in each normal section lie in the neighborhood of critical section determined by Hofer's method, but the directions of crack propagation differ in each normal section.

4. The maximum root stress of a cantilever plate due to a concentrated load is higher than that due to a uniform load under the same total load. The bending fatigue limits are almost equal regardless of the loading condition, but the time strength under a uniform load is smaller than that under a concentrated load.

5. The crack of a cantilever plate initiates at the position of maximum stress on the tensile side and propagates toward both ends with an increasing number of load cycles.

#### Fig.20 Crack length distribution on tensile side of thin rim spur gear (\( P = 1.3 \, P_d, \, N = 3.8 \times 10^5 \))

#### Fig.21 Relation between bending fatigue strength ratio \( F_{uh}/F_{us} \) and helix angle \( \beta \)}
(6) The crack of a thin rim spur gear initiates at the position of maximum root stress (just above the web) on the tensile side.
(7) The bending fatigue testing method adopted in this experiment is considered to be more similar in loading condition to the running test than to the pulsator test (loaded with a loading bar).

Acknowledgement

The authors wish to thank Mr. C. Namba of the Department of Mechanical Engineering of Tottori University for his assistance in performing this investigation.

References