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Experimental Study on Cut-Off Phenomenon for Layered Composite *

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The cut-off phenomenon of longitudinal waves propagating normal to the layers of composite is studied experimentally by measuring the attenuation of the ultrasonic waves. Present method is equivalent to measuring the imaginary part of the dispersion relation, while the conventional methods that treat the relation between phase velocity and frequency are equivalent to measuring the real part of the relation.

Constituents of composite specimens are elastic or viscoelastic, and it is shown that the effect of constituents on cut-off phenomenon is in good agreement with that predicted theoretically. For viscoelastic composite, it is clarified that the extreme attenuation observed in a specific frequency is not due to its viscosity but mainly to cut-off phenomenon.

Key words : Composite Materials, Dispersion Relation, Sound,
Cut-Off Frequency, Stress Waves

1. Introduction

A laminated composite that consists of periodic arrangement of more than two kinds of materials has dispersion characteristic because of its inhomogeneity. Especially in the case when the wave propagates perpendicular to the layers, the imaginary part of complex wave number increases suddenly in specific frequency bands and the wave attenuates greatly (cut-off phenomena) (1)(2).

In the case of wave propagation perpendicular to the layers, the exact dispersion relations were obtained by Rytov (3), and Tanaka and Konno(1) extended this result to viscoelastic composite and showed a spectrum of frequency versus complex wave number and of phase velocity versus attenuation ratio. On the other hand, many approximate theories have been proposed for elastic or viscoelastic

waves. Examples are "effective stiffness" theory by Sun-Achenbach-Herrman(4), "continuum" theory by Hegemier-Nayfeh(5), "variational method" by Kohn-Krumhansl-Lee (6) and so on for the former. "Theory of viscoelastic analogy" by Ting-Mukunoki (7) is an example for the latter.

In comparison with these theoretical studies, there are few experimental studies concerned with dispersion relations and cut-off phenomena. In the case of wave propagation perpendicular to the layers of composites that consist of two kinds of elastic materials, frequency-wave number (real part of complex wave number) spectrum was obtained and the existence of cut-off frequency bands was shown by Robinson-Leppelmeier(2). It may be said that their investigation was performed paying attention to the real part of dispersion relations.

In this paper, cut-off phenomena of longitudinal wave propagating perpendicular to the layers are studied experimentally by measuring attenuation characteristic of composites. Unlike Robinson-Leppelmeier's, this study is concerned with the imaginary part of dispersion relations. The composite structures used in this experiment are made of two kinds of layers, that is steel-aluminum, aluminum-copper and steel-copper. As well as these elastic composites, viscoelastic ones are used that consist of PVC in one layer, and the influence of the difference in material properties on cut-off phenomena is discussed.

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2. Theoretical Consideration

Consider a composite made of an infinite periodic array of two alternating layers which differ in material properties and thickness as shown in Fig.1. In the case of harmonic wave propagation perpendicular to the layers, exact solutions were obtained by Rytov(3) for elastic media, and obtained by Tanaka and Konno for viscoelastic material (3-element model). In this chapter, we show only the results of their theoretical treatment briefly and discuss the problems concerned with the energy and mode that have not been treated before.

In Fig.1, a coordinate system is placed with the x-axis perpendicular to the layers (in the direction of wave propagation) and the y,z-axis in the boundary plane of two layers perpendicular to each other. It is assumed that the problem is of plane strain and motion takes place only in the x-direction. The thicknesses of layers 1,2 are described as a,b respectively, and material parameters of layer j(j=1,2) are denoted with the subscript j.

First of all, the materials of two kinds of laminates are supposed to be linearly viscoelastic, and their constitutive equations are given as follows:

$$\sum_{m=0}^{m_0} p_{mj} \frac{\partial^m \sigma_j(x,t)}{\partial t^m} = \sum_{n=0}^{n_0} q_{nj} \frac{\partial^n \varepsilon_j(x,t)}{\partial t^n} \quad (p_{0j} \equiv 1) \quad \dots\dots\dots(1)$$

where σ and ε are the normal stress and strain, p_m and q_n are the material parameter, t is time, m and n are integer numbers, respectively.

Consider the case of harmonic wave propagation in the direction of x-axis. According to Floquet's theorem(8), the solution is described with the product of the periodic function with the period of $a+b$ and the exponential function as follows:

$$u(x,t) = v(x) \exp[i(\omega t - kx)] \quad \dots\dots\dots(2)$$

$$v(x+a+b) = v(x)$$

where u is the displacement in the x-direction, ω is angular frequency and k is a parameter of composites which has the dimension of wave number. In Eq.(2), the periodic function $v(x)$ is expressed by

$$v(x) = A_j \exp[i(k+k_j)x] + B_j \exp[i(k-k_j)x] \quad \dots\dots\dots(3)$$

Fig.1 Analytical model

where A_j and B_j are the integral constant, and k_j is the wave number of harmonic wave in layer j .

In Fig.1, we consider 1 cell of the composite which is located $-b \leq x \leq a$. According to the boundary conditions of stress and strain continuity in the boundary plane of two layers and the condition that $v(x)$ is a periodic function, the following dispersion relation is obtained

$$\left. \begin{aligned} \cos[k(a+b)] &= \cos[k_1 a] \cos[k_2 b] \\ &\quad - \frac{1+\kappa^2}{2\kappa} \sin[k_1 a] \sin[k_2 b] \end{aligned} \right\} \dots\dots\dots(4)$$

where

$$\kappa = \frac{E_2 k_2}{E_1 k_1},$$

$$E_j = \left(\sum_{n=0}^{n_0} q_{nj}(i\omega)^n \right) / \left(\sum_{m=0}^{m_0} p_{mj}(i\omega)^m \right)$$

$$\frac{k_j}{\omega} = \sqrt{\frac{\rho_j}{E_j}}$$

In the above equation, k_j and κ are real in the case when all layers are elastic, and they are complex numbers in the case of viscoelastic layers(1).

2.1 Cut-off phenomena and transported energy

As a first example, the calculated dispersion relation is shown in Fig.2, where the materials of two layers are supposed to be steel and aluminum. The material parameters used are shown in Table 1. In Fig.2, the frequency band in which k is complex is the cut-off region.

As a second example, the dispersion relation of viscoelastic composite is shown in Fig.3, where aluminum is replaced with a viscoelastic material whose constitutive equation is in the following form.

$$\sigma_2 = q_{02} \varepsilon_2 + q_{12} \dot{\varepsilon}_2$$

The dispersion relation of viscoelastic material itself is shown also in Fig.3. Comparing the imaginary parts of the dispersion relations of composite and viscoelastic materials in Fig.3, both of them show the same variation in the frequency range from 0 to 1.5MHz. But in the frequency band of more than 1.5MHz, the imaginary part of k (the composite parameter that has the dimension of wave number) increases quickly. Consequently in these frequency bands, the attenuation occurs intensively due to the cut-off phenomena together with the dissipation owing to the viscosity of the material.

The transported energy of the harmonic wave through the unit area of cross section for unit time is expressed as follows:

$$p = \frac{\omega}{2\pi} \frac{1}{2} \int_0^{2\pi/\omega} \operatorname{Re} \left[-\sigma(x,t) \frac{\partial u(x,t)}{\partial t} \right] dt \quad \dots\dots\dots(5)$$

Table 1 Material Parameter

Constituents	Density kg/m ³ ρ_j	Modulus of Elasticity φ_{0j} (Pa)	Layer thickness mm
Fe(Layer 1)	7.87×10^3	2.77×10^{11}	a:0.5
Al(Layer 2)	2.70	1.02	b:0.5

Fig.2 Dispersion relation of Fe-Al composite

Fig.3 Dispersion relation of Viscoelastic composite

where Re and $-\text{Im}$ indicate the real part and the conjugate complex respectively. In Eq.(5), integrated function is the inner product of stress vector and particle velocity vector on the complex plane, and frequency - transported energy relations depend on the phase difference between both vectors. For example, calculated results of phase difference at the location $x=0$ are shown in Fig.4 for the elastic and viscoelastic composites indicated in Fig.2 and Fig.3.

For the elastic composites, the phase difference is $\pi/2$ in cut-off bands and no

Fig.4 Phase difference between stress and particle velocity

energy is transported. But for the viscoelastic composites, there are no frequency bands where the phase difference becomes $\pi/2$. This can be explained as follows. The problem is supposed to be a stationary propagation of the harmonic waves in the infinite periodic array of two layers. In the case of viscoelastic composites, it is needed to transport the energy that is dissipated in each viscoelastic layer. Therefore, it is impossible to define cut-off bands clearly for viscoelastic composites(1).

2.2 Consideration on modes of waves

Equation(4) of dispersion relation gives many solutions k for the unique value of ω . Therefore many modes appear in Fig.2 and Fig.3. But it is proved as follows that these modes are equivalent to each other.

Substituting Eq.(3) into Eq.(2), we obtain the following displacement solution:

$${}^h u(x, t) = {}^h A_1 [{}^h C_j \exp [i(\omega t + k_j x)] + {}^h D_j \exp [i(\omega t - k_j x)]] \quad (6)$$

where

$${}^h C_j = \frac{{}^h A_j}{{}^h A_1}, \quad {}^h D_j = \frac{{}^h B_j}{{}^h A_1}$$

In Eq.(6), the subscript h denotes a parameter of the h -th mode. The first and second terms of the right hand represent a plane wave with wave number k_j travelling along the negative and positive directions of the x -axis, respectively. k is a parameter that determines ${}^h C_j$ and ${}^h D_j$ (in other words, the energy ratio of two plane waves travelling in the opposite direction to each other). ${}^h A_1$ in Eq.(6) is determined by the amplitude of waves in the composites, therefore we may take ${}^h A_1 \equiv 1$ without losing generality in the following discussion.

Now, consider the modes of waves travelling in the positive direction. From Eq.(4), the parameter ${}^h k$ of the h -th mode is obtained in terms of ${}^h k$ as follows:

$${}^h k = {}^1 k + \frac{2(h-1)\pi}{(a+b)} \quad (7)$$

According to Eq.(7), it is shown that ${}^h C_j$ and ${}^h D_j$ have the same value for every mode and so the solution shown in Eq.(6) has the same form for every h -th mode. Therefore many modes that appear in Fig.2 and Fig.3 are equal, and it is sufficient to treat only the dispersion relations expressed with a continuous line in Fig.2 and Fig.3.

In this study, measurements are performed for this mode to show the cut-off phenomena of laminated composites.

3. Experiments

3.1 Specimen

The specimens used are constituted of a periodic array of two kinds of thin plates with square cross sections as shown

Table 2 Material of Specimen

Symbol	Material
Fe	SK 3
Al	1050 H24 Aluminium
Cu	
PVC	Polyvinile carbonate

Table 3 Specimens used in the experiment

No	Combination	Layer thickness (mm)	Cell	Total length (mm)
1	Fe(0.5mm)-Fe(0.5mm)	0.506	35	35.6
2	Fe(0.5mm)-Al(0.5mm)	Fe:0.519 Al:0.508	20	20.6
3	Fe(0.5mm)-Al(0.5mm)	Fe:0.511 Al:0.515	35	36.3
4	Fe(0.7mm)-Al(0.7mm)	Fe:0.710 Al:0.707	20	28.5
5	Fe(0.5mm)-Cu(0.5mm)	Fe:0.511 Cu:0.517	20	20.5
6	Cu(0.7mm)-Al(0.7mm)	Cu:0.701 Al:0.711	20	28.1
7	Fe(0.3mm)-PVC(0.3mm)	Fe:0.304 PVC:0.309	35	21.8
8	Al(0.3mm)-PVC(0.5mm)	Al:0.301 PVC:0.515	35	28.7
9	Al(0.5mm)-PVC(0.3mm)	Al:0.500 PVC:0.305	35	28.4
10	PVC(1mm)-PVC(1mm)	0.970	10	19.4
11	PVC(1mm)-PVC(1mm)	0.970	20	39.8

Fig.5 Shape of specimen

in Fig.5. Each plate is pressure bonded to each other by using epoxy resin. Materials of the layers are shown in Table 2, and specimens examined are listed in Table 3. The average thickness of epoxy resin is less than $6\text{ }\mu\text{m}$ for all of specimens.

In addition to these composite specimens, a steel specimen 35mm square and 40mm thick is used to measure the attenuation of wave propagation in composite specimens by comparing the amplitude of a transmitted wave in the steel specimen with that of one in composite specimens.

3.2 Experimental apparatus and methods

The experimental apparatus is shown in Fig.6. Both ends of the specimen are smeared with oil, and the specimen is placed between two vibrators. With the

switch turned off, the amplitude of a signal generated by the oscillator is adjusted to a definite value. When the switch is turned on, the transmitting vibrator begins to drive the specimen and the input signal is stored in a digital memory. The harmonic wave reaches the receiving vibrator with a delay time for propagating through the specimen. The output signal from the receiving vibrator is stored in the digital memory and displayed on the oscilloscope to observe the amplitude. These operations are repeated for several definite frequencies using the steel and composite specimens as shown in Table 2.

In the present experiments, the frequency range is from 0.45 to 3.9MHz, and barium titanate piezoelectric vibrators are used. The measurements concerned with viscoelastic composites are accomplished under the condition of $20 \pm 0.5^\circ\text{C}$.

Fig.6 Block diagram of experimental apparatus

3.3 Measurements of attenuation

The modes of input and output signals are illustrated in Fig.7. The output signal has a transient part and a stationary part of a constant amplitude. In this experiment, in order to avoid the influence of frequency characteristics of vibrators upon the experimental results, attenuations are obtained as the ratio of these

Fig.7 Input and output signals

constant amplitudes of the composites to those of the steel specimens. Therefore, attenuation parameters of composites obtained in this way are not the absolute values but the values relative to the steel specimen.

For the laminated composite made of PVC layers only, two specimens of different lengths are used to determine the attenuation parameter by measuring the ratio of amplitude of the waves transferred through each specimen. Attenuation parameter that corresponds to the difference in specimen length is obtained.

4. Experimental results and discussion

Material constants used in calculation are listed in Table 4. As concerns the constitutive equation of PVC, it is supposed that m and n in Eq.(1) are equal to 2. Then the real part and the imaginary part of wave number are measured by phase comparison method(11)(12) and attenuation experiment of ultrasonic waves, and material parameters p, q in Eq.(1) are also determined by these experimental results. These parameters are determined in the frequency range of 0.5 - 1.3MHz in order to use the constitutive equation for analysis of the first transmission band.

4.1 Elastic laminated composite

The experimental results about Fe (0.5mm)- Fe(0.5mm) specimen and Fe(0.5mm)-Al(0.5mm) specimen are shown in Fig.8 and Fig.9 respectively. In Fig.8, the attenuation quantity in Fe-Fe specimen increases gradually with an increasing frequency. This gradual attenuation is supposed to be caused by epoxy resin used to bond each layer. But in Fig.9, the output waves attenuate suddenly at the frequency of 1.5 - 1.6MHz. In comparison with Fig.8, this rapid attenuation is supposed to be cut-off phenomena of composites. That is, the frequency bands that have no measuring point correspond to cut-off bands. In these bands, it is impossible to measure the amplitude of output waves because it is under the noise level. In Fig.9, the results about two specimens with different lengths are shown. Both of cut-off frequencies are 1.5 - 1.6MHz, but the attenuation quantity of 35-cells specimen is greater than that of 20-cells specimen in the cut-off band. The second transmission band is observed in only 20-cells specimen.

Fig.8 Attenuation character of Fe-Fe specimen

Fig.9 Attenuation character of Fe-Al specimen

Table 5 Ratio of acoustic impedance

	Fe-Al	Cu-Al	Fe-Cu
κ	0.354	0.395	0.897
$(1 + \kappa^2)/2\kappa$	1.59	1.46	1.01

The acoustic impedance ratio of two kinds of layers κ is very important for the cut-off phenomena of those composite materials. The dispersion relations(4) includes κ in the form of the coefficient $(\kappa + 1/\kappa)/2$. The values of κ and $(\kappa + 1/\kappa)/2$ are shown in Table 5 for the elastic composites measured in the experiment. And the experimental results of Fe(0.7mm)-Al(0.7mm) specimen, Al(0.7mm)-Cu(0.7mm) specimen and Fe(0.5mm)-Cu(0.5mm) specimen are shown in Fig.10 and Fig.11 respectively. In Table 5 and Fig.10, the acoustic impedance ratio of Fe-Al specimen is smaller (in comparison to unity) than that of Cu-Al specimen, and the cut-off frequency band of Fe-Al specimen is larger. For Fe-Cu specimen, the value of κ is much closer to unity and the cut-off band becomes much narrower. So, it was impossible to observe the cut-off region clearly.

In those results, the measured cut-off frequency tends to be a little smaller

Table 4 Material constants used in calculations

constituents	Density kg/m ³	q_0 Pa	q_1 N.s /m ²	q_2 N.s ² /m ²	p_1 s	p_2 s ²
Fe	7.87×10^3	2.77×10^{11}				
Al	2.70×10^3	1.02×10^{11}				
Cu	8.90×10^3	1.98×10^{11}				
PVC	1.40×10^3	5.55×10^9	1.70×10^4	8.27×10^{-4}	2.24×10^{-6}	1.09×10^{-13}

Fig.10 Attenuation character of Fe-Al, Al-Cu specimens

Fig.11 Attenuation character of Fe-Cu specimen

than the calculated one. This is caused as follows. In the calculation, we consider a model in which each layer spreads infinitely in the yz -plane and $\lambda + 2\mu$ (λ, μ : Lamé's constants) is used as modulus of elasticity, that is we recognize the problem as a one-dimensional one. But in the experiments, we use specimens of finite size. Strictly speaking, the problem is a multi-dimensional one with such boundary condition that stress vanishes on the side wall. When it is assumed as a one-dimensional problem, the modulus of elasticity may be smaller than $\lambda + 2\mu$. Therefore, the calculated cut-off frequency becomes a little larger than the experimental one. As an example, the calculated result with Young's modulus is shown in Fig.9 with dashed line.

4.2 Viscoelastic laminated composite

The experimental results about Fe (0.3mm)-PVC(0.3mm) specimen and Al(0.3mm)-PVC(0.5mm) specimen are shown in Fig.12 and Fig.13 respectively. A rapid attenuation is observed near the frequency of 0.8-0.9MHz in Fig.12 and near the frequency of 1MHz in Fig.13. Figure 14 indicates the result about PVC(1mm)-PVC(1mm) specimen. In this case, the attenuation quantity increase gradually with an increasing frequency, and there is no rapid attenuation as shown in Fig.12 and Fig.13. Therefore, those rapid attenuations are supposed to have been caused by the cut-off phenomena of composites. The continuous lines in Fig.12 and Fig.13 show the calculated results with respect to the first transmission band, and agree with the experimental results fairly well.

Fig.12 Attenuation character of Fe-PVC specimen

Fig.13 Attenuation character of Al-PVC specimen

Fig.14 Attenuation character of PVC specimen

In Al-PVC specimen shown in Fig.13, the second transmission band is observed. In comparison with the results about elastic composites, the gain in the second transmission band is fairly less than that in the first transmission band, because the dissipation caused by viscosity of PVC becomes larger for higher frequency. In Fe-PVC specimen shown in Fig.12, the second transmission band is not observed. Perhaps it will be beyond the measured frequency region because the acoustic impedance ratio of two layers is much less than unity, and the cut-off band becomes too wide.

5. Conclusions

With elastic composite specimens made of steel, aluminum or copper and visco-

elastic composite specimens, one of the components of which is PVC, the cut-off phenomena are studied by attenuation experiments of ultrasonic waves. This method corresponds to an experiment that pays attention to the imaginary part of dispersion relations. Unlike the traditional method to obtain the relation of phase velocity and frequency (method that pays attention to the real part), it is possible to use the method over a wide frequency range and observe the cut-off phenomena easily.

The calculated dispersion relations show the influence of the difference of material properties on cut-off phenomena. Especially with the viscoelastic composites, it is proved by comparing the attenuation characteristic of composites with that of PVC that rapid attenuation is caused not by dissipation of viscoelastic material but by cut-off phenomena.

It is concluded that the multi modes which appear in the theoretical dispersion relations are equal to each other and there is only one solution.

References

- (1) Tanaka and Konno, Trans. Japan Soc. Mech. Engrs., (in Japanese), Vol.45, No.399 (1979), p.1418.
- (2) Robinson, C.W. and Leppelmeier, G.W., Trans. ASME, Ser.E, Vol.41, No.1 (1974), p.89.
- (3) Rytov, S.M., Soviet physics Acoustics, Vol.2, No.1 (1956), p.68.
- (4) Sun, C.T., et al., Trans. ASME, Ser.E, Vol.35, No.3 (1968), p.467.
- (5) Hegemier, G.A., and Nayfeh, A.H., Trans. ASME, Ser.E, Vol.40, No.2 (1973), p.503.
- (6) Kohn, W., et al., Trans. ASME, Ser.E, Vol.39, No.2 (1972), p.327.
- (7) Ting, T.C.T. and Mukunoki, I., Trans. ASME, Ser.E, Vol.46, No.2 (1979), p.329.
- (8) Stoker, J.J., "Nonlinear Vibrations in Mechanical and Electrical Systems", (1950), p.193, Interscience Publishing Inc..
- (9) Rikagakujiiten. (in Japanese), (1970) Iwanamishoten.
- (10) Choonpagijutsubinran (in Japanese), (1978) Nikkankogyoshinbunsha.
- (11) Ting, C.S. and Sachse, W., J. Acoust. Soci. Am., Vol.64, No.3 (1978), p.852.
- (12) Lynnworth, L., et al., AMMRC Rep., CTR73-2 (1973).