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Flow Characteristics and Methods of Flow Calculation of High-speed Compressible Flow through Pipe Orifices*

By

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Flow characteristics of a compressible gas flow through an orifice are investigated experimentally at pressure ratios below the regulation values of JIS and ASME. For practical mass flow measurements, a theoretical method of mass flow estimations is extended using one-dimensional flow theory and experimental data. Using the method, the accuracy of mass flow measurements with orifice meters is about $\pm 1\%$ in the Reynolds number range of turbulent flows and also in supercritical flows. Tables of the product of flow coefficient and expansion factor are obtained by the method at various diameter ratios, pressure ratios, and specific heats.

Key words: Compressible Flow, Flow Measurement, Orifice Meter, Expansion Factor, High Speed Flow

1. Introduction

It is well known that an orifice as a device of mass flow measurements has many advantages over the other fluid meters, on account of lower manufacturing cost and easy maintenance and so on. Furthermore, with an increase in pipe sizes and flow speeds in various plants of recent years, "indirect measurement" devices (which require no calibration tests), such as orifices, nozzles and venturils, are receiving attention¹.

It is necessary to decide the value of expansion factor for mass flow measurements of any gases using orifice meters. However, the conventional empirical formulae for expansion factor are applicable to the orifice only at pressure ratio above 0.75 by JIS², partly because of an increase in the error of the formulae in the range of pressure ratios below 0.75 and shortage of reliable experimental data below the pressure ratio 0.75.

In any case, the expansion factor equation should only be used at subsonic flow regimes³. At present, satisfactory discussions on reliable expansion factor are not yet completed, and therefore, it is left as an important engineering problem to obtain a definite measuring method both at critical and supercritical flow regimes.

Mass flow characteristics through the orifices in high-speed flow regimes have been made clear to a certain extent⁴⁻⁷, but in the practical mass flow measurements under those flow conditions, a lot of problems remain to be resolved.

In this paper, the flow characteristics of a compressible flow through orifices at a pressure ratio below the regulation values of JIS² and ASME³ are investigated experimentally, and the applicable limits and the accuracy of the JIS and ASME regulations are clarified for each pressure tap system. Moreover, in the course of re-considering the physical meanings of flow coefficient and expansion factor, a theoretical method of mass flow estimations which has a required accuracy for practical purposes is developed.

2. Nomenclature

A : Cross sectional area
 A_N : Orifice opening area
 C_c : Contraction coefficient
 C_v : Velocity coefficient
 D : Pipe diameter = 50 mm
 M : Mach number
 $\dot{m}$: Mass flow rate
 p : Static pressure
 Re_D : Reynolds number
 r : Static pressure ratio across orifice
 x : Distance from inlet face of orifice
 α : Flow coefficient
 β : Diameter ratio = $\sqrt{A_N/A_1}$
 Δ, δ : Pressure difference
 ϵ : Expansion factor
 κ : Ratio of specific heats
 λ : Relative error of expansion factor
 ρ : Density
 τ_w : Shear stress at the pipe wall
 χ : Ratio of flow coefficients

Subscripts

1 : Inlet condition
 b : Dead air region
 c : Corner tap
 d : D·1/2 D tap
 $e, *$: Throat condition, sonic throat
 f : Flange tap
 i : Incompressible value
 p : Pipe tap
 v : Vena contracta tap

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Fig.1 Schematic arrangement of the experimental apparatus

3. Experimental apparatus and procedure

A schematic arrangement of the present experimental apparatus is shown in Fig.1. A steady compressed air produced by a gas supply system flows through a mass flow measuring orifice (corner tap; $\beta = 0.6$) which is installed in a 131 mm i.d. pipe, and leads to the test section of 50 mm i.d. pipe. The flows through the test orifice can be varied from subsonic to supercritical conditions, while the measuring orifice is operated under the condition of a pressure ratio above 0.98 in all test runs.

Inner surface of the pipe at the test section is smooth enough and the relative roughness reaches around 6×10^{-6} , which will be nearly equal to that of larger diameter pipes in practical use. The wall pressure taps with four holes of 0.8 mm diameter at the same cross section (only for corner taps ring slits are used) are suitably distributed in the direction of the duct axis. Shapes, dimensions of the test orifices and locations of differential pressure taps are manufactured in accordance with JIS² and ASME³ recommendations.

Present experiments are carried out after the flow attains steady conditions. The expansion factor for the test orifice is determined from the following conventional equation of mass flow rate*.

$$\dot{m} = \alpha \epsilon A_N \{2 \rho_1 p_1 (1 - r)\}^{1/2} \dots \dots \dots (1)$$

4. Experimental results and discussion

4.1 Axial distribution of wall pressures

Typical wall pressure distributions in the downstream of orifices are shown in Fig. 2. The pressure distribution curves seem to be quite similar to those of incompressible flow cases⁸. In compressible flows, however, minimum wall pressure points (indicated by broken lines in Fig.2) near the exit of the orifice vary with pressure ratio r . It has been believed that the minimum pressure point moves towards downstream with

* When diameter ratio β is 0.3, the accuracy of mass flow rate $\dot{m}$ is considerably inferior to those of larger β cases, because of small differential pressures of the flow measuring orifice (smaller than 7 mm Aq.). Therefore, the scatter of expansion factor data in the case of $\beta = 0.3$ was disregarded in the following discussions.

Fig.2 Axial distribution of wall pressures

increasing flow velocity in a compressible flow^{9,10}. However, it is seen in Fig.2 that this phenomenon of the minimum pressure point appears only at a pressure ratio larger than about 0.9, and the minimum pressure point gradually approaches the orifice with further decrease in r at any β conditions. This phenomenon could be explained as follows: The shape of vena contracta will be elongated towards the downstream, and the minimum pressure point would move to the downstream at incompressible conditions, which will be overcompensated by a stronger compressibility effect caused by a steeper expansion in the vena contracta.

A minimum throat area point generated by the jet contraction is located slightly closer to the orifice than the point of the minimum pressure¹¹. But the wall static pressures of these two points are almost the same. Therefore it is reasonable to assume that the two points are situated at the same downstream position of the orifice for mass flow measurements.

And though the variation of the minimum pressure point due to a change in the pressure ratio is a peculiar phenomenon to the compressible flow, the pressure at the same point as the minimum wall pressure in an incompressible flow will be substituted for the minimum wall pressure in high-speed flow regimes, because the substitution leads to no marked errors in mass flow measurement.

With an increased flow rate, a shock wave appears in a jet region under the conditions of supercritical flow regime. However, no significant effect of these shock waves on the wall pressure curve is recognized at the present experimental flow conditions ($r \geq 0.3$), which will be one of advantages for mass flow measurements with orifice meters at supercritical flow condi-

The value shown in parenthesis indicates the distance measured from inlet face of orifice.

Fig.3 Variation of δ/Δ with β for the typical tap positions

tions.

4.2 Relation between flow coefficient and differential pressure

In this section, the characteristics of the wall pressure distribution will be discussed in detail from the viewpoint of mass flow measurement. The values of δ and Δ defined in Fig.2 (δ, Δ are expressed in non-dimensional form, being divided by p_1) vary with the orifice pressure ratio r . It is important to know which of two factors, Reynolds number or Mach number, should be important for this variation of δ, Δ with r .

Fig.3 shows the relation between β and δ/Δ on the typical location of pressure taps. The vertical width of each plot implies the variation of the data in the pressure range $0.9 \geq r \geq 0.5$. Fig.3 also shows data by Bean et al.¹⁰ for an incompressible flow as a reference (the width of the plots seem to vary due to Re_D variation). It may be estimated from these data that hardly any influence of compressibility is recognized on the pressure distribution curve, and the variation of δ/Δ with r can be considered to come under the effect of Reynolds number Re_D .

Suppose that different kinds of differential pressure taps were used for incompressible flow measurements at the same time. Then $\alpha\sqrt{\Delta}$ of the taps should be identical [see eq.(1)] because of the identity of flow quantity. Thus the relation between α and Δ is given by equation (2) in which the value for vena contracta tap is taken as a reference value.

$$(\alpha/\alpha_c)^2 = (\Delta/\Delta_c) \quad \dots\dots\dots (2)$$

The relation will be checked here for corner tap case as an example. Equation(2) is also applicable to compressible flow case if the relation $(\epsilon_c/\epsilon_r)^2(\rho_{1c}/\rho_{1r}) \approx 1$ could be assumed.

Plots indicate the experimental result using pitot tube at a distance $-5.5D$ (o) and $-1.2D$ (x) measured from inlet face of orifice.

Fig.4 Variation of inlet Mach number with the pressure ratio.

Then, equation (2) is developed in to equation (3).

$$\chi_c = \left(\frac{\alpha_c}{\alpha} \right)^2 = \frac{\Delta_c}{\Delta} = 1 + \frac{(\delta_1 - \delta_2)\epsilon}{\Delta} \quad \dots\dots\dots (3)$$

Table 1 shows two χ_c values. One is δ/Δ calculated using experimental data and the other is derived by using α of JIS at $Re_D = 10^5$ and at 10^6 . It is confirmed by comparing those χ_c values in Table 1 that equation(3) is satisfied fairly well for compressible flow case.

4.3 Physical interpretation of expansion factor

The compressibility effect in the mass flow rate equation(1) is expressed in terms of expansion factor ϵ . The factor involves the following physical effects.

- [1] Changes of fluid states (i.e. ρ etc.).
- [2] Change of contraction coefficient due to variation of flow Mach number and r .
- [3] Deformation of the wall pressure distribution curve⁹ as described in the previous section.

However, there have been no quantitative interpretations for this each term on ϵ . It is clarified as described in 4.1 and 4.2 that the third term can be neglected. Now we consider the first term [1].

If the flow from the inlet of the orifice to the vena contracta was presumed to be one-dimensional, steady, adiabatical without friction, the following equation(4) can be derived by combining the conservation

Table 1 Value of χ_c [see equation (3)]

β		0.3	0.5	0.6	0.7	0.8
present exp.	$r=1.0$	0.994	0.995	1.005	1.030	1.070
	$r=0.6$	0.995	0.998	1.005	1.021	1.049
JIS	$Re_D=10^5$	0.991	0.998	1.011	1.033	1.069
	$Re_D=10^6$	0.988	0.993	1.002	1.024	1.064

equations of mass and energy.

$$R_1^{2/\kappa}(1-R_1^{(\kappa-1)/\kappa})=C_c^2\beta^4R_1^{2/\kappa}(1-R_1^{(\kappa-1)/\kappa})$$

$$R\equiv p/p_{e,\text{total}}, \quad (p_1=p_{e,\text{total}}) \dots\dots\dots(4)$$

By given $p_0/p_1=(R_e/R_1)(p_0/p_e)$ the inlet static-to-total pressure ratios R_1, R_e, R_b can be calculated from the following relations: $p_0/p_e=1$ for subsonic flow case, and $R_*=[2/(\kappa+1)]^{1/(\kappa-1)}$ for critical and supersonic flow cases. To evaluate the compressible contraction coefficient C_c in equation (4), Benedict's theory¹² which will be discussed later is adopted. Theoretical inlet Mach numbers of orifices and nozzles are shown in Fig. 4 calculated by equation (4) using the relation ($\bar{M}_1=C_e M_{1,\text{ideal}}$). Pitot tube traverse data using equation (5) and flow nozzle data are included in Fig. 4.

$$\bar{p}_{\text{total}}/\bar{p}=\left\{\int (p_{\text{total}}/p) \rho u dA\right\}/\dot{m} \dots\dots\dots(5)$$

The result indicate that the velocity coefficient becomes slightly less than unity due to the presence of viscous losses in a real fluid flow. The agreement between theoretical value and experimental data is fairly good at any constriction and β , if the velocity coefficient ($C_v=0.97$) is adopted.

These results show that an adiabatical change in fluid states can be assumed, and the effect of fluid viscosity is approximated by the constant velocity coefficient, $C_v=0.97$.

Next we discuss the term [2]. Benedict's theory¹² which seems to be a complete one-dimensional compressible treatment, is used as the contraction coefficient theory in this paper. Within the present experimental pressure range ($r \geq 0.3$)*, the accuracy of Benedict's theory seems to depend upon the uniformity of static pressure p_b in dead air region and the reliability of the given value of C_{ci} . The former condition is confirmed to be satisfactory by the pitot tube traverse of the present investigation.

Before discussing C_{ci} term, the experimental results of compressible contraction coefficient C_c as calculated from equation (6) putting $C_v=0.97$ are plotted in Fig. 5. The qualitative agreement in the C_c-r curves between experimental data and the theoretical prediction (indicated by solid and broken lines) is good. And it is recognized that the accuracy of Benedict's theory mainly depends on C_{ci} .

$$C_c=\dot{m}/\left[C_e A_N \left\{ \frac{2\kappa}{\kappa-1} \frac{\rho_1 p_1 (1-r_1^{(\kappa-1)/\kappa})}{1-C_c^2 \beta^4 r_1^{2/\kappa}} \right\}^{1/2}\right]$$

$$r_1=p_1/p_0, \quad (p_e=p_0)_{\text{sub.}}, \quad (p_e=p_* \neq p_0)_{\text{crit.}} \dots\dots\dots(6)$$

Solid lines in the figure indicate the theoretical results using equation (7) which is based on the orifice data shown in Table 2.

$$C_{ci}=0.604107+0.092754\beta^2+0.072685\beta^4$$

$$+0.079758\beta^6 \dots\dots\dots(7)$$

Table 2 Value of C_{ci} (extrapolated C_c for $r \rightarrow 1$)

β	present result (Fig. 5)					Benedict ⁽¹²⁾			Perry ⁽⁴⁾
	0.3	0.5	0.6	0.7	0.8	0.241	0.481	0.634	~0.1
C_{ci}	0.614	0.635	0.651	0.676	0.714	0.614	0.625	0.660	0.602

Fig. 5 Variation of C_c with r

And the broken lines indicate the theoretical result using the empirical equation introduced by Benedict⁶ based on Weisbach's data on sudden contractions. However, this result seems to contain some error for orifice case. On the other hand, the result with equation (7) is found to be quite accurate for the whole β range in the turbulent flow regime. Hereafter, for evaluating the value of C_{ci} , equation (7) is adopted in the present analytical calculations [To obtain the calculated value in Fig. 4, equation (7) is already used].

4.4 Application of the theory for practical mass flow measurements

According to a series of the results described before, we could have good prospects of analytical treatments for mass flow measurement. However, the following two problems have to be solved.

The first one is the variation of flow coefficient α with Re_D . In the range of $Re_D > 10^4$, the maximum variation of α is about 1.5% even at $\beta=0.8$. Therefore it can be assumed that α is constant in the present theory. As will be shown in next section, the error arising from this assumption lies within the present experimental variance.

The second one is a correction for the difference of wall pressure at each differential pressure tap position. As described in 4.2, the α values for each tap are considerably different from each other, therefore the effect of the difference cannot be ignored. The one-dimensional theory provides us a value of product $\alpha\epsilon$ from

* Schlieren photogram reported in ref. 13 shows that the flow near the vena contracta is less affected by expansion waves, and seems to be nearly a one-dimensional uniform jet.

(a) Vena contracta tap

(c) Corner tap

(b) Pipe tap

(d) Flange tap

(e) D · 1 / 2 D tap

Fig.6 Plot of $\lambda\%$ against r

equation (8), which is applicable only to vena contracta tap case.

$$(\alpha\epsilon)_{th} = (M_1/\beta^2) [2(1-r)/\kappa]^{-1/2} \dots\dots (8)$$

$\alpha\epsilon$ of other taps can be approximately corrected by the following equation (9) based on the results described in 4.2.

$$(\alpha\epsilon)_{s,f,d} = (\alpha\epsilon)_{v,th} \left(\frac{\alpha_{s,f,d}}{\alpha_v} \right)_{JIS, ASME} \dots\dots (9)$$

It is expected that the correction through equation (9) improves the accuracy of the mass flow measurements to a great extent, especially for corner tap case, and for flange tap case as well where the flow similarity does not hold. For pipe tap

case, it is necessary for the theoretical calculation of the pressure ratio r_p to take account of the loss of enlargement as well as the pipe friction loss. The author's analysis reported in another paper¹⁴ is available [the result for $\tau_w=0$ condition is also shown in Fig.6 (b) for reference].

4.5 Mass flow characteristics at high speed flow conditions

The applicable limits of the regulation values of JIS² and ASME³ at high-speed flow conditions, and the reliability of the theoretical value proposed in the present paper will be discussed in this section. The relative errors in the expansion factor defined by equation(10), are shown in Fig.6 (a) to (e) for each tap system.

$$\lambda = 100(1 - \epsilon_{JIS, ASME} / \epsilon_{exp., th.}) \dots \dots \dots (10)$$

Since the theoretical value of the expansion factor is derived from the relation $\epsilon_{th.} = (\alpha \epsilon)_{th.} / \alpha_{JIS, ASME}$, the theoretical curve of λ (illustrated by solid and broken lines in Fig.6) also changes with ReD . Hence, typical curves at ReD from lower limits of

the recommended turbulent flow regimes to 10^6 are shown in Fig.6. ReD of present experiments are also within the range. Of course the lines that showing the values of λ which are equal to zero (dotted lines) indicate the regulation values of JIS and ASME. It is recognized from Fig.6 that the errors of the regulation values, except for the pipe tap at $\beta \geq 0.7$, fit within $\pm 1.5\%$ in subsonic pressure range. With further decrease in r , however, the error from regulation values increases gradually. On the other hand, the theoretical value and the experimental data are in good agreement (about $\pm 1\%$) in the practical flow measurements even using any kind of taps and β at any pressure range.

It should be emphasized that the accuracy of the regulation values for corner tap is satisfactory even in supercritical flow regime, and this is supported by the trends of present theory. The accuracy of the regulation values for pipe tap is about $\pm 1.5\%$ in the range $\beta < 0.7$, and especially, this accuracy is maintained up to the supercritical flow regime at $\beta \approx 0.6$. However, it

Table 3 Numerical data on vena contracta tap $(\alpha \epsilon)_v, th.*$

$\kappa = 1.20$						
$r \backslash \beta$	0.3	0.4	0.5	0.6	0.7	0.8
0.90	0.577 2	0.586 4	0.601 7	0.627 3	0.671 1	0.752 5
0.80	0.557 9	0.566 3	0.580 4	0.604 1	0.644 8	0.721 5
0.70	0.537 5	0.545 1	0.558 0	0.579 5	0.616 4	0.685 9
0.60	0.515 8	0.522 6	0.534 1	0.553 3	0.586 0	0.646 7
0.55	0.504 4	0.510 8	0.521 6	0.539 6	0.570 1	0.626 2
0.50	0.492 8	0.498 8	0.508 9	0.525 7	0.554 2	0.606 2
0.45	0.481 2	0.486 8	0.496 3	0.512 1	0.538 8	0.587 1
0.40	0.469 9	0.475 2	0.484 1	0.498 9	0.524 0	0.569 2
0.35	0.459 1	0.464 0	0.472 4	0.486 4	0.510 0	0.552 4
0.30	0.448 7	0.453 4	0.461 3	0.474 5	0.496 9	0.536 8
$\kappa = 1.30$						
$r \backslash \beta$	0.3	0.4	0.5	0.6	0.7	0.8
0.90	0.578 6	0.587 8	0.603 3	0.629 0	0.673 0	0.754 6
0.80	0.560 7	0.569 2	0.583 5	0.607 5	0.648 6	0.726 1
0.70	0.541 8	0.549 6	0.562 7	0.584 6	0.622 3	0.693 3
0.60	0.521 6	0.528 6	0.540 5	0.560 2	0.594 0	0.656 8
0.55	0.510 9	0.517 6	0.528 7	0.547 4	0.579 1	0.637 4
0.50	0.499 9	0.506 2	0.516 7	0.534 2	0.563 9	0.618 1
0.45	0.488 8	0.494 7	0.504 6	0.521 0	0.548 9	0.599 5
0.40	0.477 9	0.483 5	0.492 8	0.508 3	0.534 5	0.581 9
0.35	0.467 4	0.472 6	0.481 3	0.496 0	0.520 7	0.565 3
0.30	0.457 2	0.462 1	0.470 4	0.484 3	0.507 7	0.549 7
$\kappa = 1.40$						
$r \backslash \beta$	0.3	0.4	0.5	0.6	0.7	0.8
0.90	0.579 8	0.589 1	0.604 6	0.630 4	0.674 6	0.756 4
0.80	0.563 1	0.571 8	0.586 2	0.610 4	0.652 0	0.730 0
0.70	0.545 5	0.553 4	0.566 7	0.589 1	0.627 4	0.699 6
0.60	0.526 6	0.533 8	0.545 9	0.566 2	0.600 9	0.665 6
0.55	0.516 6	0.523 4	0.534 9	0.554 1	0.586 9	0.647 3
0.50	0.506 2	0.512 7	0.523 5	0.541 6	0.572 4	0.628 8
0.45	0.495 6	0.501 7	0.511 9	0.529 0	0.558 0	0.610 7
0.40	0.485 1	0.490 8	0.500 5	0.516 6	0.543 9	0.593 3
0.35	0.474 8	0.480 2	0.489 3	0.504 6	0.530 4	0.576 9
0.30	0.464 8	0.469 9	0.478 6	0.493 0	0.517 5	0.561 4
$\kappa = 1.66$						
$r \backslash \beta$	0.3	0.4	0.5	0.6	0.7	0.8
0.90	0.582 2	0.591 6	0.607 3	0.633 3	0.677 8	0.760 0
0.80	0.568 1	0.576 9	0.591 7	0.616 4	0.658 7	0.737 9
0.70	0.553 0	0.561 3	0.575 1	0.598 2	0.637 9	0.712 4
0.60	0.536 9	0.544 5	0.557 3	0.578 6	0.615 2	0.683 5
0.55	0.528 3	0.535 6	0.547 8	0.568 2	0.603 1	0.667 9
0.50	0.519 3	0.526 3	0.537 9	0.557 3	0.590 5	0.651 4
0.45	0.510 0	0.516 5	0.527 6	0.546 0	0.577 4	0.634 7
0.40	0.500 4	0.506 6	0.517 1	0.534 6	0.564 3	0.618 4
0.35	0.490 8	0.496 7	0.506 7	0.523 3	0.551 5	0.602 5
0.30	0.481 4	0.487 0	0.496 5	0.512 3	0.539 1	0.587 4

* For the other taps, modification by equation (9) is necessary

should be noted that a significant error appears at other values of β . In mass flow measurements using pipe tap, especially for $\beta \geq 0.7$, the present theoretical values are recommendable for accurate measurements.

The numerical data of the theory for practical usage are shown in Table 3 and 4.

5. Conclusions

The results obtained by present investigation are summarized as follows:

[1] The effect of compressibility on the axial wall pressure distribution slightly appears at larger β as a variation of the minimum pressure point position in the exit of an orifice. But the effect is confirmed to be negligible for mass flow measurements.

[2] It is confirmed that the relation between the flow coefficient and the differential pressure for an incompressible flow is also applicable to a compressible flow. By use of the result, a theoretical method of mass flow estimation is extended using a one-dimensional flow theory.

[3] Factors which have effects on the expansion factor, the unique variable expressing the effect of compressibility in gas flow calculations, are clarified.

[4] Except for pipe tap at $\beta \geq 0.7$, it is confirmed that the errors from JIS and ASME regulation values are within $\pm 1.5\%$ up to the subsonic pressure ratio. Fortunately for corner tap, the accuracy is maintained up to supercritical flow regime.

[5] By use of the calculation formula proposed in the present paper, mass flow rates are measured within about $\pm 1\%$ accuracy in all kinds of taps, β and D including supercritical flow regime using orifice meters.

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Table 4 Numerical data on pipe tap $(\alpha \epsilon)_{p, th}^*$

$\beta = 0.3$				
r	κ	1.20	1.30	1.40
0.90		0.6068	0.6084	0.6099
0.80		0.5833	0.5867	0.5895
0.70		0.5586	0.5636	0.5680
0.60		0.5322	0.5390	0.5449
0.55		0.5186	0.5261	0.5327
0.50		0.5051	0.5131	0.5202
0.45		0.4920	0.5004	0.5079
0.40		0.4794	0.4881	0.4959
0.35		0.4675	0.4764	0.4844

$\beta = 0.4$				
r	κ	1.20	1.30	1.40
0.90		0.6410	0.6430	0.6447
0.80		0.6125	0.6164	0.6198
0.70		0.5825	0.5884	0.5935
0.65		0.5668	0.5737	0.5797
0.60		0.5509	0.5586	0.5654
0.55		0.5350	0.5434	0.5508
0.50		0.5195	0.5284	0.5363
0.45		0.5047	0.5140	0.5222
0.40		0.4908	0.5002	0.5087

$\beta = 0.5$				
r	κ	1.20	1.30	1.40
0.90		0.6921	0.6945	0.6966
0.80		0.6548	0.6597	0.6639
0.70		0.6159	0.6232	0.6294
0.65		0.5960	0.6042	0.6115
0.60		0.5762	0.5853	0.5933
0.55		0.5570	0.5667	0.5753
0.50		0.5389	0.5489	0.5579
0.45		0.5220	0.5323	0.5415

$\beta = 0.6$				
r	κ	1.20	1.30	1.40
0.90		0.7584	0.7716	0.7744
0.80		0.7154	0.7218	0.7273
0.75		0.6884	0.6963	0.7031
0.70		0.6614	0.6706	0.6786
0.65		0.6350	0.6452	0.6543
0.60		0.6098	0.6207	0.6304
0.55		0.5864	0.5977	0.6078
0.50		0.5649	0.5764	0.5867

$\beta = 0.7$				
r	κ	1.20	1.30	1.40
0.95		0.9272	0.9294	0.9313
0.90		0.8857	0.8903	0.8941
0.85		0.8442	0.8510	0.8568
0.80		0.8027	0.8117	0.8195
0.75		0.7620	0.7728	0.7822
0.70		0.7234	0.7356	0.7464
0.65		0.6874	0.7005	0.7121
0.60		0.6552	0.6686	0.6806

$\beta = 0.8$				
r	κ	1.20	1.30	1.40
0.95		1.1520	1.1548	1.1572
0.90		1.0782	1.0848	1.0903
0.85		1.0040	1.0146	1.0235
0.80		0.9329	0.9467	0.9585
0.75		0.8681	0.8843	0.8983
0.70		0.8111	0.8283	0.8434
0.65		0.7622	0.7795	0.7950

* Hydrodynamically smooth pipe at $Re_D = 10^5$