Axisymmetric Problem of a Flat Interface Annular Crack between an Elastic Layer and a Half-space*

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The singular stress problem for an elastic layer bonded to a half-space with different material properties is considered. It is assumed that the bimaterial interface contains a flat annular crack of which the surfaces are subjected to known tractions. The axisymmetric elastostatic problem is first reduced to that of solving a system of singular integral equations of the second kind. By the way of expanding the unknown function into Jacobi polynomials, the singular integral equations are further reduced to the infinite system of linear algebraic equations for determining the unknown coefficients. Numerical examples are given for an aluminum-epoxy and an aluminum-steel material combinations. The stress-intensity factor with the ratio of the inner radius to the outer one of the crack is shown graphically for several values of layer thickness.

Key Words: Annular Crack, Composite Material, Interface, Singular Integral Equation, Stress Intensity Factor, Integral Transform

1. Introduction

Concerning the problem of a brittle fracture of composite materials, the three dimensional problems of a crack being located at the interface of dissimilar layers have attracted the attention of scientists. Kassir and Bregman⁽¹⁾ solved the problem of a penny-shaped crack situated at the interface of two half-spaces. Recently Namioka, Shindo and Atsumi⁽²⁾ considered the axisymmetric problem of a penny-shaped crack at the interface of two bonded dissimilar layers under tension. By means of Hankel transforms, they reduced the problem to a singular integral equation determining an unknown auxiliary function. And the singular integral equation was regularized by Karleman-Vekua method, with the aid of an analytical solution for the case of a crack at the interface of two half-spaces. A frequently encountered crack shape in embedded cracks is a banana-shaped crack and the stress-intensity factor at the mid-points of the waist of the crack can be estimated by the stress-intensity factor of a flat annular crack. Shibuya, Nakahara and Koizumi⁽³⁾ analyzed the problem of an infinite solid containing an annular crack. They assumed that the displacement on the crack surface can be expressed by Fourier series and reduced the mixed boundary value problem to that of solving

an infinite system of simultaneous equations. But the extension of the method used there to the interface crack problems does not appear to be practicable.

In this paper, we consider the axisymmetric elastostatic problem of a flat annular crack at the interface of a layer and a half-space. By means of Hankel transforms⁽⁴⁾ the problem is first reduced to a set of simultaneous triple integral equations. By using a suitable definition of the integral transforms, the triple integral equations are further transformed into a set of simultaneous singular integral equations of the second kind. The solution of them is given in the form of the product of a series of Jacobi polynomials and their weight functions, and obtained by solving a system of linear algebraic equations for unknown coefficients⁽⁵⁾. The mechanical behaviour of the solution obtained for various elastic constants and geometric parameters of a composite material is discussed, and the stress-intensity factors are shown graphically.

2. Statement of Problem and Fundamental Equations

Fig.1 An flat annular crack at the interface of a layer and a half-space.

* Received 21st December, 1981.

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Consider a layer of thickness h and elastic constants $\mu_2, k_2 (=3-4\nu_2)$ bonded to a half-space of elastic constants $\mu_1, k_1 (=3-4\nu_1)$, as shown in Fig.1. Suppose that a flat annular crack under internal pressure $P(r)$ is located on $z=0$ and $r_1 \leq r \leq r_2$, where r_1 and r_2 are the inner and the outer radii of the crack, respectively. The boundary conditions of the present problem are

$$u_{r1} = u_{r2}, \quad 0 \leq r \leq r_1, \quad r \geq r_2 \quad (1)$$

$$u_{z1} = u_{z2}, \quad 0 \leq r \leq r_1, \quad r \geq r_2 \quad (2)$$

$$\sigma_{z1} = \sigma_{z2}, \quad 0 \leq r \leq r_1, \quad r \geq r_2 \quad (3)$$

$$\tau_{rz1} = \tau_{rz2}, \quad 0 \leq r \leq r_1, \quad r \geq r_2 \quad (4)$$

$$\sigma_{z1} = \sigma_{z2} = P(r), \quad r_1 < r < r_2 \quad (5)$$

$$\tau_{rz1} = \tau_{rz2} = 0, \quad r_1 < r < r_2 \quad (6)$$

when $z = -h$

$$\sigma_{z2} = 0, \quad 0 \leq r < \infty \quad (7)$$

$$\tau_{rz2} = 0, \quad 0 \leq r < \infty \quad (8)$$

In addition to (1)-(8), the stresses and the displacements are supposed vanish at infinity.

Here the Papkovitch-Neuber functions are used to formulate the problem, i.e., in terms of a pair of harmonic functions $\phi_k(r, z)$ and $\Psi_k(r, z)$ ($k=1, 2$), the displacement and stress components of interest may be written as

$$\begin{aligned} 2\mu_k u_{rk} &= -\frac{\partial \phi_k}{\partial r} - z \frac{\partial \Psi_k}{\partial r} \\ 2\mu_k u_{zk} &= K_k \Psi_k - \frac{\partial \phi_k}{\partial z} - z \frac{\partial \Psi_k}{\partial z} \\ \sigma_{zk} &= 2(1-\nu_k) \frac{\partial \Psi_k}{\partial z} - \frac{\partial^2 \phi_k}{\partial z^2} - z \frac{\partial^2 \Psi_k}{\partial z^2} \\ \tau_{rzk} &= \frac{\partial}{\partial r} \left[(1-2\nu_k) \Psi_k - \frac{\partial \phi_k}{\partial z} - z \frac{\partial \Psi_k}{\partial z} \right] \\ k &= 1, 2, \quad K_k = 3-4\nu_k \end{aligned} \quad (9)$$

In (9) the subscripts 1 and 2 refer to the half-space $z > 0$ and the layer $z < 0$, and μ_k, ν_k denote respectively the shear modulus and Poisson's ratio.

We will express the harmonic functions $\phi_k(r, z)$ and $\Psi_k(r, z)$ in terms of unknown functions $A_i(\lambda), B_i(\lambda)$ ($i=1, 2, 3$) and Hankel integrals as follows;

$$\begin{aligned} \phi_1(r, z) &= \int_0^\infty A_1 e^{-\lambda z} J_0(\lambda r) \frac{d\lambda}{\lambda} \\ \Psi_1(r, z) &= \int_0^\infty B_1 e^{-\lambda z} J_0(\lambda r) d\lambda \\ \phi_2(r, z) &= \int_0^\infty [A_2 \sinh \lambda(z+h) + A_3 \cosh \lambda(z+h)] \\ &\quad \cdot \frac{J_0(\lambda r)}{\lambda \sinh \lambda h} d\lambda \\ \Psi_2(r, z) &= \int_0^\infty [B_2 \cosh \lambda(z+h) \\ &\quad + B_3 \sinh \lambda(z+h)] \frac{J_0(\lambda r)}{\sinh \lambda h} d\lambda \end{aligned} \quad (10)$$

With the choice of functions as in (10), the conditions at infinity are satisfied.

3. Analysis

From equations (3) and (5) we see that $\sigma_{z1}(r, 0)$ is equal to $\sigma_{z2}(r, 0)$ for all values of r and it is easily shown that this condition is equivalent to the equation:

$$\begin{aligned} -\{A_1 + 2(1-\nu_1)B_1\} &= 2(1-\nu_2)B_2 - A_2 \\ &+ \{2(1-\nu_2)B_3 - A_3\} \coth \lambda h \end{aligned} \quad (11)$$

Similarly, we can show that equations (4) and (6) together imply the equation:

$$\begin{aligned} -\{A_1 + (1-2\nu_1)B_1\} &= A_3 - (1-2\nu_2)B_3 \\ &+ \{A_2 - (1-2\nu_2)B_3\} \coth \lambda h \end{aligned} \quad (12)$$

In addition to equations (11) and (12), from equations (7) and (8) we have the following two equations:

$$\begin{aligned} 2(1-\nu_2)B_3 - A_3 + \lambda h B_2 &= 0 \\ A_2 - (1-2\nu_2)B_2 - \lambda h B_3 &= 0 \end{aligned} \quad (13)$$

The remaining boundary conditions (1), (2), (5) and (6) will be satisfied provided A_3 and B_3 satisfy the simultaneous triple integral equations:

$$\begin{aligned} \int_0^\infty (c_{11}A_3 + c_{12}B_3) J_0(\lambda r) \lambda d\lambda &= P(r), \\ &r_1 < r < r_2 \\ \int_0^\infty (c_{21}A_3 + c_{22}B_3) J_1(\lambda r) \lambda d\lambda &= 0, \\ &r_1 < r < r_2 \\ \frac{-1}{2\mu_1\mu_2} \int_0^\infty (b_{11}A_3 + b_{12}B_3) J_0(\lambda r) d\lambda &= 0, \\ &0 \leq r \leq r_1, \quad r \geq r_2 \\ \frac{1}{2\mu_1\mu_2} \int_0^\infty (b_{21}A_3 + b_{22}B_3) J_1(\lambda r) d\lambda &= 0, \\ &0 \leq r \leq r_1, \quad r \geq r_2 \end{aligned} \quad (14)$$

where c_{ij} and b_{ij} are functions of λ . Now differentiating the last two equations of (14) with respect to r , we define the following two new unknown functions f_1 and f_2 .

$$\begin{aligned} f_1(r) &= \frac{\partial}{\partial r} (u_{z1} - u_{z2})_{z=0} \\ &= \frac{1}{2\mu_1\mu_2} \int_0^\infty (b_{11}A_3 + b_{12}B_3) J_1(\lambda r) \lambda d\lambda \\ f_2(r) &= \frac{1}{r} \frac{\partial}{\partial r} [(u_{r1} - u_{r2})_r]_{z=0} \\ &= \frac{1}{2\mu_1\mu_2} \int_0^\infty (b_{21}A_3 + b_{22}B_3) J_0(\lambda r) \lambda d\lambda \end{aligned} \quad (15)$$

Integrating the first two equations of (14) and substituting from (15) we obtain

$$\begin{aligned} -\int_{r_1}^{r_2} f_1(t) t dt \int_0^\infty e_{11} J_1(\lambda r) J_1(\lambda t) d\lambda \\ + \int_{r_1}^{r_2} f_2(t) t dt \int_0^\infty e_{12} J_1(\lambda r) J_0(\lambda t) d\lambda \\ = \frac{1+K_2}{4\mu_2} \frac{1}{r} (\int_0^\infty P(r) r dr + \text{const}) \\ - \int_{r_1}^{r_2} f_1(t) t dt \int_0^\infty e_{21} J_0(\lambda r) J_1(\lambda t) dt \\ + \int_{r_1}^{r_2} f_2(t) t dt \int_0^\infty e_{22} J_0(\lambda r) J_0(\lambda t) d\lambda \\ = \text{const}, \quad r_1 < r < r_2 \end{aligned} \quad (16)$$

where e_{ij} are given by c_{ij} and b_{ij} .

Using the result in reference⁽⁶⁾, the functions e_{ij} ($i, j=1, 2$) appearing in (16) may be expressed as

$$\begin{aligned}
e_{11}(\lambda) &= \frac{G_1(\lambda)}{G_3^2(\lambda) - G_1(\lambda)G_2(\lambda)} \\
e_{22}(\lambda) &= -\frac{G_2(\lambda)}{G_3^2(\lambda) - G_1(\lambda)G_2(\lambda)} \\
e_{12}(\lambda) &= -e_{21}(\lambda) = -\frac{G_3(\lambda)}{G_3^2(\lambda) - G_1(\lambda)G_2(\lambda)} \\
G_1(\lambda) &= \frac{1+K_1}{1+K_2} \frac{\mu_2}{\mu_1} + \frac{\coth \lambda h + \lambda h (1 - \coth^2 \lambda h)}{1 + \lambda^2 h^2 (1 - \coth^2 \lambda h)} \\
G_2(\lambda) &= \frac{1+K_1}{1+K_2} \frac{\mu_2}{\mu_1} + \frac{\coth \lambda h - \lambda h (1 - \coth^2 \lambda h)}{1 + \lambda^2 h^2 (1 - \coth^2 \lambda h)} \\
G_3(\lambda) &= \frac{2\mu_2}{1+K_2} \left(\frac{1-K_1}{2\mu_1} - \frac{1}{\mu_2} \right) + \frac{1}{1 + \lambda^2 h^2 (1 - \coth^2 \lambda h)} \quad (17)
\end{aligned}$$

It can be shown that for $\lambda \rightarrow \infty$ the functions e_{ij} have the following asymptotic behaviour:

$$\begin{aligned}
e_{11}(\infty) &= -e_{22}(\infty) = -\frac{1+K_2}{4} \left(\frac{\mu_1}{\mu_2 + \mu_1 K_2} + \frac{\mu_1}{\mu_1 + \mu_2 K_1} \right) \\
e_{12}(\infty) &= -e_{21}(\infty) = -\frac{1+K_2}{4} \left(\frac{\mu_1}{\mu_2 + \mu_1 K_2} - \frac{\mu_1}{\mu_1 + \mu_2 K_1} \right) \quad (18)
\end{aligned}$$

If we now differentiate (16) term by term with respect r , we obtain

$$\begin{aligned}
\frac{1}{\pi} \int_{r_1}^{r_2} \frac{f_1(t)}{t-r} dt - \gamma f_2(r) + \int_{r_1}^{r_2} [k_{11}(r, t) f_1(t) \\
+ k_{22}(r, t) f_2(t)] dt = \frac{1+K_2}{c\mu_2} P(r) \\
\gamma f_1(r) + \frac{1}{\pi} \int_{r_1}^{r_2} \frac{f_2(t)}{t-r} dt + \int_{r_1}^{r_2} [k_{21}(r, t) f_1(t) \\
+ k_{22}(r, t) f_2(t)] dt = 0, \quad (r_1 < r < r_2) \quad (19)
\end{aligned}$$

$$\begin{aligned}
\gamma &= \frac{(\mu_1 + \mu_2 K_1) - (\mu_2 + \mu_1 K_2)}{(\mu_1 + \mu_2 K_1) + (\mu_2 + \mu_1 K_2)} \\
c &= (1+K_2) \left(\frac{\mu_1}{\mu_2 + \mu_1 K_2} + \frac{\mu_1}{\mu_1 + \mu_2 K_1} \right) \quad (20)
\end{aligned}$$

$$\begin{aligned}
k_{11}(r, t) &= \frac{m_{11}(r, t) - 1}{\pi(t-r)} - \frac{4t}{c} \int_0^\infty [e_{11}(\lambda) \\
&\quad - e_{11}(\infty)] J_0(\lambda r) J_1(\lambda t) \lambda d\lambda \\
k_{12}(r, t) &= \frac{4t}{c} \int_0^\infty [e_{12}(\lambda) - e_{12}(\infty)] J_0(\lambda r) \\
&\quad \cdot J_0(\lambda t) \lambda d\lambda \\
k_{21}(r, t) &= \frac{4t}{c} \int_0^\infty [e_{21}(\lambda) - e_{21}(\infty)] J_1(\lambda r) J_1(\lambda t) \\
&\quad \cdot \lambda d\lambda \\
k_{22}(r, t) &= \frac{m_{22}(r, t) - 1}{\pi(t-r)} - \frac{4t}{c} \int_0^\infty [e_{22}(\lambda) \\
&\quad - e_{22}(\infty)] J_1(\lambda r) J_0(\lambda t) \lambda d\lambda \quad (21) \\
m_{11}(r, t) &= \begin{cases} 2 \left[\frac{r}{t+r} E\left(\frac{t}{r}\right) + \frac{t-r}{r} K\left(\frac{t}{r}\right) \right], & t < r \\ 2 \frac{t}{t+r} E\left(\frac{r}{t}\right), & t > r \end{cases} \\
m_{22}(r, t) &= \begin{cases} 2 \frac{t}{t+r} E\left(\frac{t}{r}\right), & t < r \\ 2 \left[\frac{t^2}{r(t+r)} E\left(\frac{r}{t}\right) - \frac{t-r}{r} K\left(\frac{r}{t}\right) \right], & t > r \end{cases} \quad (22)
\end{aligned}$$

Here $K(\cdot)$ and $E(\cdot)$ are the complete elliptic integrals of the first and the second kind, respectively.

From the last two equations of (14), we obtain the following single-valuedness conditions:

$$\begin{aligned}
\int_{r_1}^{r_2} f_1(r) dr &= 0 \\
\int_{r_1}^{r_2} f_2(r) r dr &= 0 \quad (23)
\end{aligned}$$

Under these conditions the singular integral equations of the second kind (19) must be solved.

4. The Solution

Defining the following combination

$$\Phi(r) = f_2(r) + i f_1(r) \quad (24)$$

and performing the following change of variables: $s = \frac{r_2 - r_1}{2} s + \frac{r_2 + r_1}{2}$, $t = \frac{r_2 - r_1}{2} \tau + \frac{r_2 + r_1}{2}$ (25)

(19) may be reduced to a singular integral equation with the same dominant part as (19), that is

$$\begin{aligned}
\frac{1}{\pi i} \int_{-1}^1 \frac{\Phi(\tau)}{\tau - s} d\tau - \gamma \Phi(s) + \int_{-1}^1 [L_1(s, \tau) \Phi(\tau) \\
+ L_2(s, \tau) \Phi(\tau)] d\tau = g(s), \quad |s| < 1 \quad (26)
\end{aligned}$$

where,

$$L_1(s, \tau) = \frac{1}{2} [(k_{12} - k_{21}) - i(k_{11} + k_{22})]$$

$$L_2(s, \tau) = \frac{1}{2} [(k_{12} + k_{21}) + i(k_{11} - k_{22})]$$

$$g(s) = \frac{1+K_2}{c\mu_2} P(s) \quad (27)$$

The singular integral equation (26) can be solved numerically by using the method of references (5), (6). It may easily be shown that the fundamental function of (26) is

$$\begin{aligned}
w(s) &= (1-s)^\alpha (1+s)^\beta \\
\alpha &= -\frac{1}{2} - i\omega, \quad \beta = -\frac{1}{2} + i\omega, \quad \omega = \frac{1}{2\pi} \log \left(\frac{1+\gamma}{1-\gamma} \right) \quad (28)
\end{aligned}$$

Observing that $w(s)$ is the weight function of the Jacobi polynomials $P_n^{(\alpha, \beta)}(s)$, the singularity of the integral equation (26) may be removed by expressing $\Phi(s)$ in the following infinite series, i.e.

$$\Phi(s) = \sum_{n=0}^{\infty} c_n w(s) P_n^{(\alpha, \beta)}(s) \quad (29)$$

where c_n are unknown complex constants.

Substituting (29) into (26) and integrating, we obtain the following equation.

$$\begin{aligned}
\sum_{n=0}^{\infty} \frac{(1-\gamma^2)^{1/2}}{2i} c_n P_{n-1}^{(-\alpha, -\beta)}(s) + \int_{-1}^1 \sum_{n=0}^{\infty} [c_n L_1(s, \tau) \\
\cdot w(\tau) P_n^{(\alpha, \beta)}(\tau) + \bar{c}_n L_2(s, \tau) \overline{w(\tau)} P_n^{(\alpha, \beta)}(\tau)] d\tau \\
= g(s), \quad |s| < 1 \quad (30)
\end{aligned}$$

From (30), considering the orthogonality of the Jacobi polynomials, the following equation is obtained:

$$-\frac{1}{2} (1-\gamma^2)^{1/2} i a_m c_{m+1} + \sum_{j=0}^{\infty} (c_j b_{mj} + \bar{c}_j d_{mj}) = g_m, \quad m=0, 1, 2, \dots, \infty \quad (31)$$

The interface stresses may be expressed as

$$\begin{aligned}
\frac{1+K_2}{c\mu_2} [\sigma_{31}(r, 0) - i\tau_{32}(r, 0)] &= \frac{1}{\pi i} \int_{-1}^1 \frac{\Phi(\tau)}{\tau - s} d\tau \\
+ \int_{-1}^1 [L_1(s, \tau) \Phi(\tau) + L_2(s, \tau) \overline{\Phi(\tau)}] d\tau, \quad |s| < 1 \quad (32)
\end{aligned}$$

If we now define the stress-intensity factors

K_{1r_1}, K_{1r_2} and K_{2r_1}, K_{2r_2} by

$$K_{1r_1} + iK_{2r_1} = \lim_{r \rightarrow r_1^-} \frac{\sigma_{z1}(r, 0) + i\tau_{rz1}(r, 0)}{(r_2 - r)^\alpha (r_1 - r)^\beta}$$

$$K_{1r_2} + iK_{2r_2} = \lim_{r \rightarrow r_2^+} \frac{\sigma_{z1}(r, 0) + i\tau_{rz1}(r, 0)}{(r - r_2)^\alpha (r - r_1)^\beta} \quad (33)$$

and note that

$$w(s) = i\sqrt{\frac{1-\gamma}{1+\gamma}} (s-1)^\alpha (s+1)^\beta \quad (34)$$

we obtain

$$K_{1r_1} - iK_{2r_1} = -i \frac{r_2 - r_1}{2} \frac{\sigma_{12}(1-\gamma^2)^{1/2}}{1+K_2(\alpha, \beta)} \cdot \sum_{n=0}^{\infty} c_n P_n(-1)$$

$$K_{1r_2} - iK_{2r_2} = i \frac{r_2 - r_1}{2} \frac{\sigma_{12}(1-\gamma^2)^{1/2}}{1+K_2(\alpha, \beta)} \cdot \sum_{n=0}^{\infty} c_n P_n(1) \quad (35)$$

5. Numerical Results

As an example we will consider the following case, i.e.

$$\sigma_{z1}(r, 0) = P(r) = -P_0, \quad r_1 < r < r_2 \quad (36)$$

The constants c_n are calculated by using the set of algebraic equations (31) and the single-valuedness conditions (23). The integrals defining b_{mj} are calculated as follows (7).

Substituting $\log[(1+s)/(1-s)] = \frac{\pi}{2} \sinh \theta$, $\log \cdot [(1+\tau)/(1-\tau)] = \frac{\pi}{2} \sinh \varphi$, i.e. $s = \tanh(\frac{\pi}{2} \sinh \theta)$

, $\tau = \tanh(\frac{\pi}{2} \sinh \varphi)$, and simplifying, we get

$$b_{mj} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_m^{(-\alpha, -\beta)} \left(\tanh\left(\frac{\pi}{2} \sinh \theta\right) \right) P_j^{(\alpha, \beta)} \left(\tanh\left(\frac{\pi}{2} \sinh \varphi\right) \right) L_1 \left(\tanh\left(\frac{\pi}{2} \sinh \theta\right), \tanh\left(\frac{\pi}{2} \sinh \varphi\right) \right) \frac{\cosh \theta}{\cosh\left(\frac{\pi}{2} \sinh \theta\right)} d\theta d\varphi$$

Fig. 2 Effect of the layer thickness on the normal stress-intensity factors at the outer tip of the crack with r_1/r_2 for an aluminum-epoxy and an aluminum-steel combination.

$$\cdot \frac{\cosh \varphi}{\cosh^3\left(\frac{\pi}{2} \sinh \varphi\right)} \left[\cos\left(\frac{\pi}{2} \sinh \theta\right) - i \sin\left(\frac{\pi}{2} \sinh \theta\right) \right] \left[\cos\left(\frac{\pi}{2} \sinh \varphi\right) + i \sin\left(\frac{\pi}{2} \sinh \varphi\right) \right] \cdot \frac{\pi}{4} d\theta d\varphi \quad (37)$$

Similarly $\bar{d}_{mj}$ are expressed in the form of integrals of the above type. These integrals are evaluated by using the trapezoidal formula.

To examine the influence of a layer thickness and the material constants on the stress-intensity factors, the normalized stress-intensity factors have been calculated for aluminum-epoxy ($E_1:E_2=1000:45$, $\nu_1=0.3$, $\nu_2=0.35$) and aluminum-steel ($E_1:E_2=1000:45$, $\nu_1=\nu_2=0.3$) combinations.

Fig. 2 shows the variations of the normalized normal stress-intensity factor $K_{1r_2}^*$ ($K_{1r_2}/P_0 \cdot \sqrt{r_2-r_1}$) at the outer tip of the crack with r_1/r_2 . The same kind of results for $K_{1r_1}^*$ ($=K_{1r_1}/P_0 \sqrt{r_2-r_1}$) at the inner tip of the crack is shown in Fig. 3. Fig. 4 and Fig. 5 show the normalized shear stress-intensity factors $K_{2r_1}^*$ ($=K_{2r_1}/P_0 \sqrt{r_2-r_1}$), $K_{2r_2}^*$ ($=K_{2r_2}/P_0 \cdot \sqrt{r_2-r_1}$) for aluminum-epoxy and aluminum-steel combinations, respectively. From the figures it is clear that as $h \rightarrow \infty$, the normal stress-intensity factors tend those of the interface annular crack situated between two half-spaces (1). As $h \rightarrow \infty$ and $r_1/r_2 \rightarrow 0$, the normal and shear stress-intensity factors at the outer tip of the crack approach those of the interface penny-shaped crack for two half-spaces (1). The influence of the layer thickness is perceptible when $h \leq 2$. As the layer thickness diminishes, the normal stress-intensity factors increase. On the other hand the shear stress-intensity factors strongly depend on the elastic constants. The influence of the layer thickness on the shear stress-intensity factor $K_{2r_2}^*$ is more pronounced than that of $K_{2r_1}^*$.

Fig. 3 Effect of the layer thickness on the normal stress-intensity factors at the inner tip of the crack with r_1/r_2 for an aluminum-epoxy and an aluminum-steel combination.

Fig.4 Effect of the layer thickness on the shear stress-intensity factors with r_1/r_2 for an aluminum-epoxy combination.

Fig.5 Effect of the layer thickness on the shear stress-intensity factors with r_1/r_2 for an aluminum-steel combination.

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