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Mechanism of Pseudo-shock Wave in Supersonic Jet*

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Concerning the overexpanded supersonic jet, interaction between the normal shock wave immediately downstream of the D'Laval nozzle exit and the shear layer of the jet boundary was investigated experimentally.

The following were demonstrated. (1) The normal shock wave changes into a pseudo-shock wave immediately after its occurrence in the flow, (2) the pseudo-shock wave is several jet diameters in length but somewhat shorter than the pseudo-shock wave in duct flow, (3) its pressure ratio is extremely small compared with that of the normal shock wave and nearly proportional to the Mach number immediately upstream of the pseudo-shock wave.

Key words: Supersonic Jet, Shock Wave, Jet Boundary, Schlieren Method

1. Introduction

Supersonic jet from D'Laval nozzle is roughly divided into three classes, namely, the overexpanded supersonic jet where the nozzle exit static pressure is lower, the design supersonic jet where the pressure is precisely equal to, and the underexpanded supersonic jet where the pressure is higher than the back pressure downstream of the jet.⁽¹⁾ In the case of the design and underexpanded jets which are often encountered in industrial applications, sufficient data are available concerning their boundaries, inner structures as well as their noises^{(2)~(4)}, but only a few reports have been published on the mechanism of the overexpanded supersonic jet. This class of jet is known to decelerate gradually to subsonic level through a system of normal shock waves or oblique shock waves, but this process has been generally explained only by the theory of shock wave, expansion wave and nonviscous flow in which compressibility is considered.

It has been found, however, that the actual shock wave shows a complicated pattern due to interaction first with the wall boundary layer formed in the nozzle and then with the shear layer at the jet boundary. This phenomenon cannot be explained by that theory. Some data⁽¹⁾ have been published on this subject and some authors⁽⁵⁾ have pointed out to the necessity of further investigating the phenomenon in question, but no systematic studies have been made both experimentally and theoretically.

In the previous papers the authors dealt with experimentally and theoretically the problem of interaction between the normal shock wave and the boundary layer in duct flow, that is, the mechanism⁽⁶⁾ and

oscillation⁽⁷⁾ of the pseudo-shock wave. And general equations for the pseudo-shock wave were also presented.⁽⁸⁾

In the present report, the interaction between the shock wave and the shear layer in jet will be discussed on the basis of experimental data. Supersonic jets from seven D'Laval nozzles of different design Mach numbers were filmed using the schlieren method and pressures at discrete locations were measured in order to demonstrate the mechanism of pseudo-shock wave, especially at the nozzle exit.

Nomenclature

a : sound velocity
 c : diffusion coefficient
 d, D : nozzle diameter, jet diameter
 L : length of pseudo-shock wave
 M : Mach number
 p : pressure
 r : radius
 T : temperature
 u : velocity
 w : Crocco number
 X : axial distance
 Y : radial distance
 γ : specific heat ratio
 ρ : density

Subscripts

o : stagnation point
 $*$: throat
 e : exit
 d : design
 t : pitot tube
 $1, 2$: upstream and downstream of pseudo-shock wave
 a : atmosphere

2. Experimental apparatus and procedure

Fig. 1 gives an outline of apparatus used in the present experiment. Compressed dry air (2.9 MPa, 3.7 Nm³/min) is passed through the control valve and restored to a stagnation state in the plenum chamber before being accelerated to a prescribed

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Fig. 1 Experimental apparatus

Fig. 2 D'Laval nozzle

Fig. 3 Pitot tube

Fig. 4 Static pressure tube

supersonic level and released into the atmosphere.

Seven D'Laval nozzles of different design Mach numbers, 1.0 to 2.83, were prepared. The geometry is illustrated in Fig. 2. The throat diameter is uniformly 10 mm, but the exit diameter d_e is varied according to the design Mach number.

For the experiment the plenum chamber pressure was kept constant by the control valve. The axial distribution of static

pressures on the nozzle wall and static pressures in the jet were measured. The flow pattern was filmed using the schlieren system for which a magnesium spark light source (exposure period about 10 μ s) in conjunction with two 400 mm diameter convex lenses. Fig. 3 and Fig. 4 show the pitot tube and the static pressure tube, respectively. For measuring the distributions of pressures in the jet the pitot tube or the static pressure tube was moved both axiswise and radiuswise using a traverse device. The Reynolds numbers were within the range 3.34×10^5 to 3.21×10^6 under the design flow conditions. Measurement of the sound pressure of the jet and analysis of its frequency were performed. The flow field was also filmed with the aid of a high speed camera. The results obtained are not included in the present report.

3. Experimental results and discussion

3.1 Starting process of D'Laval nozzle:

Fig. 5 gives a typical example of the axial distribution of wall static pressures observed during the starting process of a D'Laval nozzle of the design Mach number $M_d = 2.04$. It will be seen that the shock wave in the diverging portion of the nozzle moves gradually downstream as the plenum chamber pressure p_0 is increased.

Fig. 6 shows the pressure ratio dependence on p_0 . From this figure it can be noted that p^*/p_0 becomes constant at $p_0 = 0.16$ MPa, while p^*/p_0 becomes constant at $p_0 = 0.44$ MPa. Thus it can be observed that at these conditions the throat is choked and the nozzle flow begins to be supersonic. The Mach number M_1 obtained from p_e/p_0 is 1.91, which is smaller than the design Mach number. This is due to the effective area ratio between the throat and exit flows being reduced by the boundary layer developed in the nozzle.

p_a/p_e , the back pressure (atmospheric pressure)-the exit static pressure ratio, increases slowly up to $p_0 = 0.34$ MPa and then rapidly up to $p_0 = 0.44$ MPa until a maximum is reached. As can be seen from Fig. 5 and the schlieren photographs that follow, this is due to the fact that the pseudo-shock wave

Fig. 5 Wall static pressure distributions in a D'Laval nozzle ($M_1 = 1.91$)

Fig. 6 Pressure ratios to plenum chamber pressure p_0 ($M_1=1.91$)

in the nozzle then passes through the exit. The condition at which p_a/p_e achieves a maximum corresponds to that at which p_e/p_0 becomes constant and the nozzle exit flow begins to be supersonic. In the nonviscous flow theory, this condition is described as one at which the normal shock wave is formed at the nozzle exit. For brevity's sake, this condition will hereinafter be referred to as the NS condition. From its maximum value p_a/p_e decreases rapidly with an increasing p_0 until it becomes as small as 1.0 at $p_0=0.71$ MPa, suggesting that the jet becomes the design flow without any pressure wave. For brevity's sake this condition will hereinafter be referred to as the DF condition. The condition $p_a/p_e > 1$, including NS, is an overexpanded jet, and $p_a/p_e < 1$ is an underexpanded jet.

3.2 Observation of flow field by schlieren method: Fig.7 shows schlieren photographs of flow field under different plenum chamber pressures p_0 . The nozzle Mach number is 1.80 and the throat is choked in all cases. According to the nonviscous flow theory, photograph(a) shows a subsonic jet downstream of the normal shock wave in the nozzle, photograph(b) shows the NS condition, photograph(c) shows an oblique shock wave at the nozzle exit, photograph(d) shows a design jet, and photograph(e) shows an underexpanded jet. It will be clearly seen from (a), however, that strong compression waves (showing black in the photograph) and expansion waves (showing white) already exist in the jet which has been considered subsonic. The normal shock wave in the nozzle appears to have become a pseudo-shock wave through interaction with the wall boundary layer. The photograph shows the rear half of the pseudo-shock wave. The rapid but not sudden decrease of p_e (or the rapid increase of p_a/p_e) as seen at $p_0=0.34 \sim 0.44$ MPa in Fig.6 may also have been caused by movement of the pseudo-shock wave. From (b) or the NS condition it can be observed that the pseudo-shock wave now in

(a) $P_0 - P_a = 0.10$ MPa

(b) $P_0 - P_a = 0.24$ MPa (NS condition)

(c) $P_0 - P_a = 0.29$ MPa

(d) $P_0 - P_a = 0.42$ MPa (DF condition)

(e) $P_0 - P_a = 0.69$ MPa

Fig.7 Schlieren photographs in the starting process of a D'Laval nozzle ($M_1=1.80, M_2=1.70$)

the jet is formed, as in the case of duct flow, by a number of shock waves and expansion waves. Its length is several jet diameters.

According to the nonviscous flow theory, the flow of (c) is considered one in which the oblique (Mach corn) shock wave occurs at the nozzle exit. However, no significant difference can be observed between this flow and that of (b). In other words, these two flows may be regarded as phenomenally identical. The wave of (c) is considered a pseudo-shock wave and has a smaller pressure ratio and is slightly longer than that at the NS condition. In both (b) and (c) the shock waves forming the pseudo-shock wave are asymmetrical to the jet axis and strong sound waves from the shock waves are propagating in all directions. These sound waves are screeches inherent in any supersonic jet containing shock waves, (9), (10)

The flow of (d) is the design jet, which can be confirmed by the fact that the jet boundary at the nozzle exit is parallel with the axis and the waves observed in the jet are considerably weaker than those shown in the rest of the photographs. It can also be seen that the waves propagating only downstream are sound waves originating around the nozzle exit. (11), (12)

Fig. 7 (e) indicates that a strong expansion wave is formed at the nozzle exit, followed by a shock wave produced by equipressure reflection of the expansion wave at the jet boundary. This expansion-compression cycle is repeated. The sound field observed appears to contain both the screeches in (b) and (c) and the sound waves in (d).

Fig. 8 shows also photographically the NS condition jets of different Mach numbers.

The flow of (a) is one formed in a straight nozzle of $M_1=1.0$. It has been found that the flow Mach number at the nozzle exit is not exactly 1.0 but increases to 1.13 as the flow forms a throat slightly upstream of the exit owing to changes in the wall boundary layer thickness that take place near the exit. However, the pseudo-shock wave in this situation is considerably weak as shown.

Photographs (b) and (c) indicate that the shock waves forming the pseudo-shock wave are gradually intensified as M_1 increases. Photographs (d)~(g) indicate that with further increases in M_1 each of the shock waves changes in shape to X and wave-wave distance grows longer.

Comparisons of these photographs with the straight duct pseudo-shock waves discussed in the previous paper (6) indicate that particularly at higher Mach numbers the shape of the first shock wave in the straight duct further changes from X, forming a number of weak shock waves, while the shape of the first shock wave in the jet remains a fairly defined X. This is probably because the latter is fixed to its original position by the nozzle exit which acts as a clearly defined boundary.

In each of the photographs the end of the pseudo-shock wave, as determined by changes in the pressure distribution which will be discussed later, is given symbol ∇ . In all cases it can be noted that the jet boundary becomes wider downstream near the end of the pseudo-shock wave.

3.3 Total pressure and static pressure distributions on jet axis: Fig. 9(a)~(c) show logarithmically the distributions of total pressures p_t on the jet axis obtained at the NS condition for three nozzles, using the pitot tube.

Fig. 8 Pseudo-shock waves at the D'Laval nozzle exits

It will be seen from (a) that at $M_1=1.13$, p_t on the center axis remains approximately constant about three nozzle diameters and then decreases at a progressively greater rate until it reaches an asymptotical agreement with the deceleration rate given by the solution of incompressible jet diffusion (13)

$$(p_t - p_a) = \frac{1}{2} \rho u^2 \propto x^{-2} \quad \dots (1).$$

This type of change is similar to the diffusion of subsonic jet. The first region where p_t remains constant may be considered a potential core region, the second region may be considered a region where the radial velocity distribution suffers changes caused by the viscous effect of the jet boundary, and the third region expressed by Equation (1) may be considered a region where the velocity distributions at different sections show similarity (14).

Data of (b) obtained at $M_1=1.91$ show a significant axial change of p_t occurring in the first region. This is obviously due to the effects of strong shock waves in the jet. Careful comparison with the schlieren photographs will show that p_t takes a minimum downstream of the first and second shock waves in particular. This minimum includes the total pressure loss caused by the waves as well as the total pressure loss caused by the shock wave at the tip of the pitot tube inserted in a flow that has been accelerated to supersonic level, that is an apparent loss. A slight increase of p_t observed at the original position of the first shock wave can also be similarly accounted for. For the present experiment it is practically impossible to distinguish the flow at each pressure measuring point between supersonic and subsonic and to apply the Rayleigh pitot formula to the flow identified as supersonic. With this fact taken into account it has been found that application of the Rayleigh formula would not substantially affect the knowledge of flow mechanism. Therefore the measured pitot pressure p_t is applied to all situations in this report.

Data of Fig. (c) at $M_1=2.45$ show that p_t varies more significantly in the first region and its minimum is evident notably downstream of the first shock wave. Although p_t is seen to vary substantially in both (b) and (c), its average value is approximately parallel with the abscissa. It will be interesting to observe that the average total pressure on the axis of the first region still remains constant even when the pseudo-shock wave is encountered. Comparison of (b) and (c) with (a) indicates that in the second region where changes take place in the radial velocity distribution, p_t decreases first at a rate greater than given by Equation (1) and then at a reduced rate to agree with the theory. In each of the figures the end of pseudo-shock wave is given symbol ∇ .

Fig. 10(a)~(c) give the static pressure distribution on the axis at the same conditions as shown in Fig. 9(a)~(c). The abscissa indicates both the distance from the nozzle X (mm) and X/D . The ordinate indicates the difference between the static pressure p and the atmospheric pressure p_a . The wall static pressure p_1 1 mm upstream of the nozzle exit is also indicated with an arrow.

The flow being the NS conditions, all

Fig.9 Axial distributions of pitot pressure in the jets

p_1 are below the atmospheric pressure. It can be noted from the figure, however, that static pressure in the jet increases sharply immediately downstream of the nozzle exit to a value above the atmospheric. During the subsequent cycle of overcompression and overexpansion, the static pressure finally comes to agree with the atmospheric. Fig. 10(b)~(c) shows this change in the static pressure more evident with an increasing M_1 (note the difference in scale for (a)~(c)). This type of phenomenon which is not observed in the pseudo-shock wave in the duct flow may be regarded as characteristic of the pseudo-shock wave in the jet.

In a series of experiments performed by the authors, the cycle of overcompression and overexpansion has also been observed in the underexpanded jet as well as the design jet whose amplitude is relatively small (about 44 kPa at $M_1=1.91$) (15). Accordingly, it may be concluded that in the course of deceleration the supersonic jet should contain at least some shock waves and expansion waves.

In the figure symbol ∇ indicates the position determined as the end of the pseudo-shock wave.

Data obtained from the experiments suggest that the end of the pseudo-shock wave is a point where physical changes in the static pressure have almost ceased. Downstream of the pseudo-shock wave end, the static pressure once falls below the atmospheric pressure and then gradually rises from the negative side.

3.4 Radial distributions of total pressures, static pressures and local mass fluxes: Fig.11 shows the radial distribution of total pressures obtained for the NS condition jet of $M_1=1.70$ by running the pitot tube across the flow at several selected axial locations. The distribution immediately downstream of the nozzle exit is fairly flat but gradually decays in the regions further downstream.

At $X=50$ mm the flat section is shown to have almost vanished and the value at the center decreases in the regions more downstream. However, the distributions at $X=70$

mm and 90 mm appear to be similar.

Fig.12 shows the radial distribution of static pressures for same flow of Fig.11. At short distances from the nozzle exit, the distribution may show complicated patterns depending on the measuring position, while the measured values at $X=70, 90$ and 150 mm indicate that a gentle static pressure distribution curve with its minimum always at the center section is produced downstream of the pseudo-shock wave region.

Fig.13 gives the radial distribution of local mass fluxes ρu as calculated from Figs.11 and 12, using the following equation

$$\rho u = \frac{\gamma p_t}{a_o} \left(\frac{p}{p_t} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2}{\gamma-1} \left\{ 1 - \left(\frac{p}{p_t} \right)^{\frac{\gamma-1}{\gamma}} \right\}} \quad (2)$$

where a_o is the stagnation sound velocity calculated from the plenum chamber temperature. It can be pointed out that the radial distribution of mass fluxes shown is close in pattern to that of the pitot pressures of

(a) $M_1 = 1.13$

(b) $M_1 = 1.91$

(c) $M_1 = 2.45$

Fig.10 Axial distributions of static pressure in the jets

Fig.11 Radial distributions of pitot pressure ($M_1=1.70$)

Fig.12 Radial distributions of static pressure ($M_1=1.70$)

Fig.13 Radial distributions of mass flux ($M_1=1.70$)

Fig.11, while the distribution curve is a more gentle one than the total pressure distribution curve. Thus, the end of the pseudo-shock wave can be more clearly determined from the radial distributions of mass fluxes which show similarity to one another downstream of the pseudo-shock wave. At $M_1=1.70$ shown in Fig.13 the end of the pseudo-shock wave is evaluated as $X=70$ mm ($X/D=5.8$)

Fig.14 shows the radial distributions of mass fluxes at the end of the pseudo-shock waves. The ordinate indicates the radial distance y/r_1 nondimensionalized by the nozzle exit radius and the abscissa does $(\rho u)/(\rho u)_{\max}$. In the figure, the solid line represents the solution of velocity distribution in the axis-symmetrical subsonic jet

$$u/u_{\max} = \{1 + \frac{1}{4} (\frac{ky}{r_1})^2\}^{-2} \quad \dots\dots (3)$$

where k is 0.777 so that $u/u_{\max}=0.5$ at $y/r_1=\pm 1.0$. At higher M_1 the experimental value of mass flux distribution well agrees with the solution.

The mass flux was integrated over the section to calculate the radial divergence ratio of air from the nozzle between the nozzle exit and the end of pseudo-shock wave. The result is approximately 1.1 regardless of the Mach number. Therefore, diffusion of the supersonic jet in the pseudo-shock wave region is considerably small as in the case of the potential core region in subsonic jet.

3.5 Length and pressure ratio of the pseudo-shock wave: Fig.15 shows the dimensionless length L/D of pseudo-shock wave in jet with the Mach number M_1 .

For the pseudo-shock wave in straight duct the authors have proposed a diffusion model of the pseudo-shock wave with the dimensionless length written as

$$L/D = \frac{1}{c} \ln \frac{w_1^2}{w^{*2}} \quad \dots\dots (4)$$

where w is the Crocco number immediately upstream of the pseudo-shock wave, w^* is the Crocco number at $M=1$, $w^{*2}=(\gamma-1)/(\gamma+1)$ and c is a constant for diffusion in the pseudo-

shock wave. The dimensionless length is also expressed as a function of the Mach number as follows

$$L/D = \frac{1}{c} \left\{ \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \right\} \quad \dots\dots (5)$$

The two curves of Fig.15 represent the above equation for different diffusion coefficients. The length of the pseudo-shock wave in the duct flow can be expressed by $c=0.114$,⁽⁶⁾ and the length of the NS condition pseudo-shock wave in the jet can be expressed by $c=0.130$. In other words, the idea of diffusion model is also applicable for the pseudo-shock wave in the jet provided that the diffusion is more rapid than that in the duct flow, which is a probable cause of the shortness of the pseudo-shock wave in the jet. A possible explanation for the relative rapidness of the diffusion is the effects of strong over-compression and overexpansion in the pseudo-shock wave, as shown in Fig.10.

Data marked $\bullet$ are those obtained using a small D'Laval nozzle of exit diameter 3.9 mm of the previous experiment.⁽¹⁶⁾ It will be noted that the dimensionless lengths of the pseudo-shock waves show a satisfactory level of agreement regardless of a considerable difference in nozzle diameter.

Fig.16 gives the pressure ratio across the pseudo-shock wave obtained in the present investigation. Data marked $\circ$ are the values calculated with the nozzle exit static pressure as p_1 and the atmospheric pressure p_2 . Data marked ϕ are presented to show the pressure ratio across the first shock wave of the pseudo-shock wave.

It will be seen that the pressure ratio across the pseudo-shock wave in the jet is significantly low as compared with the normal shock wave shown in solid line. As compared with the case of $M_1=2.0$, the pressure ratio across the pseudo-shock wave is only 37% of the normal shock wave. Comparison with the case of the first shock wave also shows only 62% of the normal shock wave. The pressure ratio across the normal shock wave increases in proportion to M_1^2 ,

Fig.14 Mass flux distributions at the ends of pseudo-shock waves

Fig.15 Length of pseudo-shock wave

Fig.16 Pressure ratio of pseudo-shock wave

while the pressure ratio across the pseudo-shock wave in the jet increases with M_1 . This may be expressed experimentally in the form

$$p_2/p_1 = 0.66 M_1 + 0.34 \quad \dots\dots(6)$$

The pressure ratio across the pseudo-shock wave in the straight duct can be explained by modifying the normal shock wave theory considering the nonuniformity of velocities upstream and downstream of the pseudo-shock wave.⁽⁸⁾ To provide a satisfactory explanation for the results of the present experiment, however, the radial boundary conditions of the pseudo-shock wave may also have to be considered. In other words, the static pressure in the pseudo-shock wave in the straight duct rises constantly with some fluctuations, while the free boundary of the jet continues to be in contact with the atmospheric pressure which disturbs the rising static pressure so severely that the overall pressure ratio is reduced substantially. Symbol ● and symbol ○ indicate the results of the preliminary experiment⁽¹⁶⁾ which agree satisfactorily with the data of the present investigation regardless of a significant nozzle diameter difference involved.

4. Conclusions

The present experiment has clearly demonstrated the following:

- (1) The shock wave at the nozzle exit, which has been reported to be a normal shock wave, is found to be a pseudo-shock wave resulting from violent interaction with the shear layer at the jet boundary.
- (2) In the pseudo-shock region both the total pressure and the static pressure change axially several times. Especially the static pressure undergoes a cycle of violent overcompression and overexpansion until it achieves correspondence with the back pressure.
- (3) The radial distributions of total pressures and static pressures in the pseudo-shock wave suffer complicated changes due to the effects of shock

wave, expansion wave and viscosity diffusion, but the radial distribution of local mass fluxes determined from the radial distributions of total pressures and static pressures at the end of the pseudo-shock wave in general agrees with the solution based on the theory of incompressible axis-symmetrical jet diffusion.

- (4) The length of the pseudo-shock wave is expressed approximately by Equation (5) using a diffusion model which has been proposed for the pseudo-shock wave in straight duct, except that the pseudo-shock wave in the jet is shorter due to a difference in diffusion coefficient c .
- (5) The pressure ratio across the pseudo-shock wave is significantly smaller than the ratio determined for the normal shock wave. The former increases in approximate proportion with the Mach number. This result differs from that predicted using the modified theory of normal shock wave. These experimental results, especially Equation(6), require further theoretical analysis.

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