Forced Convection Heat Transfer from an Elliptic Cylinder of Axis Ratio 1:2*

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An experimental investigation has been made to clarify the heat transfer characteristics of an elliptic cylinder of axis ratio 1:2. The testing fluid was air and the Reynolds number ranged from about 5000 to 90000. The angle of attack α was varied from 0° to 90° . The local heat transfer behavior is clarified and it is quite different from that of a circular cylinder. In the region of $\alpha > 40^\circ$, the heat transfer rate from the upstream surface, on which a laminar boundary layer develops, becomes lower than that from the downstream surface in the separated flow region. The mean heat transfer coefficient Nu_m depends upon the angle of attack along with the Reynolds number. It is found that Nu_m is the highest for $\alpha = 75^\circ$ to 90° and is the lowest for $\alpha = 30^\circ$ over the whole Reynolds number range studied. The mean heat transfer rate from the elliptic cylinder investigated in the present work is higher than that from the circular cylinder at all the angles of attack over the Reynolds number range studied.

Key Words : Convective Heat Transfer, Forced Convection, Elliptic Cylinder, Experimental Study, Boundary Layer, Separated Flow

1. Introduction

For the purpose of saving the energy and making its effective use, it is very important to exploit high performance heat exchangers. Among various types of heat exchangers, those constructed of circular pipes have been used in many industries. The flow around pipes, however, is not always at right angles to the pipe axis. In such a situation, the cross section of the pipe in the flow direction becomes an ellipse. An elliptic cylinder may be regarded as a general form, which becomes a flat plate for zero eccentricity and also a circular cylinder for unity eccentricity. There have been many works on the flow around elliptic cylinders^{(1)~(9)} and the flow characteristics are found to change considerably with an angle of attack^{(5)~(9)}.

As far as the heat transfer characteristics of the elliptic cylinders are concerned, there have been only a few investigations. Seban et al.⁽¹⁰⁾ and Drake Jr. et al.⁽¹¹⁾ measured the local heat transfer coefficient on elliptic cylinders of axis ratios 1:4 and 1:3 respectively. However their main purpose seems to consist in confirming an applicability of the boundary layer theory. The angle of attack examined was very limited, say 0° to 6° , and the mean heat

transfer coefficient was not measured. Reiher⁽¹²⁾ reported the mean heat transfer coefficient for an elliptic cylinder at 0° and 90° but its configuration was obscure. On the other hand, the heat transfer characteristics may be considered to vary remarkably with the angle of attack, since the flow features change with it as previously noted.

The purpose of the present study was to clarify the local and overall heat transfer characteristics from an elliptic cylinder of axis ratio 1:2 over a relatively wide range of Reynolds number. The angle of attack is varied from 0° through 90° and the variation of the heat transfer feature with it is discussed in detail comparing with the previous results for a circular cylinder. The flow configuration examined in the present study is described in Fig.1, which includes the coordinate system employed.

Fig.1 Flow configuration and coordinate system

Nomenclature

c : length of major axis of elliptic cylinder

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d : diameter of circular cylinder of equal circumferential length
 h : heat transfer coefficient
 h_m : mean heat transfer coefficient
 Nu : local Nusselt number = hc/λ
 Nu_m : mean Nusselt number = $h_m c/\lambda$
 Nu_{md} : mean Nusselt number = $h_{md} d/\lambda$
 Re : Reynolds number = $U_\infty c/\nu$
 Re_d : Reynolds number = $U_\infty d/\nu$
 s : surface length from leading edge
 T_w : wall temperature
 T_∞ : temperature of upstream uniform flow
 U_∞ : velocity of upstream uniform flow
 x, y : coordinates
 α : angle of attack
 λ, ν : thermal conductivity and kinematic viscosity of air at T_∞

2. Experimental apparatus and technique

Experiments were carried out in a low speed open circuit wind tunnel driven by a centrifugal fan at the inlet. The working section at the tunnel exit is 150 mm wide, 400 mm high, and 800 mm long. The nozzle contraction ratio is 13.5 to 1 and the turbulence intensity of upstream uniform flow $\sqrt{u'^2}/U_\infty$ is about 0.003 over the Reynolds number range studied in the present investigation. The upstream uniform flow velocity was measured with a Pitot tube at the nozzle exit. A constant temperature hot-wire anemometer was also used to measure the uniform flow velocity at quite low speed.

The test elliptic cylinder has an axis ratio 1:2, its major axis length being 50 mm and the spanwise length 150 mm. It was made of Fiber-Reinforced Plastics (F.R.P.) by the following procedure. A basic elliptic cylinder was first constructed of a thin balsa plate ribbed with balsa plates. Next F.R.P. was piled on that cylinder and the surface was manually finished. Forty-nine copper-constantan thermocouples of 0.07 mm in diameter were embedded on the surface of the elliptic cylinder in order to measure the wall temperature. Finally a stainless steel sheet of 0.05 mm thick and 30 mm wide was wound helically and stuck on the cylinder surface as a heating plate. The inside of the cylinder was filled with rigid urethane foam to minimize the heat loss.

Heating of the cylinder surface was conducted by means of electric current to the stainless steel sheet under the condition of constant heat flux. The heat loss by conduction and radiation was neglected in the following results. The supplying heat flow rate was determined from the measured electric current and voltage. The heat transfer coefficient and the corresponding Nusselt number are defined as

$$h = q/(T_w - T_\infty) \quad \text{and} \quad Nu = hc/\lambda$$

where q denotes the heat flux per unit area and unit time and T_w the wall temperature. The properties of air such as λ and ν are calculated at the upstream uniform flow temperature T_∞ . These measuring procedures are almost the same as in the previous works by the present authors^{(13)~(15)}.

In operating the system, about one hour

was required to attain a steady state. The upstream uniform flow velocity U_∞ ranged from 1.4 m/s to 27 m/s and the corresponding Reynolds number $Re = U_\infty c/\nu$ from about 5000 to 90000. In case of a high Reynolds number, q was kept at about 1.5 kW/m², but at a low Reynolds number it was decreased to about 0.15 kW/m² in order to minimize the effects of natural convection. The angle of attack α was varied from 0° through 90°. In the preliminary experiments, effects of the heat flux were found to be negligible and the position of $\alpha = 0^\circ$ was determined with detailed measurements of the local heat transfer coefficient on the upper and lower sides of the cylinder at various Reynolds numbers.

3. Experimental results and discussion

3.1. Local heat transfer

Local heat transfer distributions around the whole circumference of the elliptic cylinder are represented for $\alpha = 0^\circ$ to 90° at various Reynolds numbers in Figs. 2 through 8.

Fig. 2 shows the results at $\alpha = 0^\circ$. It is clear that the symmetry of the local heat transfer distribution on the upper and lower surfaces is satisfactory at all the Reynolds numbers. It may be reasonable to consider that the whole Reynolds number range tested falls within the subcritical Reynolds number region since the critical Reynolds number initiated at about $Re = 100000$ ⁽⁴⁾. Therefore the heat transfer coefficient shows no essential change with the Reynolds number.

Nu attains the highest value at the leading edge and decreases steeply with development of a laminar boundary layer on the upstream face. It seems to separate at about $s/c = 0.6$, which almost coincides with the location of the minor axis. In a separated flow region, Nu again increases to the downstream and reaches a maximum at the trailing edge. However Nu shows no essential change therein at very low Reynolds number. This may be due to a very stagnant flow in the near wake.

Fig. 2 Local Nusselt number distribution, $\alpha = 0^\circ$

Fig.3 Local Nusselt number distribution, $\alpha = 15^\circ$ Fig.4 Local Nusselt number distribution, $\alpha = 30^\circ$ Fig.5 Local Nusselt number distribution, $\alpha = 45^\circ$ Fig.6 Local Nusselt number distribution, $\alpha = 60^\circ$ Fig.7 Local Nusselt number distribution, $\alpha = 75^\circ$ Fig.8 Local Nusselt number distribution, $\alpha = 90^\circ$

Represented in Fig.3 are the results at $\alpha = 15^\circ$. The laminar boundary layer separates at about $s/c = 0.3$ and -0.8 on the upper and lower surfaces respectively. The highest value of Nu locates near the leading edge. Nu decreases more steeply on the upper side than on the lower side, since the boundary layer development is limited to a very short distance on the upper side. It is of interest to note that there exists a region where Nu shows little change just behind the separation point on the upper surface and its length increases with a decrease of the Reynolds number. It may be inferred that just downstream of the separation point on the upper surface, the distance between the cylinder surface and the separated shear layer is quite short, hence the velocity of a reversed flow is very small, and it results in such a heat transfer behavior described above. The peak value of Nu in the separated flow region shifts from the trailing edge to a point on the upper side.

It is clearly understood through observing the results shown in Figs.3 to 8 that the unsymmetry of the heat transfer distribution on the upper and lower sides multiplies with an increase of α and the heat transfer coefficient on the lower surface, where the boundary layer develops, decreases with α .

Some interesting features of the heat transfer can be revealed by comparison of the results at $\alpha = 30^\circ$ and 45° . That is, in the region of $\alpha > 40^\circ$, the heat transfer rate in the separated flow region is larger than that on the upstream surface where the boundary layer develops, though some cases of very low Reynolds number are excluded. This may be due to the fact that the wake width behind the cylinder increases with the angle of attack, accompanied by a violent transversal motion of fluid near the downstream surface.

In the case of $\alpha = 30^\circ$, the flow velocity of approaching to the upstream surface

of the cylinder may decrease as compared to that for $\alpha < 30^\circ$ due to a large drag coefficient. In addition to that, the transversal motion of fluid in the near wake is not so severe as in the cases of $\alpha > 40^\circ$. It results in the lowest value of the mean Nusselt number over the whole Reynolds number range, as demonstrated later.

The flow pattern around the elliptic cylinder was visualized at various Reynolds numbers from $\alpha = 0^\circ$ to 90° with a water channel⁽¹⁶⁾, in the same way as in a previous work by the present authors⁽¹⁷⁾. It reveals that the position of the upstream stagnation point shifts with α . For example, it locates at about $s/c = -0.25$ and -0.50 for $\alpha = 45^\circ$ and 75° respectively. On the other hand, the peak value of Nu locates at about $s/c = -0.05$ and -0.15 for $\alpha = 45^\circ$ and 75° respectively. That is, the position of the maximum of Nu quite differs from that of the upstream stagnation point. Therefore it is not easy to estimate the flow feature from the local heat transfer distribution.

As far as the results at $\alpha = 90^\circ$ are concerned, Nu in the separated flow region is much higher than that on the upstream surface except in some cases of very low Reynolds number. The heat transfer coefficient is minimum at the upstream stagnation point, increases slightly with the circumferential distance, and attains a maximum at about $s/c = -0.15$. Such characteristic features of the local heat transfer are remarkably different from those of the circular cylinder. The flow seems to separate in the neighborhood of the major axis.

Fig.9 represents the results at $\alpha = 0^\circ$ shown already in Fig.2 in the form of $Nu/\sqrt{Re}$. It is very clear that the data at three values of Re fall into a single curve on the upstream surface. It implies that the laminar boundary layer develops on the upstream surface of the elliptic cylinder and the local heat transfer rate is proportional to the square root of the Reynolds number. However, in the separated flow region, the dependency of $Nu/\sqrt{Re}$ upon Re can be recognized. That is, the flow therein is very complicated and may not be regarded as laminar.

Fig.9 Local Nusselt number distribution, $\alpha = 0^\circ$

3.2. Mean heat transfer

The mean Nusselt number Nu_m over the whole circumference of the elliptic cylinder was estimated by a graphical integration. The results at $\alpha = 0^\circ$ are typically represented in Fig.10. Nu_m increases linearly with Re in logarithmic scale. The data at other angles of attack exhibit a similar trend. Therefore the present results of Nu_m at all values of angle of attack can be expressed in the following form over the whole Reynolds number range examined.

$$Nu_m = A Re^n \quad (1)$$

Values of the power n of Re and the constant A were determined by the method of least squares. The results are summarized in Tab.1. The dependency of the power n upon α is found not to be large.

It is very important to explore the heat transfer ability of the elliptic cylinder comparing with that of the circular cylinder in order to exploit high performance heat exchangers. In doing so, the characteristic length in both the Reynolds and the Nusselt numbers is to be reasonably selected. The most suitable one may be the diameter d of a circular cylinder of equal circumferential length, which can be calculated by

$$d = (2E'/\pi)c$$

where $E'(k)$ is an associated complete elliptic integral of the second kind, $k^2 = 1 -$

Fig.10 Mean Nusselt number, $\alpha = 0^\circ$

α°	n	A	A'
0	0.573	0.344	0.308
15	0.544	0.470	0.418
30	0.537	0.476	0.422
40	0.588	0.322	0.289
45	0.582	0.358	0.321
60	0.563	0.452	0.404
75	0.545	0.567	0.504
90	0.545	0.566	0.503

Tab.1 Effects of angle of attack on n , A , and A'

Fig.11 Effects of angle of attack on mean Nusselt number and comparison with data for circular cylinder

$(b/c)^2$, and b the length of the minor axis.

All the results are summarized in Fig. 11, in which the data points are omitted for clearness and their scatter is the same order as in Fig.10. The mean Nusselt and the Reynolds numbers are defined as follows.

$$Nu_{md} = h_m d / \lambda \quad \text{and} \quad Re_d = U_\infty d / \nu$$

The well-known empirical formula by Hilpert⁽¹⁸⁾ for the circular cylinder in the region of $4000 < Re_d < 40000$ is included for comparison. The results are represented by

$$Nu_{md} = A' Re_d^n \quad (2)$$

Values of A' are included in Tab.1. It is very interesting to note that the mean heat transfer coefficient does not increase monotonically with the angle of attack and is the lowest at $\alpha = 30^\circ$ over almost the whole Reynolds number range. The results at $\alpha = 75^\circ$ and 90° are nearly equal to each other and are the highest. Even the lowest value of Nu_{md} at $\alpha = 30^\circ$ is still higher than that for the circular cylinder. Although the exact configuration tested by Reiher⁽¹²⁾ is obscure, his data are in relatively good agreement with the present ones at $\alpha = 0^\circ$ but much higher than the latter at $\alpha = 90^\circ$.

The variation of Nu_m with α is clearly demonstrated in Fig.12. As noted previously, Nu_m does not increase monotonically with α . In the low Reynolds number region of about $Re < 11000$, Nu_m shows no essential change in the region of $\alpha = 0^\circ$ to 30° . Beyond $\alpha = 30^\circ$, it increases slightly with α . On the other hand, in the region of $Re > 30000$, Nu_m decreases with α in $0^\circ < \alpha < 30^\circ$, reaches a minimum around $\alpha = 30^\circ$, and subsequently increases with α . The mean heat transfer coefficient attains a maximum at about $\alpha = 75^\circ$ to 90° over the whole Reynolds number range studied. As described previous-

Fig.12 Variation of mean Nusselt number with angle of attack

ly as to the results at $\alpha = 30^\circ$ in Fig.4, a decrease of the oncoming flow velocity to the upstream surface of the cylinder brings about a relatively low value of Nu therein. Furthermore in the separated flow region Nu is not so high since the transversal motion of fluid is not so severe as in the case of $\alpha > 40^\circ$. These facts may result in the lowest value of Nu_m at $\alpha = 30^\circ$.

3.3 Discussion

It is well-known that the side walls of a wind tunnel influences upon the flow and heat transfer characteristics of bodies tested therein. In the present heat transfer study for the elliptic cylinder having the major axis length of 50 mm and the axis ratio 1:2, the results represented in the previous sections may be affected by so called blockage effects. Values of the blockage ratio are the smallest for $\alpha = 0^\circ$, 0.0625 and the largest for $\alpha = 90^\circ$, 0.125.

As far as the present authors know, there are no works on the correction rule for the blockage effect upon the heat transfer characteristics of the elliptic cylinder. An attempt, therefore, has been made to correct such the effect by following a correction formula introduced by Hiwada et al.⁽¹⁹⁾ for the circular cylinder.

The blockage correction on the mean Nusselt number Nu_m or Nu_{md} is found to be negligible for $\alpha = 0^\circ$, say smaller than 1%, and needless to say, to increase with the angle of attack. For example, it is about 3.5% for $\alpha = 90^\circ$, which is the largest in the present study. Accordingly, as far as the wall effects of the wind tunnel are concerned, the reliability of the present heat transfer data may be very satisfactory.

The pressure drag coefficient C_{Dd} for the elliptic cylinder of axis ratio 1:2 is almost equal to 0.4 at $\alpha = 0^\circ$ in the sub-critical Reynolds number region⁽⁴⁾⁽²⁰⁾. On the other hand, C_{Dd} for the circular cylinder is nearly equal to 1.0 in the same Reynolds number region. Furthermore, it can be seen from the results measured by Modi et al.⁽⁶⁾ that an increasing rate of C_{Dd} with α

for elliptic cylinders of axis ratios nearly equal to the present one is not large in the region of about $\alpha < 20^\circ$. In consideration of these facts along with the present data on the mean heat transfer coefficient demonstrated previously, the elliptic cylinder of axis ratio 1:2 examined in the present study may be said to have a higher heat transfer ability and a lower fluid dynamic drag as compared with the circular cylinder when the elliptic cylinder is operated at a relatively low angle of attack, say $\alpha < 30^\circ$.

4. Concluding remarks

Heat transfer characteristics of an elliptic cylinder having an axis ratio 1:2 are clarified through the wind tunnel experiments. The Reynolds number range examined is from about 5000 to 90000 and the angle of attack is varied from 0° to 90° .

The local heat transfer features are, in general, independent of the Reynolds number and are quite different from those of a circular cylinder. The flow behaviors, such as the upstream stagnation point, are not easily inferred from the local heat transfer distribution, especially at a higher angle of attack. In the region of $\alpha > 40^\circ$, the heat transfer rate in the separated flow region becomes higher than that on the upstream surface, where a laminar boundary layer develops.

The dependency of the mean heat transfer coefficient upon the Reynolds number and the angle of attack is clearly recognized. Nu_m is the highest at $\alpha = 75^\circ$ to 90° and the lowest at $\alpha = 30^\circ$ over the Reynolds number range examined. However, even the lowest value of Nu_m at $\alpha = 30^\circ$ is still higher than that for the circular cylinder of equal circumferential length. Therefore the elliptic cylinder of axis ratio 1:2 exhibits a superior feature of higher heat transfer capability and lower fluid dynamic drag than the circular cylinder, when the elliptic cylinder is operated at small angle of attack.

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