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Energetic Evaluation of Stability in Rotor-bearing System*

(Contribution of Each Element to Logarithmic Decrement)

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A calculation method for the contribution of each element (bearing, etc.) to the logarithmic decrement in a rotor-bearing system is shown which is based on the relation between the dissipative energy and the logarithmic decrement. By applying the method to a rotor-bearing system which is subjected to a fluid dynamic destabilizing force, the stability of the system is evaluated. The effects of the axial location of disk, the stiffness of rotor and the anisotropy of bearing characteristics on the stability of the system are described and the contribution of each element to the stability is discussed.

Vibration of Rotating Body, Stability, Journal Bearing, Energy, Logarithmic Decrement, Fluid Dynamic Force

1. Introduction

The self excited vibration of a rotor-bearing system due to oil film force of bearing and fluid dynamic destabilizing⁽¹⁾ force poses a serious problem⁽²⁾⁽³⁾ in high speed and high pressure turbomachinery. In order to design a stable rotor-bearing system, it is necessary to examine the effects of the bearing force and the fluid dynamic destabilizing force on the system stability.

The authors derived an approximate method for calculating the logarithmic decrement of a simple symmetrical rotor-bearing system, and investigated the effects of the anisotropy of bearing coefficients on the stability of the system subjected to the destabilizing force.⁽⁴⁾ Furthermore, several types of bearings which were suitable for stabilizing the system were investigated by the logarithmic decrement.⁽⁵⁾ It is, however, difficult to evaluate the effect of each element on the stability of complicated system such as asymmetrical systems and multi bearing systems. On the other hand, an energy method of evaluation of stability has been studied from a standpoint which is different from the logarithmic decrement method. Two of the authors proposed a method⁽⁶⁾⁽⁷⁾⁽⁸⁾ to find out an unstable bearing by calculating the dissipative energy for each bearing. This method is useful for clarifying the effect of each element on the system stability. But the energy is a dimensional quantity which depends on the definition of modal amplitude. Therefore, using the energy method, it is difficult to evaluate the

contribution of each element to the stability in various systems.

In this paper, a method which evaluates the contribution of each element to the system stability is derived by combining the logarithmic decrement method and the energy method. That is, the energy method is very useful in that it gives informations with regard to the dissipative energy levels in each rotor element and enables comparison. So in order to apply and combine this merit into the logarithmic decrement method which is useful in that it indicates the damping characteristics, the concept of logarithmic decrement is extended to each element of rotor system to represent the contribution of the element to the system stability, which is called "partial logarithmic decrement" in this paper. Furthermore, the stability of a rotor-bearing system subjected to the destabilizing force is discussed using the present method.

2. Nomenclature

$[M]$	: mass matrix
$[C^*]$	: damping matrix
$[K^*]$	: stiffness matrix
$[C]$	: symmetrical elements of $[C^*]$
$[\Delta C]$	: antisymmetrical elements of $[C^*]$
$[K]$	: symmetrical elements of $[K^*]$
$[\Delta K]$	: antisymmetrical elements of $[K^*]$
x	: displacement vector
ϕ	: complex mode vector ($\phi_r + i\phi_i$)
ϕ_r	: real part of ϕ
ϕ_i	: imaginary part of ϕ
λ	: eigenvalue ($= \alpha + i\beta$)
α	: real part of λ
β	: imaginary part of λ
δ	: logarithmic decrement
ω	: rotor speed
Ω_g	: natural frequency of the system on the simple support
Q	: cross coupling
t	: time
i	$= \sqrt{-1}$
$(\bar{})$	: conjugate of ()
$(\dot{})$	: $d()/dt$
$\text{Re}()$	: real part of ()

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3. Analysis

The equation of motion of a multi degree of freedom rotor system is generally represented as ;

$$[M]\ddot{x} + [C^*]\dot{x} + [K^*]x = 0 \quad (1)$$

where the rotor has a circular cross section. In Eq.(1), the mass matrix $[M]$ is symmetrical, and the stiffness matrix $[K^*]$ and the damping matrix $[C^*]$ are asymmetrical because of the oil film characteristics of bearing and gyroscopic effect. By applying the numerical calculation method such as QR method etc. to Eq.(1), the complex eigenvalues and the complex modes are obtained.

3.1 Derivation of logarithmic decrement

As the overdamped mode is trivial in the stability problem of a rotor-bearing system, this case is not considered here. In the other case, the following equations are obtained for the j th mode from Eq.(1).

$$\lambda_j^2 [M] \phi_j + \lambda_j [C^*] \phi_j + [K^*] \phi_j = 0 \quad (2)$$

$$\bar{\lambda}_j^2 [M] \bar{\phi}_j + \bar{\lambda}_j [C^*] \bar{\phi}_j + [K^*] \bar{\phi}_j = 0 \quad (3)$$

Separating $[C^*]$ and $[K^*]$ into symmetrical elements $[C]$ and $[K]$ and antisymmetrical elements $[\Delta C]$ and $[\Delta K]$, Eq.(2) and Eq.(3)

$$\lambda_j^2 [M] \phi_j + \lambda_j [C + \Delta C] \phi_j + [K + \Delta K] \phi_j = 0 \quad (4)$$

$$\bar{\lambda}_j^2 [M] \bar{\phi}_j + \bar{\lambda}_j [C + \Delta C] \bar{\phi}_j + [K + \Delta K] \bar{\phi}_j = 0 \quad (5)$$

where the diagonal elements of $[C^*]$ and $[K^*]$ are involved in $[C]$ and $[K]$ respectively, and $[\Delta C]$ and $[\Delta K]$ are defined as

$$[\Delta C] = -[\Delta C]^T, [\Delta K] = -[\Delta K]^T$$

Omitting the subscript j for simplification and premultiplying Eq.(4) and Eq.(5) by $\bar{\phi}^T$ and ϕ^T respectively, the following equations are obtained;

$$\lambda^2 m + \lambda c + \lambda \Delta c + k + \Delta k = 0 \quad (6)$$

$$\bar{\lambda}^2 m + \bar{\lambda} c - \bar{\lambda} \Delta c + k - \Delta k = 0 \quad (7)$$

where

$$\begin{aligned} \bar{\phi}^T [M] \phi &= \phi^T [M] \bar{\phi} = m \\ \bar{\phi}^T [C] \phi &= \phi^T [C] \bar{\phi} = c \end{aligned} \quad (8)$$

$$\bar{\phi}^T [\Delta C] \phi = -\phi^T [\Delta C] \bar{\phi} = \Delta c$$

$$\bar{\phi}^T [K] \phi = \phi^T [K] \bar{\phi} = k$$

$$\bar{\phi}^T [\Delta K] \phi = -\phi^T [\Delta K] \bar{\phi} = \Delta k$$

$$\begin{pmatrix} m, c, k : \text{real number} \\ \Delta c, \Delta k : \text{imaginary number} \end{pmatrix}$$

Subtracting Eq.(7) from Eq.(6),

$$(\lambda^2 - \bar{\lambda}^2)m + (\lambda - \bar{\lambda})c + (\lambda + \bar{\lambda})\Delta c + 2\Delta k = 0 \quad (9)$$

Since $\lambda = \alpha + i\beta$ and $\bar{\lambda} = \alpha - i\beta$, then we have:

$$\lambda + \bar{\lambda} = 2\alpha, \quad \lambda - \bar{\lambda} = 2i\beta, \quad \lambda^2 - \bar{\lambda}^2 = 4i\alpha\beta$$

By substituting the above equations into Eq.(9), the real part of eigenvalue is expressed as follows;

$$\alpha = -\frac{\beta c - i\Delta k}{2\beta m - i\Delta c} \quad (10)$$

Executing [Eq.(6)] $\times \bar{\lambda}$ - [Eq.(7)] $\times \lambda$, the following equation is obtained as the undamped natural frequency.

$$\alpha^2 + \beta^2 = \frac{k + i(\alpha/\beta)\Delta k}{m - i(1/\beta)\Delta c} \quad (11)$$

By using Eq.(10), the logarithmic decrement is;

$$\delta = -2\pi \frac{\alpha}{\beta} = \frac{1}{2} \left\{ \frac{\pi \beta c - i\pi \Delta k}{(1/2)\beta^2 m - i(1/4)\beta \Delta c} \right\} \quad (12)$$

Eq.(12) shows that the logarithmic decrement is determined by the natural frequency β and the space mode.

3.2 Dissipative energy

The dissipative energy per one cycle of steady vibration for j th mode is discussed below. The displacement of vibration is expressed as follows;

$$x = R e^{i\beta t} = \frac{1}{2} (\phi e^{i\beta t} + \bar{\phi} e^{-i\beta t}) \quad (13)$$

Accordingly, the velocity and acceleration of vibration are;

$$\begin{aligned} \dot{x} &= \frac{i\beta}{2} (\phi e^{i\beta t} - \bar{\phi} e^{-i\beta t}) \\ \ddot{x} &= -\frac{\beta^2}{2} (\phi e^{i\beta t} + \bar{\phi} e^{-i\beta t}) \end{aligned} \quad (14)$$

(1) Energy due to the inertial term

The change rate of energy due to the inertia term is;

$$\begin{aligned} \dot{T} &= \dot{x}^T [M] \dot{x} \\ &= -\frac{i\beta^3}{4} (\phi^T [M] \phi e^{2i\beta t} - \bar{\phi}^T [M] \bar{\phi} e^{-2i\beta t}) \end{aligned} \quad (15)$$

By integrating Eq.(15), the dissipative energy per one cycle is;

$$\begin{aligned} \int_0^{2\pi/\beta} \dot{T} dt \\ = -\frac{i\beta^3}{4} \int_0^{2\pi/\beta} (\phi^T [M] \phi e^{2i\beta t} - \bar{\phi}^T [M] \bar{\phi} e^{-2i\beta t}) dt = 0 \end{aligned} \quad (16)$$

Eq.(16) shows that the energy due to the inertia term is conservative.

(2) Energy due to the damping term

The change rate of energy due to the damping term is;

$$\begin{aligned} \dot{D} &= \dot{x}^T [C^*] \dot{x} \\ &= \frac{\beta^2}{4} (2c - \phi^T [C^*] \phi e^{2i\beta t} - \bar{\phi}^T [C^*] \bar{\phi} e^{-2i\beta t}) \end{aligned} \quad (17)$$

By integrating Eq.(17), the dissipative energy per one cycle of the vibration is;

$$\begin{aligned} \int_0^{2\pi/\beta} \dot{D} dt \\ = \frac{\beta^2}{4} \int_0^{2\pi/\beta} (2c - \phi^T [C^*] \phi e^{2i\beta t} - \bar{\phi}^T [C^*] \bar{\phi} e^{-2i\beta t}) dt = \pi \beta c \end{aligned} \quad (18)$$

The symmetrical elements of the damping matrix dissipate the energy and the anti-symmetrical elements are conservative.

(3) Energy due to the spring term

The change rate of energy due to the spring term is;

$$\begin{aligned} \dot{U} &= \dot{x}^T [K^*] x \\ &= \frac{i\beta}{4} (-2\Delta k + \phi^T [K^*] \phi e^{2i\beta t} - \bar{\phi}^T [K^*] \bar{\phi} e^{-2i\beta t}) \end{aligned} \quad (19)$$

By integrating Eq.(19), the dissipative energy per one cycle of the vibration is;

$$\begin{aligned} \int_0^{2\pi/\beta} \dot{U} dt \\ = \frac{i\beta}{4} \int_0^{2\pi/\beta} (-2\Delta k + \phi^T [K^*] \phi e^{2i\beta t} - \bar{\phi}^T [K^*] \bar{\phi} e^{-2i\beta t}) dt \\ = -i\pi \Delta k \end{aligned} \quad (20)$$

The antisymmetrical elements of the stiffness matrix dissipate the energy and the symmetrical ones are conservative.

3.3 Relation between logarithmic decrement and energy

$\pi\beta c$ and $-i\pi\Delta k$ in the numerator of Eq. (12) are equal to the dissipative energies due to the symmetrical elements of damping matrix and the asymmetrical elements of stiffness matrix. The kinetic energy and the potential energy at the time t in the steady vibration are;

$$T = \frac{1}{2} \dot{x}^T [M] \dot{x} = \frac{1}{8} \beta^2 (2m - \phi^T [M] \phi e^{2i\beta t} - \bar{\phi}^T [M] \bar{\phi} e^{-2i\beta t}) \quad (21)$$

$$U = \frac{1}{2} x^T [K^*] x = \frac{1}{8} (2k + \phi^T [K^*] \phi e^{2i\beta t} + \bar{\phi}^T [K^*] \bar{\phi} e^{-2i\beta t}) = \frac{1}{8} (2k + \phi^T [K] \phi e^{2i\beta t} + \bar{\phi}^T [K] \bar{\phi} e^{-2i\beta t}) \quad (22)$$

Summation of Eq. (21) and Eq. (22) yields;

$$E = T + U = \frac{1}{4} \beta^2 m + \frac{1}{4} k + \frac{1}{8} (-\beta^2 \phi^T [M] \phi e^{2i\beta t} + \phi^T [K] \phi e^{2i\beta t} - \beta^2 \bar{\phi}^T [M] \bar{\phi} e^{-2i\beta t} + \bar{\phi}^T [K] \bar{\phi} e^{-2i\beta t}) \quad (23)$$

When the elements of the damping matrix are relatively small comparing with the other matrices and the system damping is equally small, the following equations are obtained from Eq. (1).

$$\begin{aligned} -\beta^2 \phi^T [M] \phi e^{2i\beta t} + \phi^T [K] \phi e^{2i\beta t} &\approx 0 \\ -\beta^2 \bar{\phi}^T [M] \bar{\phi} e^{-2i\beta t} + \bar{\phi}^T [K] \bar{\phi} e^{-2i\beta t} &\approx 0 \end{aligned} \quad (24)$$

On the other hand, since $\alpha < \beta$, Eq. (11) becomes;

$$k \approx m\beta^2 - i\beta\Delta c \quad (25)$$

Substituting Eq. (24) and Eq. (25) into Eq. (23).

$$E \approx \frac{1}{2} \beta^2 m - i\frac{1}{4} \beta \Delta c \quad (26)$$

Equation (26) is coincident with the denominator of Eq. (12). Therefore, the logarithmic decrement approximately corresponds to a half ratio of the dissipative energy to the total energy.

3.4 Contribution of each element to system damping

In the rotor-bearing system, the bearings and the elements subjected to the dynamic fluid destabilizing forces generally cause a dissipative or destabilizing energy. Transforming the dissipative energy Eq. (18) and Eq. (20) into the sum of dissipative energy of each element, we have

$$\begin{aligned} c &= \bar{\phi}^T [C] \phi = \sum_{i=1}^n \bar{\phi}_{ei}^T [C_{ei}] \phi_{ei} = \sum_{i=1}^n c_{ei} \\ \Delta k &= \bar{\phi}^T [\Delta K] \phi = \sum_{i=1}^n \bar{\phi}_{ei}^T [\Delta K_{ei}] \phi_{ei} = \sum_{i=1}^n \Delta k_{ei} \end{aligned} \quad (27)$$

($i=1, 2, 3, \dots, n$)

where $[C_{ei}]$ is the symmetrical elements of $[C^*]$, $[\Delta K_{ei}]$ is the antisymmetrical elements of $[K^*]$, ϕ_{ei} and $\bar{\phi}_{ei}$ are the modes in each

element and subscript i means the element number. Rewriting Eq. (12) by using Eq. (27), we have

$$\delta = \sum_{i=1}^n \delta_{ei} \quad (28)$$

where

$$\delta_{ei} = \frac{1}{2} \left\{ \frac{\pi\beta c_{ei} - i\pi\Delta k_{ei}}{(1/2)\beta^2 m - i(1/4)\beta\Delta c} \right\} \quad (29)$$

δ_{ei} is a partial logarithmic decrement which means the contribution of each element to the logarithmic decrement. $\pi\beta c_{ei}$ and $i\pi\Delta k_{ei}$ expressed by complex modes in Eq. (29) are equivalent to the energy expressed by the amplitude and the phase of mode by two of the authors.⁽⁶⁾

Table 1 shows an example of the logarithmic decrement of the system and the contributions of each element which have been determined by Eq. (28) and Eq. (29).

Table 1 Partial logarithmic decrement

	Present method			Logarithmic decrement from eigenvalue
	Partial logarithmic decrement		Logarithmic decrement	
	Bearing 1	Bearing 2	Destabilizing force	
Fig. 3 ($Q=0$ MN/m)	0.90652	0.90652	0	1.8130
Fig. 3 ($Q=2.94$ MN/m)	0.37086	0.37086	-0.95636	-0.2146
Fig. 4 ($Q=2.94$ MN/m)	0.36082	1.12762	-0.84632	0.6421

The sum of the partial logarithmic decrements coincides with the logarithmic decrement directly calculated from the complex eigenvalue of the j th mode.

4. Evaluation of stability of rotor-bearing system

In this section, the effects of asymmetry of system and shaft stiffness on the stability of the rotor-bearing system are investigated by using a partial logarithmic decrement. Fig. 1 shows the configuration of the system. The rotor is supported at both ends by Bearing 1 and Bearing 2, and has a disk between the two bearings.

Fig. 1 Rotor-bearing system

The disk is subjected to the destabilizing force which is expressed as the cross-coupling force;

$$\begin{bmatrix} F_{qx} \\ F_{qy} \end{bmatrix} = \begin{bmatrix} 0 & Q \\ -Q & 0 \end{bmatrix} \begin{bmatrix} x_D \\ y_D \end{bmatrix}$$

The configurations of circular bearings, two-lobe bearings and tilting pad bearings used in this study are presented in Fig.2. The ratio of the bearing diameter D to the bearing length $L(= L/D)$ is set at 1.0 in every bearing. The pre-load ratio m_p of two-lobe bearing is set at 2/3 and m_p of tilting pad bearing is 0.

Fig.2 Bearing configuration

The finite element method is applied for the eigenvalue analysis of the system, and the shaft is divided at the locations marked by ● in Fig.1. The mass of each section is concentrated at nodal points.

Table 2 Material constants of shaft

Young's Modulus G Pa	206
Modulus of shear G Pa	79.2
Density t/m ³	7.84

Table 2 illustrates the material constants of the shaft, and the mass and the moment of inertia of the disk are represented in

Table 3 Mass and moment of inertia of disk

Mass kg	70
Moment of inertia kg m ²	0.317
Polar moment of inertia kg m ²	0.634

Table 3. As an example, two cases of locations of the disk are studied here. In Case 1, the disk is placed at the position ① in order to represent a symmetrical condition; and in Case 2, at the position ② for asymmetrical condition, as shown in Fig.1. The first natural frequencies Ω_s in the two cases on simple supports are shown in Table 4, and the bearing loads due to the weight of the rotor are shown in Table 5.

Table 4 Natural frequency of rotor for simple support condition

Case 1	393 rad/s
Case 2	495 rad/s

Table 5 Bearing loads

Case 1	Bearing 1	427 N
	Bearing 2	427 N
Case 2	Bearing 1	256 N
	Bearing 2	599 N

$$1\text{N} = 0.102 \text{ kgf}$$

The stiffness and damping coefficients of the bearings are calculated using the

finite difference method. The dimensions of the bearings and the oil viscosity are presented in Table 6.

Table 6 Dimensions of bearing

Diameter	40 mm
Length	40 mm
Set radial clearance	60 μm
Oil viscosity	16.7 m Pa s

$$1 \text{ m Pa s} = 0.102 \times 10^{-3} \text{ kgfs/m}^2$$

4.1 Effect of asymmetry of rotor system on stability

The relation between the logarithmic decrement δ and the cross coupling Q is shown in Fig.3 to Fig.8, where δ_{B1} , δ_{B2} and δ_Q are the partial logarithmic decrements of Bearing 1, Bearing 2 and the destabilizing force, respectively. The whirl orbits at $Q=0$ and 2.94 MN/m [3000 kgf/cm] in $\omega/\Omega_s = 1.0$ are also shown in the figures. The whirl orbits of the disk are illustrated in upper parts of the figures and the whirl orbits of Bearing 1 and Bearing 2 are done near to the curves of δ_{B1} and δ_{B2} .

Fig.3 and Fig.4 give the results of Case 1 and Case 2 which are those of circular bearings. The nondimensional rotor speeds are selected as $\omega/\Omega_s = 0.5$ and 1.0 because the system tends to be unstable due to only the oil film characteristics at high speed. In Case 1, the partial logarithmic decrements δ_{B1} and δ_{B2} decrease with an increasing cross coupling Q . In Case 2, δ_{B1} has the same tendency as that in Case 1, but δ_{B2} initially increases and then decreases. Therefore, Q_{max} in Case 2 is larger than that in Case 1. Q_{max} is a limit value of Q which makes the system unstable. Table 7 shows the partial logarithmic decrements δ_k and δ_c of the spring and damping coefficients of bearing. In Case 1, it is seen that δ_B becomes small because δ_c

Fig.3 Logarithmic decrements versus destabilizing force (Circular bearing, Case 1)

Fig. 4 Logarithmic decrement versus destabilizing force
(Circular bearing, Case 2)

Table 7 Value of partial logarithmic decrement

(a) Case 1

		Bearing 1	Bearing 2
Q = 0 MN/m	δ_B	0.907	0.907
	δ_c	2.660	2.660
	δ_k	-1.754	-1.754
Q = 2.94 MN/m	δ_B	0.371	0.371
	δ_c	1.028	1.028
	δ_k	-0.657	-0.657

(b) Case 2

		Bearing 1	Bearing 2
Q = 0 MN/m	δ_B	0.709	0.658
	δ_c	7.129	6.887
	δ_k	-6.421	-6.229
Q = 2.94 MN/m	δ_B	0.361	1.128
	δ_c	1.505	4.688
	δ_k	-1.144	-3.560

δ_k : Partial logarithmic decrement of antisymmetrical elements of oil film spring coefficients.

δ_c : Partial logarithmic decrement of symmetrical elements of oil film damping coefficients.

decreases more than δ_k at $Q = 2.4$ MN/m comparing with those at $Q = 0$ MN/m. In Case 2, the changes in δ_B , δ_c and δ_k of Bearing 1 have a similar tendency to those in Case 1, and on the contrary, δ_B of Bearing 2 becomes great because δ_k decreases more than δ_c .

Fig. 5 and Fig. 6 show the results in the cases that the rotor is supported on two-lobe bearings. Q_{max} in Case 2 is larger than that in Case 1 at every rotor speed of $\omega/\Omega_s = 1.0, 1.5$ and 2.0 , because δ_{B2} is large and the increment of δ_Q with a rise in Q is small in Case 2. The reason why the increment in δ_Q is small is that the whirl orbit of the disk is rather flat in Case 2. The anisotropy of the bearing characteristics(9) effectively makes the whirl orbit flat since the shaft stiffness is relatively high with respect to Bearing 2 in Case 2. Furthermore, δ_{B2} substantially contributes to the system stability in Case 2 because the whirl orbit of Bearing 2 is larger than that of Bearing 1.

Fig. 5 Logarithmic decrement versus destabilizing force
(Two-lobe bearing, Case 1)

Fig. 6 Logarithmic decrement versus destabilizing force
(Two-lobe bearing, Case 2)

The results in the cases that the rotor is supported on the tilting pad bearings are shown in Fig.7 and Fig.8. Q_{max} in Case 2 is about two and half times that in Case 1, because δ_{B1} is almost the same as that in Case 1, δ_{B2} is rather large and δ_Q is small in Case 2. The reason why δ_Q is small is that the whirl orbit of disk in Case 2 is more flat than that in Case 1. It is considered that the difference of the whirl orbit between Case 2 and Case 1 is attributable to the anisotropy of oil film characteristics, the location of disk and the gyroscopic action.

Fig.7 Logarithmic decrement versus destabilizing force (Tilting pad bearing, Case 1)

Fig.8 Logarithmic decrement versus destabilizing force (Tilting pad bearing, Case 2)

4.2 Effect of shaft stiffness on system stability

The shaft stiffness is varied by changing shaft diameter. The rotor is supported by the tilting pad bearings, and the change of bearing load due to the shaft weight is considered in calculating the bearing characteristics. The nondimensional rotor speed ω/Ω_s is selected as 2.0, and the calculated results of Case 1 and Case 2 are indicated

Fig.9 Effect of shaft stiffness on stability (Tilting pad bearing, Case 1)

Fig.10 Effect of shaft stiffness on stability (Tilting pad bearing, Case 2)

in Fig.9 and Fig.10. In these figures, γ means the relative flexibility which is defined as $\gamma = F_B C_B k_{es}$. k_{es} is an equivalent shaft stiffness of first mode, F_B a bearing load and C_B a radial clearance of bearing. It is seen that δ and δ_B become large with the decrease in γ , that is, an increase in shaft stiffness in Fig.9(a). In Fig.9(b) where Q is 1.96 MN/m [12000 kgf/cm], δ_Q becomes large with an increase in γ and the system falls into instability in the range $\gamma > 0.65$. The contribution of Bearing 2 to the system stability is great in Case 2 as shown in Fig.10. Therefore, the stability in Case 2 is better than that in Case 1 for all γ .

4.3 Effect of anisotropy of bearing characteristics

In order to study the effect of anisotropy of spring coefficients of bearing, the rotor of Case 2 which is an asymmetrical system is assumed to be supported by bearings without cross-coupling terms in oil film coefficients. In this calculation, Bearing 1 and Bearing 2 have same characteristics. It is assumed that the damping coefficient of bearing b_{xx} is equal to b_{yy} and it is constant, and the sum of spring coefficients k_{xx} and k_{yy} is also constant. Fig.11 presents the calculation results with this system. In the isotropic support, Case (1), δ at $Q = 0$ MN/m has rather large value, but decreases linearly and immediately with an increase in Q and δ reduces to zero at $Q = 2.2$ MN/m, while in the anisotropic supports, Case (2) to Case (4), δ at $Q = 0$ MN/m becomes small but Q_{max} becomes large depending on the amount of anisotropy. This is due to the reason why δ_Q becomes small and δ_{B2} becomes large with an increasing anisotropy of bearing.

Fig.11 Effect of anisotropy of bearing on stability

5. Conclusions

The following conclusions are obtained by investigating the relation between the dissipative energy and the logarithmic decrement in a rotor-bearing system.

- (1) A method which calculates the logarithmic decrement based on the energy conceptions has been presented. By using the presented method it becomes possible to evaluate the contributions of each element to the stability.
- (2) The stability of an asymmetrical system is better than that of a symmetrical system in the case of circular bearings, two-lobe bearings or tilting and bearings. This is by reason that the contribution of a bearing which is located near the disk is large and that in the case of the two-lobe bearings or the tilting pad bearings, the effect of the destabilizing force is small in the asymmetrical system.
- (3) The stability of the system supported by the tilting pad bearings is improved by increasing the shaft stiffness because the contributions of bearings increase and the effect of the destabilizing force decreases in both the symmetrical and the asymmetrical systems.
- (4) Increasing the anisotropy of bearing characteristics, the stability of the asymmetrical system is improved due to an increase in the contribution of a bearing which is located near the disk and a decrease in the effect of the fluid dynamic destabilizing force.

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