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Method of Calculating Optimum Angular Blade Pitches

in Fan with Unequally Pitched Blades*

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We devised a method of calculating the optimum angular pitches for blades in a high-speed fan that has unequal angular pitches, in order to reduce the level of a tonal annoying noise in such a fan. We used the influence coefficient-and least-squares methods in iterative calculations in which the harmonic spectrum associated with the angular blade pitches was made to converge to a target harmonic spectrum.

Our new method of calculation makes it possible to determine the optimum angular blade pitches and target harmonic spectrum necessary for the production of a quiet fan.

Key Words: Fluid Machine, Noise Control, Angular Blade Pitches, Numerical Calculation

1. Introduction

Aerodynamic noise is generated by a fan which rotates at high speeds. Such noise often presents problems. The results of a frequency analysis of the aerodynamic noise have shown that the noise consists of two principal components: a broad band noise and a discrete frequency noise,^{(1),(2)} the latter being especially annoying.

Sharland showed that the source of the broad band noise is pressure fluctuations on the surface of airfoils. The pressure fluctuations are generated by turbulent boundary layers, vortices shed from the trailing edges of blades and blades moving in a flow which was initially turbulent. He also showed that the source of the discrete frequency noise is the periodic pressure fluctuations generated by air as the blades pass through the air. The aim of our research was to devise a method of calculating the angular pitches for blades in a fan with unequally pitched blades, in an effort to reduce the magnitude of the tonal annoying noise generated by such a fan. We took particular notice of the discrete frequency noise in our calculations.

A paper by R.C. Mellin and G. Sovran describes the results of investigations in which the tonal annoying noise was reduced by spacing the blades unequally.⁽³⁾ The fans treated in this paper are ones for the single stage rotors. A paper by P.E. Duncan and B. Dawson discusses the tonal annoying noises of fans for the multistage

rotors and stators used in common axial flow fans.^{(4),(5)} In the above three papers, the descriptions include parameters which represent the number of blades in the fan and the degree of inequality in the angular blade pitches. However, the uniformity of the discrete distribution of the noise spectrum is apparently ignored, and this means that the information is not enough to allow the tonal annoying noise to be reduced.

In this paper, we describe a method of making the harmonic spectrum associated with the angular blade pitches to converge to a target harmonic spectrum. The iterative correction of the angular blade pitches is carried out by a method based on the influence coefficient-and least-squares methods.

2. Method of Calculations

2.1 Nomenclature

C, c_{kj}	: influence coefficient matrix, its elements
d_k	: magnitude of harmonic spectrum
e, e_k	: harmonic spectrum vector for the angular blade pitches, its elements
e_m, e_{mk}	: target harmonic spectrum vector, its elements
n	: maximum harmonic number for correction
$p(\theta)$	: sound pressure distribution near periphery of the fan considering only the angular blade pitches factor
S	: evaluation function
v, v_k	: constant vector determining the correction factor, its elements
z	: number of blades
α	: maximum magnitude of correction
Δe_m	: magnitude of correction of the target harmonic spectrum
$\Delta\theta, \Delta\theta_j$	: corrective quantity vector for the angular blade pitches vector, its elements

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- $\delta(\theta)$: delta function
 E, ϵ_k : error vector for the harmonic spectrum vector for the angular blade pitches, its elements
 ϵ : maximum tolerance for corrective quantities
 ϵ_m : constant coefficient for correcting the target harmonic spectrum
 ϵ_0 : standard deviation
 θ, θ_j : angular blade pitches vector, its elements (see Fig. 1)
 θ : angle

Subscripts

- j, k : order (integer)
 m : target
 T : transpose
 $'$: after correction

2.2 Harmonic spectrum associated with angular blade pitches

Analysis or experimental predictions of the sound field of the discrete frequency noise included in the aerodynamic noise generated by a high-speed rotating fan have been attempted. The analysis in Reference (4) indicates that the frequency characteristics of such noise except those associated with the dynamic factors can be determined with the following equations.

Fig. 1 Shape of fan

Fig. 2 Sound pressure distribution near periphery of fan

The shape of the fan used in our study is shown in Fig. 1. If we assume only the effect of the angular blade pitches factor on a sound pressure distribution shown in Fig. 2, the sound pressure distribution near periphery of the rotating fan is represented by

$$P(\theta) = \sum_{j=1}^Z \delta(\theta - \theta_j)$$

$$\theta_1 = 0, \delta(\theta): \text{delta function} \quad (1)$$

Then, if we develop Eq. (1) into a Fourier series, we obtain

$$P(\theta) = \frac{Z}{2\pi} + \frac{1}{\pi} \sum_{k=1}^{\infty} \left\{ \left(\sum_{j=1}^Z \cos k\theta_j \right) \cos k\theta + \left(\sum_{j=1}^Z \sin k\theta_j \right) \sin k\theta \right\} \quad (2)$$

From Eq. (2), the magnitude of the k th order harmonic component can be defined as

$$d_k = \frac{1}{Z} \sqrt{\left(\sum_{j=1}^Z \cos k\theta_j \right)^2 + \left(\sum_{j=1}^Z \sin k\theta_j \right)^2}$$

$$e_k = 20 \log_{10} d_k + 100 \quad (3)$$

Here, 100 is a constant value introduced for convenience in calculation. In Eq. (3), $e_k = 100$ for $k = mz$ (m : arbitrary natural number) and $e_k = -\infty$ for $k \neq mz$ when fan blades are equally spaced, and $-\infty < e_k < 100$ for $k = m$ when fan blades are unequally spaced. Let e_k represent the characteristics of the harmonic spectrum associated with the angular blade pitches. The relation between e_k and the angular position of blades θ_j is investigated numerically in the following.

2.3 Optimum angular blade pitches

One method of reducing the amount of tonal annoying noise generated by a rotating fan is to change the frequency characteristics of the noise. This can be done by making the angular pitches for blades of the fan unequal. However, the noise remains annoying unless the angular blade pitches are optimum. We intend to reduce the fan noise by devising a method of calculation in which the harmonic spectrum associated with given angular blade pitches can be brought progressively closer to a target harmonic spectrum.

2.4 Method of calculation

The method of calculation is based on the principle to arrive at optimum angular blade pitches through iteration, using the influence coefficient and least-squares methods. The harmonic spectrum vector e_k ($k=1, 2, \dots$) for the angular blade pitches should be equal to the target harmonic spectrum vector e_{mk} ($k=1, 2, \dots$). Here, the object of control is not only the angular blade pitches but also, under some conditions, the target harmonic spectrum. The method of calculation is described in detail below.

Let $e = \{e_1, e_2, \dots, e_n\}^T$ and $e' = \{e'_1, e'_2, \dots, e'_n\}^T$ be the harmonic spectrum vectors for the angular blade pitches before and after correction, and $\Delta\theta = \{\Delta\theta_2, \Delta\theta_3, \dots, \Delta\theta_n\}^T$ the corrective quantity vector for the angular blade pitches vector $\theta = \{\theta_2, \theta_3, \dots, \theta_n\}^T$. Then

$$e' = e + C\Delta\theta \quad C = [c_{kj}] \quad c_{kj} = \frac{\partial e_k}{\partial \theta_j} \quad (4)$$

can be calculated approximately, when the magnitude of correction is small comparing to changes in the size of the angular blade pitches. Here, C is the influence coefficient matrix before correction. Also, let $e_m = \{e_{m1}, e_{m2}, \dots, e_{mn}\}^T$ and $e'_m = \{e'_{m1}, e'_{m2}, \dots, e'_{mn}\}^T$ be the target harmonic spectrum vectors before and after correction, and Δe_m the magnitude of correction that takes place. Then

$$e'_m = e_m + v \Delta e_m$$

$$v = \{v_1, v_2, \dots, v_n\}^T \quad (5)$$

can be calculated. Here, v is a constant vector that is used to determine the correction factor for the target harmonic spectrum.

From the error $e' - e'_m = E = \{\epsilon_1, \epsilon_2, \dots, \epsilon_n\}^T$ generated by assuming that e' is equal to e'_m in Eqs. (4) and (5), we obtain

$$S = \sum_{k=1}^n \lambda_k |\epsilon_k|^2 \quad \lambda_k = \frac{|e_{mk} - e_k|}{\epsilon_0}$$

$$\epsilon_0 = \sqrt{\frac{1}{n} \sum_{k=1}^n (e_{mk} - e_k)^2} \quad (6)$$

as an evaluation function, where $E = C\Delta\theta - v\Delta e_m - (e_m - e)$.

The minimum conditions for $\Delta\theta$ and Δe_m are satisfied by solving the following equations.

$$\frac{\partial S}{\partial \Delta\theta_j} = 0 \quad (j=2, 3, \dots, Z), \quad \frac{\partial S}{\partial \Delta e_m} = 0 \quad (7)$$

If the relation shown on the right side of the following equation holds, then the values obtained for $\Delta\theta$ and e_m can be used as they are.

$$\max \{|\Delta\theta_2|, |\Delta\theta_3|, \dots, |\Delta\theta_Z|, |\Delta e_m|\} = \alpha \leq \epsilon \quad (8)$$

(where ϵ is the maximum tolerance for corrective quantities relating to Eq. (4)).

However, if the relation does not hold, then those values must be further corrected, as follows:

$$\Delta\theta \rightarrow \epsilon \Delta\theta / \alpha \quad \Delta e_m \rightarrow \epsilon_m \Delta e_m / \alpha \quad (9)$$

Here, ϵ_m is the constant coefficient for correcting the target harmonic spectrum, and ϵ, ϵ_m are determined by trial and error. The relation between the vectors for the angular blade pitches and target harmonic spectrum before and after correction are as follows.

$$\theta' = \theta + \Delta\theta \quad e'_m = e_m + v \Delta e_m \quad (10)$$

If we calculate θ and e_m by iteration using Eqs. (6), (7) and (8), and the expression (9), we can obtain values that converge to θ and e_m . These values will be the optimum ones for the angular blade pitches and target harmonic spectrum vector.

In Eq. (6), λ_k is a weighting factor that controls the rapidity of convergence of the harmonic spectrum vector e for the angular blade pitches and the target harmonic spectrum vector e_m .

3. Numerical Examples

3.1 Correction calculations for fixed targets

If $v_k = 0$ ($k=1, 2, \dots, n$) in Eq. (5), corrections can be calculated for a fixed target.

Fig. 3 shows the results obtained for the convergence of the harmonic spectrum in the cases where $z = 12$, $e_1 = 50$, but $e_k = 75$ (Case (A)), $e_k = 85$ (Case (B)), $e_k = 95$ (Case (C)) and $k = 2, 3, \dots, 24$ for each case.

Fig. 4 shows the results obtained for the convergence of the harmonic spectrum in the cases where $z = 13$, $e_1 = 50$, but $e_k = 95$ (Case (D)), $e_k = 0.729k + 66.04$ (Case (E)), $e_k = 1.4k + 48.6$ (Case (F)) and $k = 2, 3, \dots, 26$ for each case.

Fig. 3 Harmonic spectrum for angular blade pitches --- in fixed target ($Z = 12$)

Fig. 4 Harmonic spectrum for angular blade pitches --- in fixed target
($Z = 13$)

In Figs. 3 and 4, e_1 is made constant and smaller than e_k ($k=2, 3, \dots, n$) so that the mechanical unbalance that is characteristic of the fan in which the blades have unequal pitches may be eliminated.

As shown in both figures, the calculated harmonic spectrum is the closest to the target one when the target harmonic spectrum is represented as a curve of proper shape and height. This tendency can also be seen at $z = 5$ and $z = 29$. We assume that this is applicable for any number of z . A target harmonic spectrum which produces the optimum convergence is called an optimum target harmonic spectrum.

3.2 Correction calculations for adjustable targets

If $v_1 = 0$, $v_k = 1$ ($k=2, 3, \dots, n$), corrections can be calculated for adjustable targets, except for the first component of a target harmonic spectrum. Fig. 5 shows the results obtained for the convergence of the harmonic spectrum for (a) $z = 5$, (b) $z = 13$ and (c) $z = 29$.

These results prove that there is an optimum target harmonic spectrum as described in Section 3.1.

Table 1 shows the results obtained for the convergence of the blade pitches shown in Fig. 5. Fig. 6 shows tests for the convergence of the standard deviation, the deviations and the target harmonic spectrum obtained for $z = 13$ in the process of calculation. It is found that these various quantities converge after about 60 iterations of correction.

4. Conclusions

We devised a method of calculating the optimum angular pitches for blades in a high-speed fan with blades that have unequal angular blade pitches in order to reduce the level of the tonal annoying noise in such a fan. Particular emphasis was laid on the level of the discrete frequency noise. The following results were obtained.

(1) Using the influence coefficient-and least-squares methods we devised such a method of calculation that the harmonic

Table 1 Results obtained in calculations of angular blade pitches
(in adjustable target)

Number of blades	Angular blade pitches (degree)									
	98.4	29.0	108.3	51.8	72.5	—	—	—	—	—
5	26.3	23.7	17.6	25.3	32.0	55.1	9.0	12.3	38.4	32.8
	12.2	36.7	38.6	—	—	—	—	—	—	—
13	4.6	21.6	4.5	18.6	10.4	7.6	12.3	7.3	16.1	7.1
	11.8	28.9	8.5	26.9	3.3	4.5	3.8	12.5	17.8	17.7
	5.2	10.8	12.6	18.4	13.7	12.5	13.7	10.3	17.0	—

Fig.5 Harmonic spectrum for angular blade pitches --- in adjustable target

Fig.6 Test for convergence of various calculated values ($z=13$)

spectrum associated with the angular blade pitches converges to a target harmonic spectrum.

(2) We calculated the optimum angular blade pitches and the optimum target harmonic spectrum for a quiet fan.

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