Breakup of a Thin Liquid Layer Heated from Below*

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When a thin liquid layer, which is at rest under the action of gravity on a horizontal wall, is locally heated from below, a flow is induced in the layer by the surface-tension force. The flow in such a case is studied theoretically by using the Stokes approximation and the thin film approximation. This flow is governed by two dimensionless parameters, Nusselt number N and thermal Weber number M . When the liquid layer is heated from below, the free surface is always depressed. When the heating is strong, the liquid layer breaks up. The critical value of the thermal Weber number, above which the liquid layer breaks up, is found as a function of N . The shapes of the surface just when a breakup occurs are calculated. The critical thermal Weber numbers are experimentally found for the case of an oil sheet. These values are in agreement with the predictions from the theory.

1. Introduction

If a thin liquid layer on a horizontal wall is locally heated from below, there usually happens a breakup in the liquid layer. This phenomenon is called dry-out. It is closely related with an abnormal chemical reaction. Then, from what cause a film breakup arises and under what condition it first appears are the most important problems to be answered.

The breakup of a thin liquid layer heated from below is often considered to arise from evaporation. However, in the chemical processes, the liquid layers are frequently used under a high pressure state. Then, the evaporation is not always the main cause of film breakup. It is well known that the surface-tension variation caused by a difference of the temperature or the concentration of a surface-active material on a free surface induces easily a fluid motion in the liquid layer⁽¹⁾⁽²⁾. Then it is firstly investigated whether or not the forces induced thermally by surface-tension variation cause a film breakup of a thin liquid layer. Supposing that they do, secondly, a critical condition in which the film breakup occurs is studied theoretically and experimentally.

The motion of a liquid film induced by a surface-tension variation is analyzed by Levich, under such conditions that the values of surface tension coefficient are a given function of the position on the free surface and the Stokes approximation holds⁽³⁾. It is pointed out by Yih that the presence of a pressure gradient in the horizontal direction due to a spatial variation of the thickness of the liquid layer is not negligible in such a fluid flow⁽⁴⁾. Levich's analysis is re-examined by Yih. In this paper, a fluid motion in a thin horizontal layer

induced by a local heating from below is analyzed by assuming the surface tension coefficient to be a function of temperature and solving the equations of motion and of energy simultaneously. The critical condition in which a film breakup occurs is found theoretically. This result is compared with the experimental result obtained about an oil film heated from below.

2. Basic Equations

Consider the fluid motion of a thin liquid layer of infinite extent on a horizontal wall locally heated from below. If a fluid layer of a constant thickness H with the density ρ and the kinematic viscosity ν , which is at rest under the action of gravity, is locally heated from below in a finite extent of the characteristic spatial length L , a fluid motion is, in general, induced by thermal expansion and thermo-capillary force. The fluid motion, which arises from these two causes, is governed by Marangoni number and Rayleigh number. The former is proportional to the second power of the layer thickness, but the latter is proportional to the fourth power of it. Therefore the flow in a thin liquid layer considered here is mainly governed by Marangoni number, that is, thermo-capillary force⁽⁵⁾. Thus it is assumed that the thermal expansion effect is negligible and the surface tension coefficient σ is a function of only temperature T . Furthermore, the Stokes approximation is applied to analysis of this problem, since the Reynolds number when the layer thickness is adopted as a characteristics length is very small. Then, the convection terms in energy equation are assumed to be negligible.

The cylindrical coordinate (r, θ, z) in which z axis is in the vertically upward direction from the horizontal wall is taken as shown in Fig.1. For simplicity, heating of the liquid film from the wall is assumed to be steady and axi-symmetric, that is, the flow is independent of θ . If u and w are the velocity components in the directions of r and z , p is the pressure, and T is the temperature, the flow in the liquid film is governed by

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Fig.1 Schematic diagram and coordinate.

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} \right) = 0 \quad (1)$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) - g = 0 \quad (2)$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (4)$$

Now, consider the boundary conditions for the above differential equations. On the horizontal wall, no slip condition holds and temperature is prescribed with a function of r :

$$z=0: u=w=0 \quad (5)$$

$$z=0: T=T_w(r) \quad (6)$$

If the thickness of the liquid film heated from the horizontal wall is denoted by $h(r)$, the outward normal vector becomes $(-\partial h/\partial r/\sqrt{1+(\partial h/\partial r)^2}, 0, 1/\sqrt{1+(\partial h/\partial r)^2})$. Taking into account that the pressure is the atmospheric pressure and the surface tension coefficient is a function of only the temperature, the conditions that the forces in the normal direction and in the tangential direction balance at the free surface⁽⁶⁾ are obtained as

$$\begin{aligned} z=h: \\ p = \frac{2\sigma\nu}{1+(\partial h/\partial r)^2} \left\{ \frac{\partial u}{\partial r} \left(\frac{\partial h}{\partial r} \right)^2 + \frac{\partial w}{\partial z} - \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right) \frac{\partial h}{\partial r} \right\} \\ - \sigma \frac{(\partial^2 h/\partial r^2) + (1/r)(\partial h/\partial r) + (1/r)(\partial h/\partial r)^3}{(1+(\partial h/\partial r)^2)^{3/2}} \quad (7) \\ \rho\nu \left\{ -2 \left(\frac{\partial u}{\partial r} - \frac{\partial w}{\partial z} \right) \frac{\partial h}{\partial r} + \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right) \left[1 - \left(\frac{\partial h}{\partial r} \right)^2 \right] \right\} \\ - \frac{d\sigma}{dT} \left(\frac{\partial T}{\partial r} + \frac{\partial T}{\partial z} \frac{\partial h}{\partial r} \right) \sqrt{1 + \left(\frac{\partial h}{\partial r} \right)^2} = 0 \quad (8) \end{aligned}$$

The kinematic condition⁽⁷⁾ is written as

$$z=h: u \frac{\partial h}{\partial r} = w \quad (9)$$

If the atmospheric temperature, the thermal conductivity and the heat transfer coefficient are denoted by T_a , k and r , respectively, the boundary condition for heat flow at the free surface is from Newton's cooling law⁽⁸⁾ as follows

$$z=h: -\frac{k}{\sqrt{1+(\partial h/\partial r)^2}} \left(\frac{\partial T}{\partial z} - \frac{\partial T}{\partial r} \frac{\partial h}{\partial r} \right) = r(T - T_a) \quad (10)$$

Here, the kinematic viscosity ν and the thermal conductivity k are assumed to be

independent of the temperature, because a fluid motion is scarcely induced by the temperature dependency of these coefficients.

3. Nondimensionalized Differential System

and Thin Film Approximation

Although the differential system, Eqs. (1)-(10), is derived by applying the Stokes approximation, it is still complex. In particular, the boundary conditions are so. The relative importances of the various terms in the differential equations considered are examined by nondimensionalizing them.

Estimate the characteristic pressure, velocity and temperature which are necessary for the equations to be nondimensionalized. The characteristic length in the r direction and that in the z direction are apparently the horizontal extent of the heating region L and the thickness of the liquid layer H in the isothermal state. The magnitude of the pressure P is estimated as the hydrostatic pressure $\rho g H$, because $(1/\rho)(\partial p/\partial z) \approx g$ from Eq. (2). In such a thin liquid layer that a film breakup can occur, L is very large compared with H . Then the leading one of the viscous terms in the equation of motion is estimated as $\nu(\partial^2 u/\partial z^2) \approx \nu(U/H^2)$, where U is the magnitude of the velocity component in the r direction. The pressure term in that equation is estimated as $(1/\rho)(\partial p/\partial r) \approx P/\rho L \approx gH/L$. Thus, equating the magnitude of the viscous term to that of the pressure term, the magnitude U of the velocity component in the r direction is estimated as $U \approx gH^3/\nu L$. Since $\partial u/\partial r \approx \partial w/\partial z$ from the equation of continuity, the magnitude W of the velocity component in the z direction is estimated as $W \approx HU/L \approx gH^4/\nu L^2$. The magnitude of the temperature T is clearly estimated as the maximum value of the difference between the temperature of the wall heated and the atmospheric temperature, $T \sim |T_w - T_a|_{\max}$. Therefore the nondimensional variables with symbol $*$ are introduced as

$$\begin{aligned} r &= Lr^*, \quad z = Hz^*, \quad h = Hh^*, \quad u = (gH^3/\nu L)u^*, \\ w &= (gH^4/\nu L^2)w^*, \quad p = \rho g H p^*, \\ T &= |T_w - T_a|_{\max} T^* + T_a \end{aligned}$$

Rewriting the differential system, Eqs. (1)-(10), in the previous section with the nondimensional variables, the following equations are obtained:

$$-\frac{\partial p}{\partial r^*} + \frac{\partial^2 u}{\partial z^{*2}} + \left(\frac{H}{L} \right)^2 \left(\frac{\partial^2 u}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial u}{\partial r^*} - \frac{u}{r^{*2}} \right) = 0 \quad (1)'$$

$$-\frac{\partial p}{\partial z^*} + \left(\frac{H}{L} \right)^2 \frac{\partial^2 w}{\partial z^{*2}} + \left(\frac{H}{L} \right)^4 \left(\frac{\partial^2 w}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial w}{\partial r^*} \right) - 1 = 0 \quad (2)'$$

$$\frac{\partial u}{\partial r^*} + \frac{u}{r^*} + \frac{\partial w}{\partial z^*} = 0 \quad (3)'$$

$$\frac{\partial^2 T}{\partial z^{*2}} + \left(\frac{H}{L} \right)^2 \left(\frac{\partial^2 T}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial T}{\partial r^*} \right) = 0 \quad (4)'$$

$$z=0: u=w=0 \quad (5)'$$

$$z=0: T=T_w \quad (6)'$$

$$z=h: \quad (7)'$$

$$\begin{aligned} p &= \left(\frac{H}{L} \right)^2 \frac{2}{1+(H/L)^2(\partial h/\partial r)^2} \left[\left(\frac{H}{L} \right)^2 \frac{\partial u}{\partial r^*} \left(\frac{\partial h}{\partial r^*} \right)^2 + \frac{\partial w}{\partial z^*} \right. \\ &\quad \left. - \left(\left(\frac{H}{L} \right)^2 \frac{\partial w}{\partial r^*} + \frac{\partial u}{\partial z^*} \right) \frac{\partial h}{\partial r^*} \right] \\ &\quad - \frac{\sigma}{\rho g H^2} \left(\frac{H}{L} \right)^2 \frac{(\partial^2 h/\partial r^{*2}) + (1/r^*)(\partial h/\partial r^*) + (H/L)^2(1/r^*)(\partial h/\partial r^*)^3}{(1+(H/L)^2(\partial h/\partial r^*)^2)^{3/2}} \quad (7)' \end{aligned}$$

$$-2\left(\frac{H}{L}\right)^2\left(\frac{\partial u}{\partial r}-\frac{\partial w}{\partial z}\right)\frac{\partial h}{\partial r}+\left\{\left(\frac{H}{L}\right)^2\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right\}\left\{1-\left(\frac{H}{L}\right)^2\left(\frac{\partial h}{\partial r}\right)^2\right\}$$

$$-M\left(\frac{\partial T}{\partial r}+\frac{\partial T}{\partial z}\frac{\partial h}{\partial r}\right)\left\{1+\left(\frac{H}{L}\right)^2\left(\frac{\partial h}{\partial r}\right)^2\right\}^{1/2}=0$$

$$\dots\dots\dots(8)'$$

$$u\frac{\partial h}{\partial r}=w\dots\dots\dots(9)'$$

$$\frac{1}{\sqrt{1+(H/L)^2(\partial h/\partial r)^2}}\left\{\frac{\partial T}{\partial z}-\left(\frac{H}{L}\right)^2\frac{\partial T}{\partial r}\frac{\partial h}{\partial r}\right\}=NT$$

$$\dots\dots\dots(10)'$$

where, M is the Weber number due to thermo-capillary force $(d\sigma/dT)(|T_w-T_a|_{\max})/\rho g H^2$ and N is the Nusselt number $\gamma H/k$. Hereafter the symbol * denoting the nondimensional variable is omitted for simplicity.

Since the ratio H/L is very small in such a thin liquid layer that a film breakup can occur, Eqs. (1)-(10)' obtained above are approximated with a high degree of accuracy as follows:

$$\frac{\partial^2 u}{\partial z^2}-\frac{\partial p}{\partial r}=0\dots\dots\dots(11)$$

$$\frac{\partial p}{\partial z}+1=0\dots\dots\dots(12)$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0\dots\dots\dots(13)$$

$$\frac{\partial^2 T}{\partial z^2}=0\dots\dots\dots(14)$$

$$z=0: u=w=0\dots\dots\dots(15)$$

$$T=T_w\dots\dots\dots(16)$$

$$z=h: p=0\dots\dots\dots(17)$$

$$\frac{\partial u}{\partial z}-M\left(\frac{\partial T}{\partial r}+\frac{\partial T}{\partial z}\frac{\partial h}{\partial r}\right)=0\dots\dots\dots(18)$$

$$-\frac{\partial T}{\partial z}=NT\dots\dots\dots(19)$$

$$u\frac{\partial h}{\partial r}=w\dots\dots\dots(20)$$

This differential system, Eqs.(11)-(20), is solved in the following section.

4. Solution

The pressure p is given from Eqs.(12) and (17) as

$$p=h-z\dots\dots\dots(21)$$

and the temperature T is given from Eqs. (14), (16) and (19) as

$$T=-N\frac{T_w}{1+Nh}z+T_w\dots\dots\dots(22)$$

The pressure depends on r through the height of the liquid layer h and the temperature depends on r through h and the wall temperature T_w . The second term in the left-hand side of Eq.(18), denoting the thermo-capillary force, is rewritten by a derivative of the temperature at the free surface with respect to r , because the temperature T is obtained as a function of r and z . Then, solving Eq.(11) under the boundary conditions, Eqs.(15) and (18), the velocity component in the r direction u is obtained as

$$u=\frac{1}{2}\frac{dh}{dr}z^2+\frac{d}{dr}\left(MT_w-\frac{h^2}{2}\right)z\dots\dots\dots(23)$$

Solving Eq.(13), into which this solution is substituted, under the boundary condition, Eq.(15), the velocity component in the z direction w is obtained as

$$w=-\frac{1}{6}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)hz^3-\frac{1}{2}\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(MT_w-\frac{1}{2}h^2\right)z^2$$

$$\dots\dots\dots(24)$$

If the stream function ψ which is defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial z}, \quad w=-\frac{1}{r}\frac{\partial\psi}{\partial r}\dots\dots\dots(25)$$

is introduced, it is given from Eqs.(23) and (24) as

$$\psi=r\left\{\frac{1}{6}\frac{dh}{dr}z^3+\frac{1}{2}\frac{d}{dr}\left(MT_w-\frac{h^2}{2}\right)z^2\right\}$$

$$\dots\dots\dots(26)$$

Thus, the velocity, the pressure, the temperature and the stream function, which are necessary for the determination of the state of the flow, are obtained in the form containing the unknown variable $h(r)$ which denotes the profile of the free surface.

5. Height Equation and Its Solution

If Eqs.(23) and (24) are substituted into the kinematic condition at the free surface, Eq.(20), the unknown variable $h(r)$ can be obtained. Since this flow is axi-symmetric, there can not exist a global flow. Then, the value of the stream function on the wall is equal to that at the free surface and these values can be taken as zero, that is, $\psi|_{z=h}=0$. Thus, the equation governing the height of the liquid layer is found from Eq. (26), instead of Eq.(20), as follows:

$$\frac{d}{dr}(h^2-3MT_w)=0\dots\dots\dots(27)$$

Since the extent of the heating region is assumed to be finite, the wall temperature is equal to the atmospheric temperature at a point infinitely far from the origin, that is,

$$r\rightarrow\infty: h\rightarrow 1, \quad T_w\rightarrow 0\dots\dots\dots(28)$$

Integrating Eq.(27) under this boundary condition, Eq.(28), the temperature at the free surface, T , is obtained using Eq.(22) as

$$T_w=\frac{T_w}{1+Nh}\dots\dots\dots(29)$$

therefore, the height of the liquid layer is found as

$$1-h^2=-3M\frac{T_w}{1+Nh}\dots\dots\dots(30)$$

Thus, if the wall temperature T_w is given, the height of the layer is concretely obtained from Eq.(30). It depends on two dimensionless variables, that is, the Weber number due to the thermo-capillary force, M , and the Nusselt number, N .

6. Breakup Condition

Consider now whether a film breakup occurs or not. In general, the derivatives of the surface tension coefficient with respect to the temperature are negative. Then, $-M$ has a positive value. If the liquid layer is heated from below, the right-hand side of Eq.(30) becomes positive. Thus, $h^2 < 1$ holds. It is, therefore, found that the thickness of a liquid layer heated from below always diminishes.

In practice, various types of tempera-

ture distribution $T_w(r)$ of the heated wall are conceivable. Most of the practical temperature distributions are, however, axi-symmetric with respect to z -axis and have a maximum value at its axis. Then, $T_w(r)$ is assumed to be such a temperature distribution.

Film breakup should occur at the maximum temperature. Then, if the cubic equation with respect to the film height h , Eq.(30), has not a positive real root at $r=0$, the liquid layer is regarded as broken up. Taking $T_w|_{r=0}=1$ into account, the critical condition at which a film breakup occurs is found in terms of Weber number due to thermo-capillary force as

$$-M_{cr} = \frac{1}{3}(1 + Nh_0 - h_0^2 - Nh_0^3) \equiv f(N) \quad (31)$$

$$h_0 = (-1 + \sqrt{1 + 3N^2})/3N \quad (32)$$

If $-M$ is larger than the value of $-M_{cr}$, the liquid layer breaks up. Thus, the critical Weber number due to thermo-capillary force M_{cr} is determined as a function of the Nusselt number N , $f(N)$. The critical condition for film breakup is shown in M - N plane of Fig.2. The solid line in this figure is the critical curve on which the liquid layer breaks up. The liquid film breaks up in the upper side of the solid line, but it does not in the lower side of it. The values of $-M_{cr}$ increase as the values of the Nusselt number increase. Thus, when there exists a heat flow at the free surface from the liquid layer to the ambient air, it becomes difficult for a film to break up. If there exists no heat flow at the free surface, that is, $N \rightarrow 0$, the temperature of the free surface becomes equal to that of a heated wall. Then, the critical condition is found as

$$-M_{cr} = \frac{1}{3} \quad (33)$$

The point at which the vertical axis intersects the solid line corresponds to this condition.

Now, rewriting the critical condition, Eq.(31), in a dimensionless form, yields

$$|T_w - T_0|_{\max} = f(N) \frac{g\rho}{(-d\sigma/dT)H^2} \quad (34)$$

Therefore, the critical temperature difference between the heated wall and the ambient air at which a film breakup occurs is found to be proportional to the second power of the

Fig.2 The critical curve of film breakup.

thickness of the liquid layer. This critical temperature difference depends only on the liquid density and the derivative of the surface tension coefficient with respect to the temperature, but does not on the viscosity, the surface tension coefficient and the profile of the temperature distribution of the heated wall.

7. Shapes of liquid film

If the wall temperature $T_w(r)$ is concretely given, the shapes of the liquid film heated or cooled from below are found from Eq.(30). Assuming the wall temperature to be a probability curve, that is,

$$T_w = \pm e^{-r^2} \quad (35)$$

the shapes of the liquid layer are found as shown in Figs. 3 and 4. In these figures, $MT_w|_{r=0}$ is denoted by M . Then, when the value of M is negative, the liquid layer is heated from the wall. When it is positive, the liquid layer is cooled from it. It is found from these figures that a liquid layer on the heated wall becomes concave and one on the cooled wall becomes convex. It is also found from Fig.4 that the depression decreases as the heat flow from the liquid layer to the ambient air increases and the liquid film just breaks up at $N=0$. Of course, the shapes of the liquid layer shown in these

Fig.3 The shape of the heated or cooled liquid layer (the effect of the Weber number due to thermo-capillary).

Fig.4 The shape of the heated liquid layer (the effect of the Nusselt number).

figures depend on the temperature distribution of the heated or cooled wall. However, the critical condition at which a film break-up occurs does not depend on it.

8. Comparison with Experimental results

A thin salad oil layer is at rest on a horizontal steel plate 17 cm $\times$ 28 cm with thickness 5 mm. The central part of the plate is locally heated from below by a gas burner. The thickness of the salad oil layer is measured by a micrometer with a needle to be in the range from 0.3 mm to 1 mm. The maximum value of the temperature on the upper surface of the steel plate is below 170°C. The evaporation of the salad oil scarcely takes place in that temperature range. The temperature on the upper surface of the plate is measured with a surface thermometer with a contact area of 5 mm in diameter. Since the air temperature in the vertical direction abruptly changes near the free surface, the temperature of the ambient air is measured at a position 1 cm apart from the free surface of the liquid layer. The kinematic viscosity and the density of the salad oil used are $0.906 \times 10^{-4} \text{ m}^2/\text{s}$ and 924 kg/m^3 at 15°C, respectively. The temperature dependency of the surface tension coefficient, which is the most important material property in this problem, is shown in Fig. 5. The surface tension coefficient linearly decreases as the temperature increases. The derivative of the surface tension with respect to the temperature $d\sigma/dT$ is $-0.133 \times 10^{-3} \text{ N/m}^2\text{C}$. These experimental data are obtained by Du Noüy's method.

The behavior of the depressed free surface is observed using the distorted patterns of the shadows of the regular mesh cast on the free surface. The results of such observations are shown in Figs. 6-8. Fig. 6 is the case when the liquid layer is 0.76 mm in thickness and the free surface is steadily depressed a little by a constant heating from below. As the degree of heating increases, the depression of the free surface increases. When the degree of heating increases further, the oil film breaks up at last as shown in Fig. 7. The critical state in which the liquid layer breaks up does not become steady, even if heating is constant. The liquid layer continues to break up until the extent of the breaking region becomes a few millimeters in diameter as shown in Fig. 8. The state shown in this figure is steady. In this state, the maximum temperature difference between the wall temperature and the ambient air is 86°C and the wall temperature at the center is 119°C. The critical state in which the layer breaks up is unstable as described above. Then, in practice, the critical steady state given theoretically by Eq. (31) is impossible to exist.

Since the critical state in which a film breaks up is unsteady as mentioned above, the measurement with high accuracy can not be accomplished. However, the time rate of unsteadiness is comparatively small in case of such a highly viscous fluid as the salad oil. Then, the wall temperatures T_w near the critical state in which the film breaks up are shown in Fig. 9. These temperature distributions are very similar to the probability curve which is assumed in the theory.

Fig. 5 The temperature dependency of the surface tension.

In the critical state, the maximum differences $|T_w - T_a|_{\max}$ between the wall temperature at the center of the heating region and the ambient air temperature depend on the thickness H of the liquid layer in the isothermal state as shown in Fig. 10. In this figure, the solid line is drawn by assuming the maximum temperature difference to be proportional to the second power of the layer thickness. Since there exist experimental data on or near this straight line, the critical condition anticipated theoretically by Eq. (34), that is, the maximum temperature difference being proportional to the second power of layer thickness, is ensured practically.

The temperature difference divided by the second power of the thickness, $|T_w - T_a|_{\max}/H^2$, is shown in Fig. 11 for the isothermal liquid layer H . From this figure, the values of $|T_w - T_a|_{\max}/H^2$ are found almost equal to a constant, that is, $1.31 \times 10^4 \text{ C/cm}^2$. This value divided by $g\rho/(-d\sigma/dT)$ is regarded as the critical Weber number due to thermo-capillary force $-M_{cr}$ which is obtained experimentally. Since $g\rho/(-d\sigma/dT) = 6.810 \text{ C/cm}^2$ in this experiment, the value of $-M_{cr}$ is found to be 1.92. The Nusselt number has not been measured experimentally. Therefore it is impossible to examine whether these data of the Weber number due to thermo-capillary force fit the critical curve in Fig. 2 or not. This experimental datum is about 5.76 times the minimum Weber number due to thermo-capillary force obtained theoretically, $1/3$, and is surely in the breakup region. If the Nusselt number corresponding to this experimental datum is calculated from Eqs. (31) and (32), it is equal to 12.4. The experiment is performed in a wide wall temperature range from 35°C to 170°C. Therefore there must be large differences in the viscosity and the temperature of the liquid in each experiment. In spite of this fact, the Weber numbers due to thermo-capillary force found experimentally are almost equal to a constant. Thus the critical state is experimentally confirmed to depend on only the derivative of the surface tension coefficient with respect to the temperature and the liquid density, and not to depend on the viscosity and the temperature distribution of the heating wall. Thus the theoretical result is in good agreement with the experimental one.

Fig.6 The free surface depressed by the heating wall. ($H = 0.76\text{mm}$)

Fig.7 The state near the critical condition in which the liquid layer is breaking up. ($H = 0.76\text{mm}$)

Fig.8 The steady state in which the liquid layer has been broken up.
($h = 0.76\text{mm}$, $T_w = 119\text{ }^\circ\text{C}$, $T_w - T_s = 86\text{ }^\circ\text{C}$)

Fig.9 The wall temperature distribution in the critical state in which the layer just breaks up.

Fig.10 The relationship between the maximum temperature difference and the layer thickness at the critical state.

9. Conclusions

When a thin viscous layer on a horizontal wall is heated from below, whether the liquid layer breaks up or not is examined theoretically and experimentally. In theoretical analysis, solutions are found by using the equation of motion and the energy equation simultaneously, and applying the Stokes approximation. They are governed by only two dimensionless parameters, i.e., the Weber number due to thermo-capillary force M and the Nusselt number N . The height h of the liquid layer is found as a function of

Fig.11 The ratio of the maximum temperature difference to the second power of the layer thickness at the critical state.

the wall temperature containing M and N parametrically. If the liquid layer is heated from below, the free surface of the layer is always depressed. The degree of the depression decreases as the Nusselt number increases. The critical curve on which the film breaks up is found in $M-N$ plane. The layer can break up in the region of $-M > 1/3$. The temperature difference between the wall and the ambient air depends on only the derivative of the surface tension coefficient with respect to the temperature, the Nusselt number and the liquid density, and does not on the viscosity and the wall temperature distribution. These theoretical results are experimentally verified by using a heated liquid layer of the salad oil. The steady solution corresponding to the critical state in which the film breaks up is experimentally found to be unstable. Thus the critical steady state is impossible to exist practically.

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