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Trajectory Interpolation of a Robot Hand*

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For smoothing the motion of a robot hand, a simple method for trajectory interpolation of the robot hand has been developed, in which successive straight line trajectories are connected continuously by parabolas. Only a linear interpolation function of computation is required for this method and it makes the computation time so short that the trajectory interpolation can be performed real time while the robot is in motion. Determination of a closed loop spatial mechanism with single mobility which is equivalent to the robot arm with a specified hand location is also described in the paper for the purpose of analyzing joint displacements of the robot arm.

Key Words: Industrial Machine, Automatic Machine, Robot, Trajectory, Spatial Mechanism

1. Introduction

Industrial robots have been mainly driven with Point-to-Point (PTP) control. However, since they came recently to be used for arc welding and assembly processes, the control method has been changed to Continuous-Path (CP) control, which provides excellent functions. CP control includes the memory storage of joint displacement data at every sampling time. When a robot is driven over a long period of time, a large memory is required when this method is utilized. In order to overcome this disadvantage, the following PTP teaching and CP control method is generally employed.

The coordinates of some points are stored during PTP teaching and when a robot is driven, the space between the points is interpolated and a continuous trajectory is generated in this case. The trajectory is discontinuous at the points that were taught when the space between the points is simply interpolated and the hand cannot pass through the points with a uniform speed. Therefore, to allow the hand to pass through the vicinity of the points without stopping, it is necessary to correct the trajectory around these points and execute continuous motion.

A method was previously suggested in which line segments are made continuous by circular arcs. However, the volume of computation required for trajectory generation is considerably increased when this method is utilized. These computations could not be

performed in the control computer utilized for an industrial robot during the real time in which the robot is in motion. The stopping or the deceleration of robots near the points is one of the most serious primary factors which prolongs the time required for robot operation.

In this paper, a trajectory interpolation method is developed in which a number of line segments are continuously connected by parabolas using only the linear interpolation utilized in a conventional industrial robot. Because computations can be performed only with conventional operating functions when this method is used, additional programs for robot control and, therefore, new memory area are not required. Also, only small amounts of computation are required and real time computation can be performed while a robot is in motion.

2. Position and orientation of a robot hand

The position and orientation of a robot hand in three-dimensional space can be expressed by three vectors $\underline{p}$, $\underline{f}$ and $\underline{g}$. $\underline{p}$ is the position vector of the reference point on the hand while $\underline{f}$ and $\underline{g}$ are direction cosines of the two axes of the hand which orthogonally intersect.

Let the two hand locations be provided by $(\underline{p}_a, \underline{f}_a, \underline{g}_a)$ and $(\underline{p}_b, \underline{f}_b, \underline{g}_b)$, where all the vectors are expressed in the fixed frame $O^*-X^*-Y^*-Z^*$ as illustrated in Fig. 1. In this case, the hand can be shifted between two locations by translational motion along a straight line and rotational motion about an axis with constant direction. The straight line $\underline{P}$ and the axis $\underline{E}$ are obtained by the following expressions.

$$\underline{P} = \underline{p}_a - \underline{p}_b + \epsilon \underline{p}_a \times \underline{p}_b \quad \text{-----}(1)$$

$$\underline{E} = (\underline{f}_b - \underline{f}_a) \times (\underline{g}_b - \underline{g}_a) + \epsilon \underline{p} \times (\underline{f}_b - \underline{f}_a) \times (\underline{g}_b - \underline{g}_a) \quad \text{-----}(2)$$

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$\underline{P}$ and $\underline{E}$ are motor representations of straight lines and $\underline{e}$ is a dual operator. $\underline{p}$, which changes according to the hand movement, indicates the hand position on the straight line. From Equ. (2), the direction cosines of the rotational axis $\underline{e}$ of the hand can be determined as follows.

$$\underline{e} = \frac{(\underline{f}_b - \underline{f}_a) \times (\underline{g}_b - \underline{g}_a)}{|(\underline{f}_b - \underline{f}_a) \times (\underline{g}_b - \underline{g}_a)|} \quad (3)$$

The angle θ_{ab} of the hand rotation about $\underline{e}$ from $(\underline{f}_a, \underline{g}_a)$ to $(\underline{f}_b, \underline{g}_b)$ can be determined by the following expressions.

$$\cos \theta_{ab} = \{ \underline{f}_a \cdot \underline{f}_b - (\underline{e} \cdot \underline{f}_a)^2 \} / \{ 1 - (\underline{e} \cdot \underline{f}_a)^2 \} \quad (4)$$

$$\sin \theta_{ab} = \{ \underline{e} \cdot (\underline{f}_a \times \underline{f}_b) \} / \{ 1 - (\underline{e} \cdot \underline{f}_a)^2 \} \quad (5)$$

Linear interpolation is done such that the translational and rotational displacements $\underline{P}_b - \underline{P}_a$ and θ_{ab} are equally divided according to a prescribed velocity and each point is used as datum for the hand location at each sampling time. In other words, when the hand is driven between two locations with the time t , the number of samplings, which is necessary for shifting the hand, is the minimum integer which is greater than t/T (T : sampling time), and the displacements are equally divided by the number of samplings.

3. Interpolation method for a uniform speed along line segments

The interpolation method which allows the hand to be driven among three given points with a uniform speed along two line segments (excluding a parabola) will now be explained.

When a robot hand is driven at a uniform speed along polygonal lines which connect three points A, B and C as illustrated in Fig. 2, the trajectory is discontinuous at point B. To make it continuous, the space between D, B and E is parabola-connected so that the hand acceleration becomes uniform. In Fig. 2, the origin of the Cartesian coordinates x-y is located at point B and y axis is fixed with point B toward the direction with which the angle ABC is divided into

Fig. 1 Hand position and orientation

a pair of equal parts and the x axis is fixed orthogonally to y axis. Then the equation of the parabola is expressed as

$$y = a x^2 + b \quad (6)$$

The tangent of the parabola can be written in a differential equation as follows.

$$y = \frac{dy}{dx} x + c \left(\frac{dy}{dx} \right)^2 + d \quad (7)$$

Differentiating Equ. (7) with x and obtaining its singular solution give

$$\frac{dy}{dx} = -\frac{x}{2c} \quad (8)$$

Substituting Equ. (8) into Equ. (7) gives

$$y = -\frac{x^2}{4c} + d \quad (9)$$

Therefore, from Eqs. (6) and (9), the following relationships can be obtained.

$$a = -1/4c \quad (10)$$

$$b = d \quad (11)$$

Let the length of segments $\overline{DB}$ and $\overline{BE}$ be l . The tangent of the parabola at points D and E will pass the origin. Moreover, if the angle ABC is denoted by θ , the coordinates of the points D and E are $(\pm l \sin(\theta/2), -l \cos(\theta/2))$ and the inclination of the tangent at these points is $\mp l/\tan(\theta/2)$. From these conditions, the following relations are obtained.

$$c \left(\frac{dy}{dx} \right)^2 + d = \frac{c}{\tan^2 \theta/2} + d = 0 \quad (12)$$

$$a = -\frac{\cos \theta/2}{2l \sin^2 \theta/2} \quad (13)$$

Therefore, the equation of the parabola becomes as follows

$$y = \frac{-\cos \theta/2}{2l \sin^2 \theta/2} x^2 - \frac{l}{2} \cos \theta/2 \quad (14)$$

Fig. 2 Trajectory continuation by a parabola

Because the robot control is a sampling control, the number of samplings from point D to point E must be an integer. Moreover, in the linear interpolation, the numbers of samplings between D and B, and between B and E have the same value. Hence, the number of samplings between D and E must be an even number. Let the number of samplings between D and E be $2n$. Because of the uniform speed operation in x axis direction, the hand position on the parabola after m samplings from D is

$$x_m = \left(\frac{m}{n} - 1 \right) \{ \sin \theta/2 \} \quad (15)$$

$$y_m = -\left\{ \left(\frac{m}{n} - 1 \right)^2 + 1 \right\} \frac{1}{2} \cos \theta/2 \quad (16)$$

Also, the inclination of the tangent of parabola at point (x_m, y_m) is $-(m/n-1)\cot(\theta/2)$. The point of intersection (x'_m, y'_m) of the tangent and segment BC is

$$\begin{aligned} (x'_m, y'_m) &= \left(\frac{m}{2n} \{ \sin \theta/2, \right. \\ &\quad \left. - \frac{m}{2n} \{ \cos \theta/2 \} \right) \quad (17) \end{aligned}$$

When m is changed such as $1, 2, 3, \dots, n$, these points are plotted on segment BC with equal spacing. The ratio of the x component distances between point (x_{m-1}, y_{m-1}) and point (x'_m, y'_m) and between point (x_{m-1}, y_{m-1}) and point (x_m, y_m) is

$$\frac{x_m - x_{m-1}}{x'_m - x_{m-1}} = \frac{1}{n+1-m/2} \quad (18)$$

Thus, x_m is determined by dividing the segment which connects point (x_{m-1}, y_{m-1}) and point (x_m, y_m) into equal parts of $n+1-m/2$. The value of y_m can be approximately obtained on the segment (tangent) if each divided part of the segment is small.

This result can be applied to trajectory interpolation for the hand along the parabola. Divide segment BE into $2n$ equal parts and arrange them as $(x'_1, y'_1), (x'_2, y'_2), \dots, (x'_{2n}, y'_{2n})$, starting from the point which is close to point B. The last point (x'_{2n}, y'_{2n}) will coincide with point E. Let point D be (x_0, y_0) . Divide the segment which connects (x_0, y_0) and (x'_1, y'_1) into $n+1/2$ equal parts and arrange the point

Fig. 3 An example of trajectory interpolation

next to (x_0, y_0) as (x_1, y_1) . Arrange it as (x_2, y_2) by dividing the segment which connects (x_1, y_1) and (x'_2, y'_2) into n equal parts and as (x_3, y_3) by dividing segment (x_2, y_2) and (x'_3, y'_3) into $n-1/2$ equal parts. Then the $2n$ times point coincides with point E and the curved line which connects $(x_1, y_1), (x_2, y_2), \dots, (x_{2n}, y_{2n})$ becomes a parabola. Parabolic interpolation can be performed by a simple combination of linear interpolation which divides the segment with equal spacing as mentioned above and no complicated operation is required.

Fig. 3 shows an example of parabolic interpolation when $n=3$. $E_1, E_2, \dots, E_6$ are the points at which segment BE was divided into six equal parts. First, fix point P_1 by dividing segment DE_1 into 3.5 equal parts. Then fix P_2 by dividing segment P_1E_2 into 3 equal parts and fix $P_3, P_4, P_5, P_6(=E_6)$ by dividing $P_2E_3, P_3E_4, P_4E_5, P_5E_6$ into 2.5, 2, 1.5, 1 equal parts, respectively.

The relationship between the interpolated points by using the above mentioned method and the parabola based on Equ. (14) is illustrated in Fig. 4, where $n=10$ and $\theta=60$ degrees. Circles in the figure indicate the interpolated points and the broken line indicates the parabola. Both are nearly in accord with each other. The difference between the interpolated points and the parabola was caused because interpolated points were set on the tangent of the parabola.

4. Applications to general cases

The method for creating a parabolic trajectory when the robot hand moves along a polygonal line with a uniform speed was explained in the previous section. This method can be utilized as an interpolating method for general robot hand acceleration and deceleration control.

Suppose the method explained in the previous section is applied when shifting is along AB and BC at a uniform

Fig. 4 Interpolation result

speed as shown in Fig. 5. The kinetic component along segment $\overline{AC'}$ which includes segment $\overline{AB}$ is considered in this case. Fig. 5 indicates this when $n=3$. A point at which point E_i ($i=1,2,\dots,6$) was projected to $\overline{AC'}$ is supposed to be E'_i . If the speed of the robot hand between points A and D, as well as points E'_6 and C, is v at that time, the velocity component between E'_6 and C' along segment $\overline{AC'}$ is $v\sin\theta$. If the method in the previous section is applied along with $\overline{AC'}$, from point D to E'_6 , the hand is uniformly decelerated or accelerated. In other words, the relationship of the interpolated points can be proportionally maintained in the projection of results of the parabolic interpolation of the polygonal line ABC to $\overline{AC'}$. Also, the interpolating method along with $\overline{AC'}$ is in accord with the method in the previous section because the interpolating method is linear. In Fig. 5, when the hand is shifted along straight line $\overline{AC'}$ and is driven with a velocity of v_{ab} in $\overline{AD}$ and with a velocity v'_{bc} in $\overline{E'_6C'}$ by accelerating or decelerating around point B (the ratio $|DB|/|BE'_6|$ is v_{ab}/v'_{bc}), divide $\overline{BE'_6}$ into six equal parts in which $E'_1, E'_2, \dots, E'_6$ are arranged. First, arrange P'_1 by dividing $\overline{DE'_1}$ into 3.5 equal parts and then arrange P'_2 by dividing $\overline{P'_1E'_2}$ into 3 equal parts. By this procedure, P'_3, P'_4, P'_5 , and P'_6 can be determined.

Especially when $v_{ab}=0$, the hand corresponds to the motion which accelerates uniformly from a stationary position to a uniform velocity status in $2n$ samplings. Also, when $v'_{bc}=0$, it contrarily corresponds to a motion which stops with deceleration.

Fig. 6 shows a result of the interpolation when a hand is accelerated from a stationary position according to the above mentioned method. Circles in the figure indicate interpolated points while the solid line indicates the parabola in a uniform accelerated motion. Both are nearly in accord with each other. In this example, $n=10$.

Fig. 7 shows an example of interpolation when velocities along a polygonal line differ. The velocity in BC is half that in AB and $n=10$ in this example.

5. Interpolation algorithm for a robot hand positions and orientations

The method for interpolation of the robot hand position discussed above can be applied to interpolation of the orientation. The interpolation in which three points A, B and C are given as well as an orientation for each point is discussed.

Assume that the robot hand is driven with a translational velocity v_{ab} along segment $\overline{AB}$ and with v_{bc} along segment $\overline{BC}$. Also, it is accelerated from a stationary position at point A. Then it is driven along the parabola around point B and stops at point C after

deceleration.

Let the hand positions and orientations at points A, B and C be (p_a, f_a, g_a) , (p_b, f_b, g_b) and (p_c, f_c, g_c) respectively. The unit vector $\underline{e}_{ab}$ along the axis of a rotational motion of the hand in $\overline{AB}$ is determined on the basis of four vectors $\underline{f}_a, \underline{g}_a, \underline{f}_b$ and $\underline{g}_b$ (see Equ.(3)). Therefore, it is expressed as follows.

$$\underline{e}_{ab} = e(\underline{f}_a, \underline{g}_a, \underline{f}_b, \underline{g}_b) \quad \text{----- (19)}$$

The rotational angle θ_{ab} from $(\underline{f}_a, \underline{g}_a)$ to $(\underline{f}_b, \underline{g}_b)$ about $\underline{e}_{ab}$ is expressed as follows.

$$\theta_{ab} = \theta(\underline{f}_a, \underline{g}_a, \underline{f}_b, \underline{g}_b) \quad \text{----- (20)}$$

Fig. 5 Velocity component along a straight line

Fig. 6 Displacement during acceleration

Fig. 7 Interpolation result

On the other hand, hand orientation (f, g), when a rotational angle θ about e_{ab} is given, can be calculated as follows.

$$\begin{aligned} f &= f(\theta, e_{ab}, f_a) \\ &= \cos\theta f_a + (1-\cos\theta)e_{ab} \cdot f_a e_{ab} \\ &\quad + \sin\theta e_{ab} \times f_a \end{aligned} \quad (21)$$

$$g = g(\theta, e_{ab}, g_a) \quad (22)$$

Also, the length s_{ab} of segment $\overline{AB}$ is as follows.

$$s_{ab} = s(p_a, p_b) = |p_a - p_b| \quad (23)$$

Suppose $\overline{AB}$ and $\overline{BC}$ are equally divided into the values n_{ab} and n_{bc} where $n_{ab} = \lceil s_{ab}/v_{ab}T \rceil$ and $n_{bc} = \lceil s_{bc}/v_{bc}T \rceil$. $\lceil s_{ab}/v_{ab}T \rceil$ is a minimum integer which is greater than or equal to $s_{ab}/v_{ab}T$ and T is a sampling time. Also, at point A the hand is accelerated in $2n_a$ sampling periods, parabolic-interpolated in $2n_b$ sampling periods and it stops at point C after deceleration in $2n_c$ sampling periods. In this case, the time required for shifting the hand from point A to point C is $(n_{ab} + n_{bc} + n_a + n_c)T$ under the following conditions.

$$n_{ab} \geq n_a + n_b \quad (24)$$

$$n_{bc} \geq n_b + n_c \quad (25)$$

Let the hand position and orientation at time nT be expressed as $p(nT)$, $f(nT)$ and $g(nT)$. First, determine point D_1 on segment $\overline{AB}$ such that

$$|AD_1| = \frac{n_a}{n_{ab}} |AB| \quad (26)$$

Divide segment $\overline{AD_1}$ into $2n_a$ equal parts and let each point be denoted by position vectors $d_1^a, d_2^a, \dots, d_{2n_a}^a$.

If $n=0$,

$$p(0) = p_a, f(0) = f_a, g(0) = g_a \quad (27)$$

The hand position at time nT ($0 \leq n \leq 2n_a$) during the acceleration from point A is determined by dividing the space between the hand position at $(n-1)T$ and point d_n^a into equal parts of $n_a + 1 - n/2$. Therefore, the hand position can be determined by the following expressions.

$$p(nT) = \frac{d_n^a - p(n-1T)}{n_a + 1 - n/2} + p(n-1T) \quad (28)$$

The rotational angle θ_{ab} of the hand from point A to point B is given by Equ. (20). Therefore, interpolating θ_{ab} in the same manner as Equ. (28), $\theta(nT)$ can be obtained as follows.

$$\begin{aligned} \theta(nT) &= \frac{n\theta_{ab}/2n_{ab} - \theta(n-1T)}{n_a + 1 - n/2} \\ &\quad + \theta(n-1T) \end{aligned} \quad (29)$$

The orientation of the hand at time nT is computed by the following expressions.

$$f(nT) = f(\theta(nT), e_{ab}, f_a) \quad (30)$$

$$g(nT) = g(\theta(nT), e_{ab}, g_a) \quad (31)$$

At time $2n_aT$, the hand arrives at

point D_1 with the orientation

$$f(n_a\theta_{ab}/n_{ab}, e_{ab}, f_a)$$

and

$$f(n_a\theta_{ab}/n_{ab}, e_{ab}, g_a).$$

The hand moves with a uniform speed along segment $\overline{D_1D_2}$ until the time $(n_{ab} + n_a - n_b)T$, where D_2 is determined on segment $\overline{AB}$ such that

$$|D_2B| = \frac{n_b}{n_{ab}} |AB| \quad (32)$$

Therefore, a point at each sampling time can be obtained by dividing segment $\overline{D_1D_2}$ into equal parts of $n_{ab} - n_a - n_b$. After this procedure, the hand is shifted on the parabola from segment $\overline{AB}$ to segment $\overline{BC}$. The interpolation in this case can be written as follows.

First, point E is determined on segment $\overline{BC}$ such that

$$|BE| = \frac{n_b}{n_{bc}} |BC| \quad (33)$$

and $\overline{BE}$ is divided into $2n_b$ equal parts and points are fixed from the point next to point B as $d_1^b, d_2^b, \dots, d_{2n_b}^b$. At this time, the hand position at time nT ($n_{ab} + n_a - n_b \leq n \leq n_{ab} + n_a + n_b$) is given by

$$p(nT) = \frac{d_{m_b}^b - p(n-1T)}{n_b + 1 - m_b/2} + p(n-1T) \quad (34)$$

where,

$$m_b = n - n_{ab} - n_a + n_b \quad (35)$$

on the other hand, interpolation of the orientation is performed as follows.

First, the orientation (f_m, g_m) is obtained by dividing the space between (f_b, g_b) and (f_c, g_c) into $2n_{bc}$ equal parts.

$$f_{m_b}^b = f\left(\frac{m_b}{2n_{bc}}\theta_{bc}, e_{bc}, f_b\right) \quad (36)$$

$$g_{m_b}^b = g\left(\frac{m_b}{2n_{bc}}\theta_{bc}, e_{bc}, g_b\right) \quad (37)$$

where,

$$\theta_{bc} = \theta(f_b, g_b, f_c, g_c) \quad (38)$$

$$e_{bc} = e(f_b, g_b, f_c, g_c) \quad (39)$$

Then the orientation at time nT can be obtained by dividing the space between ($f(n-1T), g(n-1T)$) and ($f_{m_b}^b, g_{m_b}^b$) into equal parts of $n_b + 1 - m_b/2$.

$$\theta = \frac{\theta(f(n-1T), g(n-1T), f_{m_b}^b, g_{m_b}^b)}{n_b + 1 - m_b/2} \quad (40)$$

$$e = e(f(n-1T), g(n-1T), f_{m_b}^b, g_{m_b}^b) \quad (41)$$

$$f(nT) = f(\theta, e, f(n-1T)) \quad (42)$$

$$g(nT) = g(\theta, e, g(n-1T)) \quad (43)$$

Based on the above, the hand reaches segment $\overline{BC}$ at time $(n_{ab} + n_a + n_b)T$ and the orientation is ($f_{2n_b}^b, g_{2n_b}^b$). It shifts until $(n_{ab} + n_{bc} + n_a - n_c)T$ according to the result in which (p_b, f_b, g_b) and (p_c, f_c, g_c) are divided into n_{bc} equal parts along segment $\overline{BC}$.

Finally, the hand is decelerated towards point C after time $(n_{ab} + n_{bc} + n_a - n_c)T$. In this case, the hand position at time nT ($n_{ab} + n_{bc} + n_a - n_c \leq n \leq n_{ab} + n_{bc} + n_a + n_c$) is

$$\underline{p}(nT) = \frac{\underline{p}_c - \underline{p}(n-1T)}{n_c + 1 - m_c/2} + \underline{p}(n-1T) \quad (44)$$

where,

$$m_c = n - n_{ab} - n_{bc} - n_a + n_c \quad (45)$$

The rotational angle of the hand about the axis $\underline{e}_{bc}$ is

$$\theta(nT) = \frac{\theta_{bc} - \theta(n-1T)}{n_c + 1 - m_c/2} + \theta(n-1T) \quad (46)$$

where the initial value of the rotational angle is given by the following equation.

$$\theta(n_{ab} + n_{bc} + n_a - n_c T) = \frac{n_{bc} - n_c}{n_{bc}} \theta_{bc} \quad (47)$$

The hand orientation at time nT is obtained by the following expressions.

$$\underline{f}(nT) = \underline{f}(\theta(nT), \underline{e}_{bc}, \underline{f}_b) \quad (48)$$

$$\underline{g}(nT) = \underline{g}(\theta(nT), \underline{e}_{bc}, \underline{g}_b) \quad (49)$$

As a result, the hand reaches $(\underline{p}_c, \underline{f}_c, \underline{g}_c)$ at time $(n_{ab} + n_{bc} + n_a + n_c)T$ and can stop at this point.

6. Determination of a Closed Loop Spatial Mechanism for Displacement Analysis

Determination of joint displacements of a robot based on the position and orientation data of the robot hand is necessary for trajectory control of the robot. Conventionally, the matrix method in which homogeneous coordinates in the reference frame on the hand are transformed into those in the fixed frame, has been utilized for this purpose and an algebraic solution could not be obtained except in a very special case⁽²⁾. However, a displacement analysis for the closed loop spatial mechanisms was recently established in which the vector equation and spherical trigonometry are utilized. As a result, a method has been proposed in which the robot mechanism is replaced with a closed loop spatial mechanism and the displacement analysis is performed⁽³⁾. In this section, a closed loop mechanism which is equivalent to a robot mechanism is derived from the location data of the hand $(\underline{p}, \underline{f}, \underline{g})$. This derivation makes it possible to apply the displacement analysis of closed loop spatial mechanisms to the joint displacements computation for robot control.

The dimensions of the spatial mechanism, including robot mechanisms, are expressed as illustrated in Fig. 8. $\underline{s}_i$ is a unit vector along the i th joint axis and $\underline{a}_{ij}$ is a unit vector along a common perpendicular line of the i th and j th joint axes. The link length a_{ij} is a length of the common perpendicular line and the offset s_i is a length along the

i th joint axis. The angles θ_j and α_{ij} are formed by $\underline{a}_{ij}$ and $\underline{s}_{jk}$ and by $\underline{s}_i$ and $\underline{s}_j$ respectively. The angles θ_j and α_{ij} are measured in the clockwise direction from small to large numbers about $\underline{s}_j$ and $\underline{a}_{ij}$ respectively.

In order to locate the hand at an arbitrary position with an arbitrary orientation, the robot is required to have six degrees of freedom which generally consist of six kinematic pairs, each having a single degree of freedom. Therefore, the dimensions of the robot mechanism is expressed by 23 parameters, i.e., $s_1, s_2 - s_5, s_6, a_{12} - a_{56}, a_{67}, \phi_1, \theta_2 - \theta_5, \phi_6$ and $\alpha_{12} - \alpha_{56}$.

In this case, s_1 and ϕ_1 determine the location of the first joint of the robot in the fixed frame

$O^*-X^*-Y^*-Z^*$. The origin O^* is located on the first joint axis and Z^* axis is collinearly located with the first joint axis as illustrated in Fig. 8. The angle ϕ_1 is an angle between X^* axis and $\underline{a}_{12}$. a_{67} and s_6 represent the relation between the sixth joint and the reference point on the hand and ϕ_6 is an angle between $\underline{a}_{56}$ and $\underline{a}_{67}$ where $\underline{a}_{67}$ is a line passing through the reference point and intersecting sixth joint axis orthogonally.

The closed loop mechanism can be obtained by introducing a hypothetical revolute pair $\underline{s}_7$ and by connecting $\underline{s}_6$ and $\underline{s}_7$, and, $\underline{s}_7$ and $\underline{s}_1$ by hypothetical links $\underline{a}_{67}$ and $\underline{a}_{71}$ respectively. By computing $\theta_7, s_6, a_{67}, \alpha_{67}, \theta_6 - \phi_6, s_7, a_{71}, \alpha_{71}, s_1$ and $\theta_1 - \phi_1$ based on the data $(\underline{p}, \underline{f}, \underline{g})$, all the dimensions of the closed loop mechanism including its input θ_7 are determined.

Fig. 8 Robot mechanism

Fig. 9 shows the dimensions of the hypothetical joint. When the position and orientation of the hand are given by (p, f, g) , the position vector q of the sixth joint and s_6 and s'_6 can be computed by the following equations.

$$q = p + \lambda_{q1}f + \lambda_{q2}g + \lambda_{q3}fxg \quad (50)$$

$$s_6 = \lambda_{s1}f + \lambda_{s2}g + \lambda_{s3}fxg \quad (51)$$

$$s'_6 = \lambda_{a1}f + \lambda_{a2}g + \lambda_{a3}fxg \quad (52)$$

where λ has a constant value determined by the relation between the sixth joint and the hand. If the hypothetical joint s_7 is determined such that it may be orthogonal to both s_6 and s_1 and may pass through the point q , the following equations are obtained.

$$s_7 = \frac{s_6 \times s_1}{|s_6 \times s_1|} \quad (53)$$

$$s_7 = 0, a_{67} = 0, \alpha_{67} = \pi/2, \alpha_{71} = \pi/2 \quad (54)$$

s_7 cannot be determined by Equ. (53) when s_6 is parallel to s_1 . In this case, s_7 can be arbitrarily set parallel to the X^*-Y^* plane. However, the computation volume can be reduced if it is fixed parallel to X^* or Y^* axis.

Link a_{67} is $s_6 \times s_7$. therefore, the angle $\theta_6 - \phi_6$ formed by links a_{67} and s_6 is

$$\begin{aligned} \theta_6 - \phi_6 &= 2 \tan^{-1} \frac{a_{67} \times s_6 \cdot a'_{67}}{a'_{67} \cdot a_{67} + 1} \\ &= 2 \tan^{-1} \frac{s_7 \cdot a_{67}}{a'_{67} \cdot a_{67} + 1} \quad (55) \end{aligned}$$

Also, since link a_{71} is $s_7 \times s_1$, the angle $\theta_1 - \phi_1$ formed by a_{71} and the X^* axis is

$$\begin{aligned} \theta_1 - \phi_1 &= 2 \tan^{-1} \frac{i \times s_1 \cdot a_{71}}{a_{71} \cdot i + 1} \\ &= 2 \tan^{-1} \frac{s_7 \cdot i}{a_{71} \cdot i + 1} \quad (56) \end{aligned}$$

where i is a unit vector along X^* axis. The angle θ_7 formed by a_{67} and a_{71} is

Fig. 9 Determination of hypothetical joint

$$\begin{aligned} \theta_7 &= 2 \tan^{-1} \frac{a_{71} \times s_7 \cdot a_{67}}{a_{67} \cdot a_{71} + 1} \\ &= 2 \tan^{-1} \frac{s_1 \cdot a_{67}}{a_{67} \cdot a_{71} + 1} \quad (57) \end{aligned}$$

On the other hand, putting s_1^* as $q \cdot s_1$, the following vector equation is obtained.

$$s_7 \cdot s_7 + a_{71} \cdot a_{71} = s_1^* \cdot s_1 - q \cdot q \quad (58)$$

From the inner products of Equ. (58) and s_7 and a_{71} , the following relations are obtained.

$$s_7 = s_7 \cdot (s_1^* \cdot s_1 - q \cdot q) \quad (59)$$

$$a_{71} = a_{71} \cdot (s_1^* \cdot s_1 - q \cdot q) \quad (60)$$

The offset of the closed loop mechanism along s_1 is

$$s_1 = s'_1 - s_1^* \quad (61)$$

Thus the dimensions of the closed loop mechanism are determined and the joint displacements can be computed by the displacement analysis of closed loop mechanisms, which is fully described in the reference (3).

7. Conclusions

In this paper, the following conclusions are obtained.

- (1) A method is developed in which a polygonal line trajectory of a robot hand is continued with a parabola. This continuation method can only be executed with the linear interpolation function provided for conventional industrial robots. The computation time is short enough for the real time operation. Also, this method can be used when the hand is accelerated or decelerated along a straight line.
- (2) In order to compute the joint displacements of a robot, the method for determining an equivalent closed loop spatial mechanism of a robot in accord with each specified hand location is derived. This derivation makes it possible to utilize the displacement analysis of closed loop mechanisms for the joint displacement analysis of a robot.

References

- (1) L. Brand: "Vector and Tensor Analysis", John Wiley and Sons, Inc. (1964)
- (2) D. L. Pieper: Stanford Artificial Intelligence Project, Memo No. 72. (1968)
- (3) J. Duffy: "Analysis of Mechanisms and Robot Manipulators", Edward Arnold. (1980)