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An Experimental Investigation of Viscoelastic Flow past a Circular Cylinder*

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The drag forces of circular cylinders of infinite or finite span are examined in both Newtonian fluids and dilute polymer solutions. Using some circular cylinders with different ratios of length (l) to diameter (d), the drag forces in the Re range of 1 to 5×10^4 and the pressure distributions in the Re range of 1.7×10^4 to 5×10^4 are measured. The results make it clear that (1) the drag forces decrease at $Re < 20$ and increase at $Re > 20$ with an increase of l/d , (2) the drag forces for dilute polymer solutions reduce more than those for Newtonian fluids at $Re > 1 \times 10^4$.

Key Words: Non-Newtonian Fluid, Dilute Polymer Solution, Wake, Finite Length Cylinder, Separation, Drag Reduction

1. Introduction

Many studies have been reported on the flow past a circular cylinder of infinite span in Newtonian fluids.

On the other hand, few studies have dealt with the drag force of a circular cylinder in non-Newtonian fluids. Kotake⁽¹⁾ analyzed for the velocity distributions in a wake of the flow field by using a power law model, and Mizushima et al.⁽²⁾ measured the pressure distributions and the separation points in pseudo-plastic fluids.

For flows of viscoelastic fluids including polymer solutions, Gadd⁽³⁾ measured the Strouhal numbers, James et al.⁽⁴⁾ measured the drag forces and visualized a flow around a cylinder, Mochimaru et al.⁽⁵⁾ obtained analytically the drag forces and the velocity distributions in a wake at low Reynolds numbers, and Sarpkaya et al.⁽⁶⁾ measured the drag forces, pressure distributions and the separation points in the critical range of Reynolds numbers.

On the other hand, for flow past a cylinder of finite span, Wieselsberger⁽⁷⁾ and Christiansen et al.⁽⁸⁾ measured the drag forces and Okamoto et al.⁽⁹⁾ measured the drag forces and the pressure distributions only in Newtonian fluids.

In this study, the effects of viscoelasticity of the polymer solutions on the drag force and the pressure distribution of a circular cylinder are considered from the experimental results of the drag forces for the cylinders of finite span in the Re range of 1 to 5×10^4 and the pressure distributions around the cylinders of infinite or finite span in the Re range of 1.7×10^4 to 5×10^4 .

ders of infinite or finite span in the Re range of 1.7×10^4 to 5×10^4 .

2. Nomenclature

- C_D : coefficient of drag force
 $= 2F / \rho_f u_\infty^2 dl$
 $C_{D'}$: modified coefficient of drag force
 $= C_D ((\rho_c - \rho_f) / \rho_f)^{0.05}$
 $C_{D'}$ or $C_{Dw'}$ means $C_{D'}$ for Peo aqueous solutions or water, respectively
 $\bar{C}_D$: mean value of drag coefficient
 $= \frac{1}{h_0} \int_0^{h_0} \left\{ \int_0^\pi C_P(\theta, z) \cos \theta d\theta \right\} dz$
 C_P : coefficient of pressure
 $= 2(P_0 - P_\theta) / \rho_f u_\infty^2$
 C_θ : coefficient of local pressure drag reduction
 $= 2(P_{\theta w} - P_{\theta f}) \cos \theta / \rho_f u_\infty^2$
 d : diameter (m)
 f : frequency of periodic pressure fluctuation (Hz)
 F : drag force of a circular cylinder (N)
 h_0 : wetted length in fluid (m)
 l : length of a circular cylinder (m)
 n : exponent in power law model
 N_{We} : Weissenberg number
 $= 2\lambda(u_\infty/d)^{1-n}/\mu$
 P_0 : static pressure of free stream (Pa)
 P_θ : total pressure on the wall of a cylinder at the angle of θ (Pa)
 $P_{\theta f}$ or $P_{\theta w}$ means P_θ in Peo aqueous solutions or water respectively
 Re : Reynolds number $= du_\infty/\nu$
 Re^* : generalized Reynolds number
 $= \rho_f u_\infty^{2-n} d^n / \mu$
 s : exponent in Denn model
 St : Strouhal number $= fd/u_\infty$
 u_∞ : free stream velocity (m/s)
 z : length in the axial direction measured from the end face of the cylinder (m)
 θ : angle from a forward stagnation point (deg)

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- κ : rate of drag reduction
 $=1-C_{DP}/C_{DW}'$
 κ_z : rate of local drag reduction
 in the axial direction
 λ : rheological constant in Denn
 model (Pa·s²)
 μ : rheological constant in Denn
 model (Pa·sⁿ)
 ν : kinetic viscosity of Newto-
 nian fluid (m²/s)
 ρ_c : density of a circular cylinder
 (kg/m³)
 ρ_f : density of fluid (kg/m³)

3. Experimental Apparatus and Procedures

The settling tanks were used for measuring the drag forces, namely, one tank 0.12 m wide, 2 m long and 0.4 m high in the Re range of 1 to 1×10^3 , and the other 0.48 m wide, 0.48 m long and 1.2 m high in the Re range of 1×10^3 to 5×10^4 . Cylinders used were made of brass, steel or duralumin. Their dimensions are listed in Table 1.

In the falling cylinder experiment, cylinders were dropped into a fluid filled tank with their axes horizontal. The terminal velocity was measured by a photo-electric time counter. The test section decided by the calculations and the photographic measurements was from 0.1 m to 0.2 m under the surface of water in the Re range of 1 to 1×10^3 or from 0.55 m to 0.85 m in the Re range of 1×10^3 to 5×10^4 .

A towing tank was used for measuring the pressure distributions, because it was suitable for reducing the mechanical degradation of dilute

polymer solutions. A vertical section of the experimental apparatus is shown in Fig. 1. The size of the tank was 0.48 m in width, 0.515 m in height and 10 m in length. The carriage was taken in tow by a steel rope which was wound round a drum and by a motor. The jointless and smooth finished rails were used. One of them had vertical and horizontal planes and the other had a inclined plane (angle of 15°) which served for pushing the carriage against a vertical plane of the other rail. The linearity of the rails was very good, and the static relative errors between the carriage and rails for the horizontality were less than 0.2 mm.

Acrylic resin pipes of 100 mm in diameter were used for measuring the pressure distributions. To substitute the pipe for a cylinder of infinite span, acrylic resin disks (500 mm in diameter) were attached on either side of the pipe ($h_0 = 0.3$ m). On the other hand, pipes ($h_0 = 0.1, 0.2$ m) with one disk on the upper side were used as the cylinders of finite span. A few pressure holes (0.5 mm in diameter) were drilled normally to the wall of the cylinders. These holes kept the interval of 120 degrees to avoid a mutual interference.

The local pressure was measured by a pressure transducer. The total pressure was picked up from the pressure holes, and the static pressure was from a pitot tube which was set up on the carriage.

Glycerine aqueous solutions and water were used as Newtonian fluids, and polyethyleneoxide (Peo) aqueous solutions were used as viscoelastic fluids. Viscosities of Newtonian fluids and Peo dilute solutions were measured by a Ubbelohde viscometer, because the non-Newtonian viscous effect of Peo dilute solutions could be disregarded in the range of shear

material	ρ_c kg/m ³	d m	l/d
brass	8470	0.012~0.026	2
steel	7830	0.004~0.006	1
		0.004~0.015	2
		0.002~0.006	5
		0.002~0.006	10
		0.002~0.006	50
duralumin	2700	0.002	500
		0.004~0.006	1
		0.004~0.015	2
		0.004~0.006	5
		0.004~0.006	10
		0.004~0.006	50

Table 1 Dimensions of the cylinder

Fig. 1 Vertical section of a towing tank

polymer	mol. wt.	conc. ppm	temp. °C	λ/μ s ²⁻ⁿ	n	s
Peo 10	2×10^6	100	11.0	5.51×10^{-3}	1.0	2.0
		200	8.5	2.09×10^{-2}	1.0	2.0
		300	11.0	3.15×10^{-2}	1.0	2.0
Peo 16	3×10^6	50	21.0	4.02×10^{-2}	1.0	1.85
		100	21.5	9.60×10^{-2}	1.0	1.83
		200	19.8	2.86×10^{-1}	1.0	1.73
Peo 18	4×10^6	30	10.0	6.43×10^{-1}	1.0	1.69
		50	13.0	7.45×10^{-1}	1.0	1.66
		100	14.0	1.01×10^0	1.0	1.62
		200	17.5	1.42×10^0	1.0	1.58

Table 2 Rheological constants of Peo dilute solutions

polymer	mol. wt.	conc. %	temp. °C	μ Pa·s ⁿ	n
Peo 16	3×10^6	0.4	11.5	1.8	0.65
		0.5	11.5	3.2	0.59
Peo 18	4×10^6	0.2	10.0	0.30	0.83

Table 3 Rheological constants of Peo high concentrated solutions

rates of this experiment. The rheological constants of elasticity could not be measured directly for Peo dilute solutions. Therefore, they were decided by using the values measured for Peo high concentrated solutions, namely, by using Denn model⁽¹⁰⁾ which expressed the elastic behaviour comparatively well, the rheological constants in the model λ and μ were measured by a Weissenberg rheogoniometer for Peo high concentrated solutions, and the constants of Peo dilute solutions were determined from those values by extrapolation for the concentrations and the shear rates. These values are listed in Table 2. On the other hand, Peo high concentrated solutions exhibited a non-Newtonian viscosity. Therefore, these rheological constants n and μ included in Denn model were measured by a coaxial cylindrical viscometer. These values are listed in Table 3.

4. Experimental Results and Discussions

4.1 Drag force of a circular cylinder

Fig. 2 shows the experimental results for Newtonian fluids. The results by Wieselsberger⁽⁷⁾ who used a cylinder with $l/d = \infty$ or 5 (noted as W_∞ or W_5 curve respectively in this paper) are shown in this figure, together with the results by Tritton⁽¹¹⁾ who used $l/d = 1400, 1600$ or 2000 in the range of $Re < 10$. According to this figure, all results of cylinders with $l/d = 2, 5, 10$ and 50 cross W_∞ curve at $Re \approx 20$, and the drag forces increase with a decrease of l/d in the Re range of 1 to 20 (de-

fined as A region in this paper) and decrease in the Re range of 20 to 1×10^3 (defined as B region). It is considered that the increase of the drag forces in A region is due to the increase of the skin friction caused by the flow passing around both end faces of cylinders and the decrease in B region is due to the decrease of the form drag caused by a rise of the back pressure. Next, the result of a cylinder with $l/d = 500$ agrees with W_∞ curve in B region, but it is higher than W_∞ curve in A region. The results which were obtained by Tritton are also higher than this curve in A region. On the other hand, the curve of a cylinder with $l/d = 1$ deviates from those of other cylinders, and crosses W_∞ curve at $Re \approx 50$.

Next, the influence of l/d on C_D is summarized as follows. Imai⁽¹²⁾ analyzed a flow past a cylindrical body at low Reynolds numbers, and obtained the following equation.

$$C_D^{1/2} - C_{D\infty}^{1/2} = k R_e^{-1/2} \quad (1)$$

where, $C_{D\infty}$ is equal to $1/2$ for a circular cylinder.⁽¹³⁾ A following equation which is extended from Eq. (1) is assumed for the drag forces of the cylinders of finite span.

$$C_D^{1/2} - \frac{1}{\sqrt{2}} = \alpha R_e^{-1/2} \quad (2)$$

where, the values of α and β can be decided from the experimental results. Moreover, the region of l/d is divided between $l/d = 2, 5, 10$ and $l/d = 50, 500$, because Okamoto et al.⁽⁹⁾ reported that the former was clearly different from the latter in the influence of the flow passing around the end face of the cylinder. The values of α and β are listed in Table 4. Eq. (4) in this table for the cylinder with $l/d = 1 \times 10^4$ almost a-

Fig. 2 Drag coefficients of a cylinder of finite span in Newtonian fluids

greets with W_{∞} curve. This fact is clear from the result of the cylinder with $l/d=500$ and also the results which were obtained by Tritton. These experimental equations are shown in Fig.2. The differences between these equations and the experimental values are less than 5%.

Fig.3 shows the experimental results for Peo high concentrated solutions. The cylinders with $l/d=2$ were used conveniently, because it was considered that the influence of l/d on the drag force was not so different from that in Newtonian fluids. According to this figure, the elastic effect of Peo high concentrated solutions has little influence on the drag force, because C_D can be expressed only by Re^* . In general, C_D is a function of Re^* and $n^{(4)}$ in non-Newtonian fluids in which the elastic effect can be ignored. However, C_D depends only on Re^* in this experiment, and the equation is as follows.

$$C_D^{1/2} - \frac{1}{\sqrt{2}} = 4.0 Re^{*-1/2.0} \quad (7)$$

l/d	A region	B region
2, 5, 10	$\alpha = 4.4(l/d)^{-0.15}$	$\beta = 1.9(l/d)^{-0.083}$ Eq.(3)
50, 500	$\alpha = 3.0(l/d)^{-0.027}$ $\beta = 2.7$ } Eq.(4)	$\alpha = 1.7$ $\beta = 3.1(l/d)^{-0.054}$ } Eq.(5)
1	$\alpha = 5.0$	$\beta = 2.0$ Eq.(6)

Table 4 The values of α and β in Eq.(2)

Fig. 3 Drag coefficients of a cylinder ($l/d=2$) in Peo high concentrated solutions

Fig. 4 Drag coefficients of a cylinder ($l/d=2$) in Newtonian fluids

Eq.(7) is shown by a solid line in Fig.3. In the case of $n=1$ this equation agrees with Eq.(2) for the cylinder with $l/d=2$.

As is analogized from the experimental results of cylinders with $l/d=2$, the drag coefficients C_D of other cylinders can possibly be arranged by the following equation which is obtained by replacing Re in Eq.(2) with Re^* .

$$C_D^{1/2} - \frac{1}{\sqrt{2}} = \alpha Re^{*-1/\beta} \quad (1 < Re^* < 1 \times 10^2) \quad (8)$$

where, for the values of α and β , Eqs.(3),(4),(5) and (6) for Newtonians are applied to Peo solutions under the assumption that these values are independent of n .

In the next place, the drag forces of cylinders with $l/d=2$ were measured in the Re range of 1×10^3 to 5×10^4 . Figs.4(a) and (b) show the results for water. The results which were obtained by Christiansen et al.⁽⁸⁾ are shown in Fig.4(a). They measured the drag forces of cylinders ($l/d=2.5$) which were dropped into the fluid filled tank. In this figure, the experimental results almost agree with the results which were obtained by them, and C_D depends on the material of the cylinder. They considered that

Fig. 5 Drag coefficients of a cylinder ($l/d=2$) in Peo dilute solutions

Fig. 6 Rate of drag reduction

this phenomenon was caused by rotation and oscillation of cylinders, and they rearranged the experimental values by the following modified equation in which the inertial and rotational moment were introduced.

$$C_D' = C_D \left(\frac{\rho_o - \rho_f}{\rho_f} \right)^{0.05} \quad (9)$$

Fig.4(b) shows the experimental results rearranged by Eq.(9).

Figs.5(a), (b) and (c) show the results for Peo dilute solutions. According to this figure, the drag forces reduce considerably as compared with those in water. The maximum values of the rate of drag reduction $\kappa_{\max}$ are 23%, 32% and 33% in Peo 10, Peo 16 and Peo 18, respectively.

Fig.6 gives the results expressed by a non-dimensional parameter N_{we} , which shows the ratio of the elastic force to the viscous force. It is clear that κ has a maximum value at $N_{we} = 20$, and κ is little affected by the Reynolds number in the range of $Re > 8 \times 10^3$. The solid lines in this figure indicate the experimental equations as follow.

$$\kappa = (9.10 N_{we} + 0.0121 N_{we}^{0.194} Re) \times 10^{-3} \quad (10)$$

$(2 \times 10^3 < Re < 8 \times 10^3, 0.5 < N_{we} < 20)$

$$\kappa = (9.10 N_{we} + 96.8 N_{we}^{0.194}) \times 10^{-3} \quad (11)$$

$(8 \times 10^3 < Re < 4 \times 10^4, 0.5 < N_{we} < 20)$

$$\kappa = 30.0 \times 10^{-3} N_{we}^{-4.50} \quad (12)$$

$(8 \times 10^3 < Re < 4 \times 10^4, N_{we} > 35)$

4.2 Pressure distribution around a cylinder of infinite span

As was stated in section 4.1, the drag forces in Peo dilute solutions reduced considerably as compared with those for water in the range of 1×10^3 to 5×10^4 . From this fact, it was considered that the elastic effect in Peo dilute solutions had much influence on the pressure distributions, and so these distributions were measured in both water and Peo dilute solutions for the cylinder of infinite span. The experiment was made at $Re = 1.7 \times 10^4$, 2.5×10^4 , 4.0×10^4 and 5.0×10^4 . The results for water are shown in Fig.7. The results by Asanuma et al.⁽¹⁵⁾ who

Fig. 7 Pressure distributions around a cylinder of infinite span in water

measured using a manometer at $Re = 2 \times 10^4$ are also shown in this figure. The results at $Re = 2.5 \times 10^4$ agree with the results which were obtained by them except in the neighbourhood of $\theta = 160^\circ$. The separation points (marked s in this figure) which are assumed to be the point of inflection of the pressure distributions almost agree with the results by Son and Hanratty⁽¹⁶⁾. From this fact, it is considered that these experimental results for water are reasonable.

Fig.8 shows some of the experimental results for Peo dilute solutions. The pressure distribution in 30 ppm solution almost agrees with that in water in the whole region of θ . On the other hand, the pressure distribution in 50 ppm solution is considerably different from that in water in the θ region of 70° to 180° . The back pressure rises about 30%, and the separation delays about 10 degrees as compared with that in water. The same results are obtained in another experimental range of Reynolds numbers.

Fig.9 shows the drag coefficients calculated by the equation, $C_D = \int_0^\pi C_F \cos \theta d\theta$. The friction force is ignored. The sign * in this figure indicates the values obtained from the pressure distributions which were measured by Asanuma et al.⁽¹⁵⁾ The calculated values for water increase by 2 to 8% as compared with the results which were obtained by Wieselsberger⁽⁷⁾, but almost agree with the results by Asanuma et al. On the other hand, C_D for 50 or 100 ppm solution reduces considerably as compared with that for water, and the values of the rate of drag reduction κ are from 15 to 21%. But, these values are smaller than those which are obtained in Fig.5 for the cylinders of finite span. This difference will be discussed in section 4.3.

Fig.10 shows some of the results of the periodic pressure fluctuations recorded by an electromagnetic oscillograph. The amplitude of the fluctuation is small at $\theta = 0^\circ$, but the large periodic fluctuations which seem to be caused by the vortices

Fig. 8 Pressure distributions around a cylinder of infinite span in Peo dilute solutions

behind a cylinder occur in the θ region of 60° to 150° . The Strouhal numbers St for these periodic fluctuations are shown in Fig. 9. The results which were obtained by Itaya et al.⁽¹⁷⁾ and Okajima et al.⁽¹⁸⁾ are also shown in this figure. The former obtained the Strouhal numbers for the frequency of the vortices which grew behind a cylinder, and the latter obtained them for the fluctuations of the lift force. In this figure, the results for water almost agree with those which were obtained by Itaya et al. On the other hand, St for 50 or 100 ppm solution increases by 9 to 11% as compared with that for water. Itaya et al. reported that a decrease of C_D produced an increase of St . The same fact can be seen in Fig. 9, namely, C_D for Peo solutions decreases and St increases as compared with those for water, and St increases with the decrease of C_D .

These experimental results make it clear that the decrease of C_D and the increase of St depend on the rise of the back pressure and the delay of the separation.

Fig. 9 Drag coefficients and Strouhal numbers of a cylinder of infinite span

Fig. 10 Periodic pressure fluctuations

4.3 Pressure distribution around a cylinder of finite span

As was stated in section 4.2, the drag force reduced considerably in the case of Peo dilute solution. But, the rate of drag reduction for the cylinder of finite span was larger than that for the cylinder of infinite span. It was considered that this difference was caused by the flow passing around both end faces of the cylinder of finite span. Therefore, the pressure distribution around the cylinder of finite span was measured.

The experiment were made at $Re = 4.0 \times 10^4$ using the cylinders with $h_0/d = 1$ ($l/d = 2$)⁽⁹⁾ and $h_0/d = 2$. Fig. 11 shows some of the results for water. It is found that C_p decreases with a decrease of z/d in the neighbourhood of $\theta = 0^\circ$. Fig. 12 shows an example of the results for Peo dilute solutions. According to this figure, the pressure decreases considerably on the front side of the cylinder as compared with that in water.

Figs. 13 and 14 show the influence of the elastic effect of Peo dilute solutions. The results for water show that the influence of the flow passing around the end face of the cylinder reaches the region of $z/d \leq 0.2$ or 1.0 for the cylinder with $h_0/d = 1$ or 2, respectively (Fig. 13(a)), and both the separation points θ_{sep}

Fig. 11 Pressure distributions around a cylinder of finite span in water

Fig. 12 Pressure distributions around a cylinder of finite span in Peo dilute solutions

and the points of minimum pressure $\theta(C_p=\min)$ move forward with an increase of z/d (Fig.14(a) and (b)). These values almost agree with the results by Okamoto et al.⁽⁶⁾ who measured using a wind tunnel at $Re=1.33 \times 10^4$. In the next place, the results for Peo dilute solutions indicate that the pressure at $\theta=0^\circ$ decreases considerably as compared with that for water and becomes negative at $z/d=0.05$, and the pressure at $\theta=180^\circ$

Fig. 13 Stagnation pressures or minimum pressures

Fig. 14 Angle of minimum pressure or separation

increases over the full length of the cylinder (Fig.13(a)). In addition, these results mean that the minimum pressure $C_{p,\min}$ decreases over the full length of the cylinder (Fig.13(b)), $\theta(C_p=\min)$ moves little except in the neighbourhood of the end face of the cylinder (Fig.14(a)), and θ_{sep} delays considerably over the full length of the cylinder, in particular, the delay is beyond 10 degrees in the neighbourhood of the end face of the cylinder (Fig.14(b)).

The drag coefficient C_D calculated by using the experimental results of the pressure distributions is shown in Table 5. The experimental values which were obtained by Okamoto et al.⁽⁶⁾ and Wieselsberger⁽⁷⁾ are also given in this table. The latter measured the drag forces by setting both end faces of the cylinders into the flow. The results for water almost agree with their values. On the other hand, C_D for Peo dilute solutions decreases considerably as compared with that for water. The rate of drag reduction κ is equal to 34% or 30% for the cylinder with $h_0/d=1$ or 2, respectively. κ for the cylinder with $h_0/d=1$ almost agrees with that for the cylinder with $l/d=2$ which was dropped into the fluid filled tank.

	Re	h_0/d	C_D	
			water	Peo 18 100ppm
Authoers	4.0×10^4	1	0.74	0.49
		2	0.79	0.55
Okamoto et al. ⁽⁶⁾	1.33×10^4	1	0.68	—
		2	0.75	—
Wieselsberger ⁽⁷⁾	8.8×10^4	1($l/d=2$)	0.70	—
		2($l/d=4$)	0.76	—

Table 5 Drag coefficients of a cylinder of finite span

Fig. 15 Local drag reduction at θ or z

Here, the influence of the flow passing around the end face of the cylinder is discussed. The pressure distributions and the local pressure drag reduction of the cylinder of infinite span are compared with those of the cylinders of finite span.

When comparing Fig.8 with Fig. 12-14, the back pressure rises about 30% and the separation delays about 10 degrees in the case of Peo dilute solutions for both cylinders of infinite and finite span, and the pressure in the θ region of 0° to 90° of the cylinders of finite span decrease considerably as compared with that of the cylinder of infinite span. This decrease seems to be caused by the elastic effect which has an influence on the flow passing around the end face of the cylinder. Fig.15 shows the local pressure drag reduction at $Re = 4.0 \times 10^4$. C_d in Fig.15(a) denotes the coefficient of the pressure drag reduction at the angle of θ . For the cylinders of finite span, the mean value of the pressure at the angle of θ is used. κ in Fig.15(b) denotes the rate of pressure drag reduction at the point of κ . According to Fig. 15(a), the drag reduction occurs only in the θ region of 90° to 180° for the cylinder of infinite span, but it occurs in the whole region of θ for the cylinders of finite span. According to Fig.15(b), a large drag reduction occurs in the neighbourhood of the end face of the cylinders. The sign * in this figure indicates the value of the cylinder of infinite span.

These experimental results make it clear that the increase of the rate of drag reduction for the cylinders of finite span happen due to a large pressure drag reduction which occurs in the θ region of 0° to 90° and in the neighbourhood of the end face of the cylinder.

5. Conclusions

The drag forces and the pressure distributions of the cylinder were measured in both Newtonian fluids and polymer solutions, and the following fact were made clear.

- (A) On the drag force of a cylinder
 1. The drag forces of the cylinders of finite span increase in the Re range of 1 to 20, and decrease in the Re range of 20 to 1×10^3 with a decrease of l/d in Newtonian fluids. These results are shown by Eqs.(2) - (6).
 2. The drag forces in Peo high concentrated solutions are little affected by the elastic effect in the Re^* range of 1 to 1×10^2 . This result is shown by Eq.(7).
 3. The drag forces in Peo dilute solutions reduce considerably as compared with those in water in the Re range of 1×10^3 to 5×10^4 . The rate of drag reduction is shown by Eqs.(10),(11) and (12).

(B) On the pressure distribution around a cylinder

1. In the case of the cylinder of infinite span, the back pressure in Peo dilute solutions rises and the separation delays as compared with those in water in the Re range of 1.7×10^4 to 4.0×10^4 .
2. In the case of the cylinders of finite span ($h_0/d = 1, 2$), the pressure in Peo dilute solutions decreases in the θ region of 0° to 90° and in the neighbourhood of the end face of cylinders, the back pressure rises and the separation delays as compared with those in water at $Re = 4 \times 10^4$.

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