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Analysis of Forced Vibration by Reduced Impedance Method*

(Part.4 Proposition and application of multiple
reduced impedance method)By Akio NAGAMATSU**, Masaaki OOKUMA***, Yutaka FUJITA[†],Terutaka IKEUCHI^{††} and Masaaki SHIRAI^{†††}

A multiple reduced impedance method (MRIM) is proposed as an advanced type of the reduced impedance method (RIM) for analyzing the vibration of a mechanical structure so complex that the usual methods, for example the finite element method, can not be applied directly on it. The natural frequency, the natural mode and the dynamic response of the supporting base of a diesel generator package are analyzed by MRIM. The analytical results by MRIM are compared with the analytical results by RIM and the experimental results. These three results agree well with one another. It is clear that the vibration of a complex structure can be analyzed by MRIM with less calculation time and with smaller memory storage than by RIM.

Key words: Vibration, Finite Element Method, Natural Frequency,
Multiple Reduced Impedance Method, Substructure

1. Preface

The reduced impedance method (RIM in brief) is a general method of vibration analysis proposed by the authors for machines and structures so complex that direct use of usual methods, for instance the finite element method (FEM in brief) or the transfer matrix method, is difficult. In this series of research concerning RIM by the authors^{(1)~(6)}, introduction of the method and basic investigation are done in the 1st report, a new technique of curve-fitting used in RIM is proposed in the 2nd report, and a way of analysis of damping vibration by RIM is described in the 3rd report.

In the present report, it is tried to use RIM in multiple degrees and a multiple reduced impedance method (MRIM in brief) is proposed as an advanced type of RIM. MRIM is a general method of analysis of vibration originally intended for so complex structures that application of RIM is difficult, but even for usual structures, the calculation time is less and the memory storage is smaller by MRIM than by ordinary RIM.

The natural frequency, the natural mode and the dynamic response of a model of the supporting base of a diesel generator package are analyzed by MRIM, and the result is compared with the analytical result by ordinary RIM and the experimental result.

2. Method of analysis by MRIM

A total structure is divided into some substructures or parts. Each substructure is divided into some parts of 2nd degree. If necessary, this process of division is repeated, but the dual RIM is given here as a most simple example of MRIM.

A part or substructure of 2nd degree is separated from all other parts, and the following equation of motion is made for this part only by FEM etc.

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + [K]\{\delta\} = \{F\} \quad (1)$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, $\{\delta\}$ is the displacement vector and $\{F\}$ is the force vector. If $\{F\}$ is a harmonic force whose angular frequency is ω , $\{\delta\}$ becomes harmonic, too.

$$\{F\} = \{F_0\}e^{j\omega t} \quad (2)$$

$$\{\delta\} = \{\delta_0\}e^{j\omega t} \quad (3)$$

Substituting Eqs.(2) and (3) into Eq.(1),

$$[A_0]\{\delta_0\} = \{F_0\} \quad (4)$$

$$[A_0] = [K] + j\omega[C] - \omega^2[M] \quad (5)$$

This part is divided into two regions d and a . Region d is a region connective with other parts or substructures, which contains

* Received 13th November, 1981.

** Associate Professor,
Faculty of Engineering,
Tokyo Institute of Technology,
2-12-1 Ohokayama Meguro-ku Tokyo.

*** Graduate Student,
Tokyo Institute of Technology.

[†] Nissan Motor Co.Ltd.

^{††} Manager, Nippon Kohkan Co.Ltd.

^{†††} Nippon Kohkan Co.Ltd.

exciting points and detecting points if they exist. Region a is an internal region except the region d. As no exciting force exists in the region a, Eq.(4) is rewritten as

$$\begin{bmatrix} A_{aa} & A_{ad} \\ A_{da} & A_{dd} \end{bmatrix} \begin{Bmatrix} \delta_{0a} \\ \delta_{0d} \end{Bmatrix} = [A] \begin{Bmatrix} \delta_{0a} \\ \delta_{0d} \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_{0d} \end{Bmatrix} \quad \dots (6)$$

Eliminating $\{\delta_{0a}\}$ from Eq.(6)

$$[Z]\{\delta_{0d}\} = \{F_{0d}\} \quad \dots (7)$$

$$[Z] = [A_{dd} - A_{da}A_{aa}^{-1}A_{ad}] \quad \dots (8)$$

$[Z]$ in Eq.(8) is called a reduced impedance matrix of this part, though strictly speaking it is a dynamic stiffness.

As calculation of $[A_{aa}]^{-1}$ in Eq.(8) is difficult when the freedoms of region a are numerous, the process of Gauss elimination is applied to the upper half of Eq.(6), and we have

$$\begin{bmatrix} A'_{aa} & A'_{ad} \\ 0 & A'_{dd} \end{bmatrix} \begin{Bmatrix} \delta_{0a} \\ \delta_{0d} \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_{0d} \end{Bmatrix} \quad \dots (9)$$

Comparing Eq.(7) with Eq.(9),

$$[A'_{dd}] = [Z] \quad \dots (10)$$

Thus, the reduced impedance matrix $[Z]$ can be made simply without any troublesome calculation of the inverse matrix.

Next process is synthesis of $[Z]$ of all parts. The total structure in Fig.1 with 2 substructures composed of 2 parts of 2nd degree respectively is taken as an example. In this case, region d is divided into regions b and c. In Fig.1, b1 and b2 are regions connecting parts 1 and 2 in substructures (sub. in brief) 1 and 2 respectively, c1 and c2 are regions connecting sub.1 and 2, c3 is an exciting region and c4 is a detecting region.

Fig.1 Total structure composed of 2 substructures, each composed of 2 parts

$[Z]$ of part 1 and part 2 of sub.1 are written as follows respectively.

$$[Z_{11}] = \begin{bmatrix} Z_{b1}^{11} & Z_{b1c1}^{11} & Z_{b1c3}^{11} \\ Z_{c1b1}^{11} & Z_{c1}^{11} & Z_{c1c3}^{11} \\ Z_{c3b1}^{11} & Z_{c3c1}^{11} & Z_{c3}^{11} \end{bmatrix} \quad \dots (11)$$

$$[Z_{12}] = \begin{bmatrix} Z_{b1}^{12} & Z_{b1c2}^{12} \\ Z_{c2b1}^{12} & Z_{c2}^{12} \end{bmatrix} \quad \dots (12)$$

Synthesizing Eqs.(11) and (12), $[Z]$ of

$$[Z_{g1}] = \begin{bmatrix} Z_{b1}^{11} + Z_{b1}^{12} & Z_{b1c1}^{11} & Z_{b1c2}^{12} & Z_{b1c3}^{11} \\ Z_{c1b1}^{11} & Z_{c1}^{11} & 0 & Z_{c1c3}^{11} \\ Z_{c2b1}^{12} & 0 & Z_{c2}^{12} & 0 \\ Z_{c3b1}^{11} & Z_{c3c1}^{11} & 0 & Z_{c3}^{11} \end{bmatrix} \quad \dots (13)$$

Eliminating the terms for region b1 from Eq.(13) by Gauss's process, we have

$$[Z_1] = \begin{bmatrix} Z_{c1}^{11} & Z_{c1c2}^{11} & Z_{c1c3}^{11} \\ Z_{c2c1}^{11} & Z_{c2}^{11} & Z_{c2c3}^{11} \\ Z_{c3c1}^{11} & Z_{c3c2}^{11} & Z_{c3}^{11} \end{bmatrix} \quad \dots (14)$$

Similarly for sub.2,

$$[Z_2] = \begin{bmatrix} Z_{c1}^{12} & Z_{c1c2}^{12} & Z_{c1c4}^{12} \\ Z_{c2c1}^{12} & Z_{c2}^{12} & Z_{c2c4}^{12} \\ Z_{c4c1}^{12} & Z_{c4c2}^{12} & Z_{c4}^{12} \end{bmatrix} \quad \dots (15)$$

Synthesizing Eqs.(14) and (15),

$$[Z_g] = \begin{bmatrix} Z_{c1}^{11} + Z_{c1}^{12} & Z_{c1c2}^{11} + Z_{c1c2}^{12} & Z_{c1c3}^{11} & Z_{c1c4}^{12} \\ Z_{c2c1}^{11} + Z_{c2c1}^{12} & Z_{c2}^{11} + Z_{c2}^{12} & Z_{c2c3}^{11} & Z_{c2c4}^{12} \\ Z_{c3c1}^{11} & Z_{c3c2}^{11} & Z_{c3}^{11} & 0 \\ Z_{c4c1}^{12} & Z_{c4c2}^{12} & 0 & Z_{c4}^{12} \end{bmatrix} \quad \dots (16)$$

Eliminating the terms for regions c1 and c2 from Eq.(16) by Gauss's process,

$$[Z_c] \begin{Bmatrix} \delta_{0c3} \\ \delta_{0c4} \end{Bmatrix} = \begin{Bmatrix} F_{0e} \\ 0 \end{Bmatrix} \quad \dots (17)$$

Inverting Eq.(17),

$$\begin{Bmatrix} \delta_{0c3} \\ \delta_{0c4} \end{Bmatrix} = [Z_c]^{-1} \begin{Bmatrix} F_{0e} \\ 0 \end{Bmatrix} \quad \dots (18)$$

The complex amplitude of vibration $\{\delta_{0c4}\}$ at region c4 is obtained from Eq.(18).

3. Application

3.1 Rectangular plate

A rectangular steel plate with one fixed side shown in Fig.2 is divided into 2 substructures, and each substructure is divided again into 4 parts. The calculated result by MRIM is compared with the experimental one in Table 1, and both results are in agreement with each other within an error of 6.9%. Figs.3 and 4 show the compliances of this plate. The two analytical results by MRIM and RIM agree perfectly with each

other, and they are shown as the solid lines in Figs.3 and 4. The dotted lines in Figs.3 and 4 are the experimental compliances, and agree with the analytical ones.

The calculation time necessary for compliance of one frequency point is 4.7 seconds by RIM and 4.3 seconds by MRIM. The necessary memory storage is 400KB by RIM and 200KB by MRIM.

Fig.2 Rectangular plate with one fixed side

Table 1 Natural frequency of plate

order	MRIM (Hz)	Experiment (Hz)	error (%)
1	11	11	0.0
2	50	51	-2.0
3	66	62	6.5
4	161	157	2.5
5	185	173	6.9

Fig.3 Compliance of plate between F and A

Fig.4 Compliance of plate between F and B

3.2 Model structure

The 1/5 reduced model of the supporting base of a diesel generator package shown in Fig.5 is divided into 4 subs., and each sub. is divided again into 7 parts, as shown in Figs.6 and 7. In Fig.6, P1 is the exciting point and P1 and P2 are the detecting points. The natural frequency, the natural modes and the damping response are calculated by real MRIM and by real RIM⁽⁴⁾. The experimental values of the modal damping ratio adopted in these analyses are written in Table 2. The natural frequency calculated by MRIM is compared with the experimental one in Table 3. The natural frequency calculated by RIM agrees perfectly with the one by MRIM. In Figs.8 and 9, the damping responses calculated by MRIM are compared with the experimental ones. The damping responses calculated by RIM agree perfectly with the ones by MRIM.

The calculation time necessary for compliance at one frequency point is 27 seconds by RIM and 17 seconds by MRIM. The necessary memory storage is 1400KB by RIM and 300KB by MRIM. Thus, MRIM is more advantageous than RIM in both the calculation time and the memory storage.

Fig.5 1/5 reduced model of supporting base (unit is mm)

Fig.6 Division of supporting base into 4 substructures and parts

Fig.7 Finite element division of each part of supporting base

Table 2 Modal damping ratio ζ_n of supporting base

order	ζ_n
1	0.0043
2	0.1000
3	0.0031
4	0.0043

Table 3 Natural frequency of supporting base

order	MRIM (Hz)	Experiment (Hz)	error (%)
1	64	71	-9.9
2	108	120	-10.0
3	335	298	12.4
4	386	353	9.3

Fig.8 Point compliance of supporting base of P1

Fig.9 Compliance of supporting base between P1 and P2

4. Conclusions

Multiple reduced impedance method is proposed as a general method of vibration analysis. The natural frequency, the natural mode and the response of a rectangular plate and a model of the supporting base of a diesel generator package are calculated by MRIM. The result by MRIM is compared with the analytical result by ordinary RIM and the experimental result. Accuracy of calculation by MRIM is the same as by ordinary RIM, and MRIM is more advantageous than ordinary RIM in both the calculation time and the memory storage.

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