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A Simple Method to Calculate the Force Ratio of Bolted Joints*

(The case where clamped parts are T-flanges)

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In designing a bolted joint, it is important to estimate the ratio of a bolt axial force to a load when a load is applied on an assembly, that is the force ratio. In this paper, in order to establish a simple method to calculate the force ratio of general bolted joints, the force ratio of T-flange is discussed. At first, in order to analyse more exactly, the T-flange is replaced by a finite strip. In the next place, in order to calculate simply, the T-flange is replaced by a beam on an elastic foundation. Moreover a bending moment is introduced in the bolt when the load is applied. In order to analyse this bending moment, a calculation method is proposed by defining newly the bending compliance of T-flanges and bolts. Then, to verify these analyses, experiments are carried out. The analyses are in a fairly good agreement with the experiments.

Key Words: Fixing Element, Machine Element, Bolt,
Force Ratio, T-flange, Structural analysis

1. Introduction

The fastening with bolts has been usually used in the mechanical industry and in many other kinds of industries, but the design of bolted joints mainly depends on experiences. Recently, in view of the tendency toward more precise and lighter machines, the prevention of accidents and the tendency toward the computer aided design of bolted joints with the development of electronics computers, it is necessary to make the design of bolted joints more rational and systematic. In designing the bolted joints it is necessary to estimate the force ratio (ratio of an increment of a bolt axial force to a load) more precisely^{(1)~(3)}. This force ratio has been calculated by the formula proposed by Thum⁽⁴⁾ using the spring constant K_c for the bolt-nut system and the compressive spring constant K_t for clamped parts. Recently, it has been clarified that the value by Thum's formula is greater than the actual value and Junker⁽⁵⁾ has proposed that the value obtained by Thum's formula be corrected. But the proposition by Junker mainly depends on experiences and the theoretical ground is not clear.

In a series of investigations on the force ratio of bolted joints, first of all, in the case where two hollow cylinders were

clamped, by defining newly the tensile spring constant K_{pt} for clamped parts besides the compressive spring constant K_c , a calculation method, which was able to decide the force ratio more precisely, was proposed and it was shown that the value of K_c/K_{pt} corresponded to the correction factor proposed by Junker. Moreover, applying this method on the force ratio of connecting rods and flanged connections, which were important examples of bolted joints and analysing the correction factor K_c/K_{pt} , it was shown that numerical results were in a fairly good agreement with experimental ones. But these bolted joints are examples of mechanical structures and the methods of the analyses are not applied to other joints. Furthermore, since the analyses are pretty complex, they can not meet the tendency toward the computer aided design of bolted joints.

In practice, it is common that axial loads act on bolted joints and as a result, an increment of bolt axial force and a bending moment are simultaneously produced in bolts, namely bolted joints are subjected to an eccentric load. In order to establish the calculation method for the force ratio of general bolted joints, it is necessary to discuss these eccentrically loaded bolted joints. From these viewpoints investigations have been made by Kowalskie⁽⁹⁾, Agatonovic⁽¹⁰⁾ and Weber⁽¹¹⁾, but there are some problems.

In this paper, as a basic study to establish a more simple and precise calculation method for the force ratio of common bolted joints, discussion is made on the force ratio of T-flanges, which is an example of eccentrically loaded joints, by analysing the correction factor K_c/K_{pt} . For this purpose, first of all, replacing T-flanges by a finite strip (two-dimensional model) and analysing the correction factor

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K_c/K_{pt} , the force ratio is obtained. In the next place, for simplicity, by replacing T-flanges with a beam on an elastic foundation (beam model), the method to analyse the force ratio is shown in the same way mentioned above. In addition, a bending moment is produced in bolts with an application of the load and the stress due to the bending moment is added to the stress due to the increment of the bolt axial force. In order to obtain the bending moment, the bending compliance for clamped parts and for bolts are defined newly, then a calculation method for the bending moment produced in bolts is proposed. In order to verify the above theories, experiments are carried out. Then, comparisons and discussions are made.

2. Theoretical Analysis

Let's consider a bolted joint in which two hollow cylinders are fastened with an initial clamping force F_i shown in Fig.1(a). When the load W is applied on the bolted joint as shown in Fig.1(b), an increment of the bolt axial force F_t is produced and the clamping force F_c is lost from the interface of clamped parts. In order to find the values F_t and F_c for the load W , in the reference (6) denoting the spring constant for the bolt-nut system by K_t and the compressive spring constant for clamped parts by K_c , the tensile spring constant K_{pt} for clamped parts is defined by $W/(2\delta_2)$ by using the mean displacement of the bearing surface when the force acts on the hollow cylinder as shown in Fig.2(a) and (b). By using these spring constants, the ratio of the increment F_t of the bolt axial force to the load W , that is the force ratio is given by the following equation.

$$\phi = \frac{F_t}{W} = \frac{K_t}{K_t + K_c} \left(\frac{K_c}{K_{pt}} \right) \quad (1)$$

The force ratio proposed by Thum is expressed as follows.

$$\phi_0 = \frac{K_t}{K_t + K_c} \quad (2)$$

Accordingly, the value of K_c/K_{pt} corresponds

Fig.1 Bolted joints in the case where two hollow cylinders are fastened

Fig.2 Definition of the tensile spring constant K_{pt}

Fig.3 Bolted joints in the case where T-flanges are fastened

Fig.4 Two dimensional model of T-flanges

to the correction factor proposed by Junker⁽¹¹⁾. Since it is very difficult to analyse the force ratio of bolted joints in practice, in this paper by analysing the correction factor K_t/K_{pt} the method to calculate the force ratio of T-flanges is adopted.

2.1 Analysis of Force Ratio

The load W is applied on T-flanges fastened by two row bolts with an initial clamping force F_t as shown in Fig.3. As a result, the increment of the bolt axial force F_t is produced in the bolts. The force ratio ϕ of this joint is given by the equation $\phi = F_t/(W/2)$. In this paper, the force ratio is discussed in the case where the interface is not separated.

2.1.1 Two dimensional mode

For the case of Fig.4(a)

$$\left. \begin{aligned} x = \pm l: \sigma_x = \tau_{xy} = 0 \\ y = \pm h: \tau_{xy} = 0, \quad \sigma_y = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x \end{aligned} \right\} \dots\dots\dots (3)$$

Where

$$a_0 = \frac{b}{l} P_1, \quad a_n = \frac{2P_1}{n\pi} \sin \frac{n\pi}{l} b$$

For the case of Fig. 4(b)

$$\left. \begin{aligned} x = \pm l: \sigma_x = \tau_{xy} = 0 \\ y = \pm h: \tau_{xy} = 0, \quad \sigma_y = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{l} x \end{aligned} \right\} \dots\dots\dots (4)$$

Where

$$a_0 = \frac{2a}{l} P_2, \quad a_n = \frac{2P_2}{n\pi} \left[\sin \left\{ \frac{(c+a)}{l} n\pi \right\} - \sin \left\{ \frac{(c-a)}{l} n\pi \right\} \right]$$

In the analysis of the finite strip having the boundary conditions Eqs.(3) and (4), Airy's stress function χ is used. The stress and the displacements are expressed by the following equations. Where the displacement in x direction is denoted by u and in y direction by v .

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \chi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \dots\dots\dots (5)$$

$$2Gu = -\frac{\partial \chi}{\partial x} + \frac{1}{1+\nu} \frac{\partial \varphi}{\partial y}, \quad 2Gv = -\frac{\partial \chi}{\partial y} + \frac{1}{1+\nu} \frac{\partial \varphi}{\partial x} \dots\dots\dots (6)$$

Where

$$\nabla^4 \chi = 0 \left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right), \quad \nabla^2 \varphi = 0, \quad \nabla^2 \chi = \frac{\partial^2 \varphi}{\partial x \partial y}$$

Where ν is Poisson's ratio, G is modulus of shear. Considering the boundary conditions, χ is taken as follows. Where A_0 , $\bar{A}_n$ and $\bar{B}_s$ are unknown coefficients.

$$\begin{aligned} \chi = & \frac{1}{2} A_0 x^2 + \sum_{n=1}^{\infty} \frac{\bar{A}_n}{\alpha_n^2 \Delta_n} [\{ \sinh(\alpha_n l) + \alpha_n l \cosh(\alpha_n l) \} \cosh(\alpha_n x) \\ & - \sinh(\alpha_n l) \alpha_n x \sinh(\alpha_n x)] \cos(\alpha_n y) + \sum_{s=1}^{\infty} \frac{\bar{B}_s}{\lambda_s^2 Q_s} [\{ \sinh(\lambda_s h) \\ & + \lambda_s h \cosh(\lambda_s h) \} \cosh(\lambda_s y) - \sinh(\lambda_s h) \lambda_s y \sinh(\lambda_s y)] \cos(\lambda_s x) \dots\dots\dots (7) \end{aligned}$$

Where

$$\alpha_n = n\pi/h, \quad \lambda_s = s\pi/l (s=1, 2, 3, \dots), \quad \Delta_n = \cosh(\alpha_n l) \sinh(\alpha_n l) + \alpha_n l, \quad Q_s = \cosh(\lambda_s h) \sinh(\lambda_s h) + \lambda_s h$$

Substituting Eq.(7) into Eq.(5), the stresses are obtained. Then, equating these stresses to Eqs.(3) and (4), the following equations are obtained

$$\bar{A}_n + \sum_{s=1}^{\infty} P_{ns} \bar{B}_s = 0, \quad \sum_{n=1}^{\infty} Q_{ns} \bar{A}_n + \bar{B}_s = a_s, \quad A_0 = a_0 \dots\dots\dots (8)$$

Where

$$P_{ns} = \frac{(-1)^{s+n} 4\lambda_s h \sinh^2(\lambda_s h) n^2 \pi^2}{Q_s \{ (n\pi)^2 + (\lambda_s h)^2 \}^2}, \quad Q_{ns} = \frac{(-1)^{s+n} 4\alpha_n l \sinh^2(\alpha_n l) s^2 \pi^2}{\Delta_n \{ (s\pi)^2 + (\alpha_n l)^2 \}^2}$$

By using the coefficients A_0 , $\bar{A}_n$ and $\bar{B}_s$ determined by Eq.(8) and substituting Eq.(7) into Eq.(6), displacements are obtained. Moreover, by using these displacements the spring constants K_{pt} and K_t' are obtained.

T-flanges shown in Fig.3 are replaced by the finite strip shown in Fig.4. The tensile spring constant K_{pt} in the correction factor K_t/K_{pt} is obtained by the equation $K_{pt} = (W/2)/(2\epsilon_2)$ using the mean displacement ϵ_2 when the load W acts uniformly as shown in Fig.4(a). The compressive spring constant K_t in K_t/K_{pt} is rewritten by K_t' and obtained by the equation $K_t' = F_t/(2\epsilon_1)$ using the mean displacement ϵ_1 on the bearing surface when the force F_t acts uniformly on the bearing surface as shown in Fig.4(b). The correction factor is hereinafter referred to as K_t'/K_{pt} . The value of K_t in $K_t/(K_t + K_t')$ in Eq.(1) is obtained by using the formula proposed by Shibahara⁽¹²⁾. The value of K_t is taken from the reference (13).

The analyses of K_{pt} and K_t' shown in Fig.4 are done by the two dimensional theory of elasticity. By developing the boundary conditions of Fig.4(a) and (b), the following Eqs.(3) and (4) are obtained.

2.1.2 Beam model

In the next place, replacing T-flanges by the beam on the elastic foundation shown in Fig.5, it is tried to simplify the analysis of the correction factor K_c'/K_{pt} . The displacement at the bolt axis is denoted by $\varepsilon_1-\varepsilon_2$ when the load W and the force P shown in Fig.5(a) are applied. As a result, the value of K_{pt} is obtained by the equation $K_{pt}=(W/2)/(\varepsilon_1-\varepsilon_2)$ and the value of K_c' by the equation $K_c'=P/(2\varepsilon_2)$. The values of K_{pt} and K_c' are obtained by the following equations, respectively in the case where the interface is not separated.

$$K_{pt} = \frac{k}{2\beta} \frac{1}{e^{-\beta C}(\cos \beta C + \sin \beta C)} \quad \dots\dots\dots (9)$$

$$K_c' = \frac{k}{\beta} \frac{1}{1 + e^{-2\beta C}(\cos 2\beta C + \sin 2\beta C)} \quad \dots\dots\dots (10)$$

Therefore, the correction factor K_c'/K_{pt} is expressed as follows.

$$K_c'/K_{pt} = \frac{2e^{-\beta C}(\cos \beta C + \sin \beta C)}{1 + e^{-2\beta C}(\cos 2\beta C + \sin 2\beta C)} = 2e^{-\beta C}(\cos \beta C + \sin \beta C) \quad \dots\dots\dots (11)$$

Where

$$\beta = \sqrt{\frac{k}{4EI_x}} \quad \dots\dots\dots (12)$$

Where E : Young's modulus of the beam
 I_x : second moment of area of the beam
 k : modulus of the foundation

2.2 Analysis of Bending Moment Produced in Bolt

2.2.1 Two dimensional model

When the load W is applied, the bearing surface is inclined by θ_p shown in Fig.6(a) and as a result, the bending moment M_b is produced in the bolts. In order to obtain the moment M_b for the load W , the bending

compliance γ_p of the clamped part with the application of the load is defined by the equation $\gamma_p = (W/2)/\theta_p$. Moreover, the reaction moment M_b acts on the bearing surface as shown in Fig.6(b). Therefore, the bending compliance γ_c of the clamped part is defined by the equation $\gamma_c = M_b/\theta_c$ using the inclination θ_c of the bearing surface. By using the inclination θ_c of the bearing surface due to the bending moment M_b shown in Fig.6(c), the bending compliance γ_c of bolts is defined by the equation $\gamma_c = M_b/\theta_c$. Where, in the calculation of θ_c , the shank of the bolt is replaced by a beam having a length $2h$. The bolt head is replaced by a circular plate and analysed by the theory of plates⁽¹⁵⁾. Then, taking the inclination of the bolt head into consideration, this inclination is added to that of the shank of the bolt. By defining bending compliances as above mentioned, Eq.(13) is obtained from the condition that the inclination of the bearing surface of the bolt equals to that of clamped parts. From Eq.(13), Eq.(14) is obtained.

$$\frac{W/2}{\gamma_p} - \frac{M_b}{\gamma_c} = \frac{M_b}{\gamma_b} \quad \dots\dots\dots (13)$$

$$\frac{M_b}{W/2} = \frac{\gamma_b}{\gamma_b + \gamma_c} \left(\frac{\gamma_c}{\gamma_p} \right) \quad \dots\dots\dots (14)$$

Accordingly, when bending compliances are obtained, the bending moment produced in bolts is obtained from Eq.(14). Denoting

Fig.5 Beam model of T-flanges

Fig.6 Analysis of bending moment produced in bolts by two dimensional model

Fig.7 Analysis of bending moment produced in bolts by beam model

the value of Eq.(14) by ϕ , the maximum increment of the stress produced in the bolts when the load is applied is expressed as follows.

$$\sigma_{tmax} = \left(\frac{\phi}{A} + \frac{\phi}{z} \right) \left(\frac{W}{2} \right) \dots (15)$$

Where A : cross sectional area of bolt
 z : modulus of section of bolt

2.2.2 Beam model

The analysis of the bending moment by the beam model is carried out by analysing the bending compliances γ_p and γ_c in the case of Fig.7. Namely, Fig.7(a) shows the method to obtain the bending compliance γ_p for clamped parts. By eliminating θ'_i in Fig.5(b), γ_p is expressed by Eq.(16).

$$\gamma_p = \frac{k}{\beta^2} \frac{e^{\beta C}}{\sin(\beta C)} \dots (16)$$

Then, the bending compliance γ_c shown in Fig.7(b) is expressed by Eq.(17).

$$\gamma_c = \frac{k}{\beta^3} \frac{1}{1 + e^{-2\beta C} (\sin 2\beta C - \cos 2\beta C)} \approx \frac{k}{\beta^3} \dots (17)$$

2.3 Estimation of load when interface starts to separate

With an increasing of the load W , the interface stress decreases and when the load reaches a specific value, the interface starts to separate from the origin 0 shown in Fig.4. When bolted joints in practice in which the interface is separated are subjected to repeated loads, threads of bolts easily get loose and joints are likely to become a source of leakage accidents. Therefore, a separation of the interface is not desirable for bolted joints. In designing bolted joints it is important to estimate the load W for the initial clamping force F_i when the interface starts to separate. Then, in the case of the two dimensional model by analysing the cases of Fig.4(a) and (b) and by superposing the interface stresses, the load when the interface starts to separate is determined such that the stress at the origin 0 is zero.

3 Experimental Method

Figure 8 shows T-flanges and the bolt used in experiments. In order to examine the effect of the distance C from the center of T-flanges to that of the bolts on the force ratio, three kinds of T-flanges, in which C is 27.5, 35.0 and 42.5mm, are prepared. Figure 8(b) shows a bolt by which T-flanges are joined and on the shank of the bolt two strain gauges are glued at the positions different in phase by 180 degrees. The force ratio is measured by 4-gauge method in which two dummy gauges are used, and the maximum stress produced in the bolt is measured by 2-gauge method, where in strain gauges glued on the shank are previously calibrated. In order to apply the load on T-flanges, after clamping two

Fig.8 Dimensions of T-flanges and bolts used in experiments

Fig.9 Sketch of experimental apparatus

T-flanges by two bolts, the apparatus with attachments is assembled as shown in Fig.9. Setting this apparatus on a material testing machine and applying a compression, a tension is caused on the T-flange connection. The load is picked up by a load cell put on the apparatus. The output signals of strain gauges glued on the bolt shaft and the load cell are recorded by an X-Y recorder through dynamic strain amplifiers.

4 Comparisons of Analytical Results with Experimental Ones and Discussions

4.1 Comparisons of analytical results with experimental ones

Numerical computations are carried out for the case of a two dimensional model. Corresponding to experiments, dimensions are taken as $l=60\text{mm}$, $h=30\text{mm}$, $b=9\text{mm}$, $2a=24\text{mm}$,

$c=27, 5, 35.0$ and 42.5 mm. The number of series in Eq. (7) is 30 terms, Poisson's ratio ν is 0.3, and Young's modulus E is 206 GPa. Analytical results for the case of Fig. 4 are shown in Figs. 10 and 11. From Fig. 10(a), the tensile spring constant K_{pt} is obtained for various values of C . From Fig. 11(a), the compressive spring constant K_c is obtained for various values of C . The value of K_c is much the same regardless of the value of C . By superposing Fig. 10(b) and Fig. 11(b), which show the interface stress distribution, the load when the interface starts to separate can be estimated. In Fig. 11(b), however, in the case where C is 42.5 mm, it is shown that a part of the interface is already separated in the initial clamping. As mentioned above, the correction factor K_c'/K_{pt} is obtained and the load when the interface starts to separate is estimated for various values of C . By using the correction factor K_c'/K_{pt} , the force ratio is obtained. Now, the value of K_c in $K_c/(K_c+K_{pt})$ in Eq. (1) is calculated by Shibahara's formula⁽¹²⁾. In this calculation, clamped parts are replaced by a

finite hollow cylinder the outer diameter of which is equated to the width of T-flanges used in experiments, that is 30 mm. The value of K_c is calculated by the reference (13). As a result, the value of K_c is obtained as $444.9 \text{ N}/\mu\text{m}$, K_c as $1419 \text{ N}/\mu\text{m}$. The force ratio Φ_0 by Thum's formula is obtained as 0.2386.

In the next place, the correction factor K_c'/K_{pt} in the case of the beam model is calculated. The modulus of the foundation k in β of Eq. (12) is obtained as follows; Denoting the spring constant for the unit area by k_0 and the width of T-flanges by d' , the value of k is obtained by the equation $k=k_0 d'$. In the beam theory the displacement at the neutral axis is used, therefore in the calculation of k_0 , by replacing clamped parts by a hollow cylinder in which half of the height equals to $h/2$, the spring constant K_c'' is calculated⁽¹²⁾. Since the spring constant of the beam on the elastic foundation equals to $2K_c''$, the value of k_0 is calculated by $2K_c''/A$. Where, A is the area of the bearing surface calculated by the equation $A=(\pi/4)(2d^2-17d^2)$, $d'=30$ mm. As a result, we have $k=836.8 \text{ kN}/\text{mm} \cdot 1/\text{mm}$, $\beta=0.06229 \text{ 1}/\text{mm}$. Using the above values, the correction factor is obtained by Eq. (11). Table 1 shows the correction factor K_c'/K_{pt} and the force ratio obtained by the two dimensional model and the beam model. In the case where C is 35.0 mm, the value of K_c'/K_{pt} by the beam model is smaller than that by the two dimensional model but they are in a fairly good agreement.

Figure 12 shows the comparisons of the numerical results with the experimental ones with respect to the force ratio. Fig. 12(a) is the case of $C=27.5$ mm, (b) is the case of $C=35.0$ mm. The abscissa is the load $W/2$ and the ordinate is the bolt axial force F_f+F_t . The initial clamping force F_f is 19.6 kN

Table 1. Comparisons of values of K_c'/K_{pt} and Φ obtained by the two dimensional model with those obtained by the beam model

C		27.5	35.0
K_c'/K_{pt}	Two-Dim	0.256 6	0.077 7
	Beam	0.305 8	0.055 9
Φ	Two-Dim	0.061 2	0.018 5
	Beam	0.072 9	0.013 3
	Thum	0.238 6	0.238 6

(a) displacement ($y=h$)

(b) stress distribution on interface ($y=0$)

Fig. 10 Analytical result for the case of Fig. 4(a)

(a) displacement ($y=h$)

(b) stress distribution on interface ($y=0$)

Fig. 11 Analytical result for the case of Fig. 4(b)

(2000kgf). With an increase of the load, the increment of the bolt axial force by the experiments is increased linearly, then become a curved line when the interface starts to separate. The inclination of the straight line indicates the force ratio. The value of the force ratio by experiments is 0.043 for the case of $C=27.5\text{mm}$, and 0.013 for the case of $C=35.0\text{mm}$. Comparing the numerical values with the experimental ones, they are in a fairly good agreement. It is shown that the value obtained by Thum's formula is much greater than that by experimental ones, with an increase of C the force ratio becomes small and that the load when the interface starts to separate also becomes small. As mentioned above, it is thought that by the analysis of the beam model the force ratio can be fairly well estimated.

Furthermore, by the method mentioned in the section 2.2, the bending moment M , produced in bolts is obtained and taking this moment into consideration, comparisons of the maximum and the minimum stresses, which are produced in the bolts, obtained by Eq.(15) with the experimental ones are shown in Fig.13. The abscissa is the load $W/2$ and the ordinate is the stress. The solid line shows the experimental values and the initial clamping force F_i is 19.6 kN (2000kgf). Comparing these results, the numerical results are in a fairly good agreement with the experimental ones. Therefore, it is thought that by the beam model the maximum stress can be fairly well estimated.

4.2 Effects of distance from the center of T-flanges to the bolt axis and the width of T-flanges on the force ratio

(a) case of $c=27.5\text{mm}$

(b) case of $c=35.0\text{mm}$

Fig.12 Comparisons of numerical value with experimental ones with respect to force ratio

Figure 14 shows the effects of the distance from the center of T-flanges used in experiments to the bolt axis on the correction factor K'_c/K_{pt} . It is shown that the value of K'_c/K_{pt} rapidly decreases with an increase of C . When the value of C is large, it is shown that the value of K'_c/K_{pt} obtained by the beam model is smaller than that by the two dimensional model and that for the most part the value obtained by the two dimensional model is slightly larger than that by the beam model. But it can be said that by a simple calculation the force ratio is fairly well estimated.

In the next place, in order to examine the effects of the width of T-flanges on the force ratio, for the T-flanges of which the width is 48mm and 24mm keeping the value of C constant, experiments are carried out. The measured value of the force ratio is 0.038 for the width 48mm, and 0.062 for the

(a) case of $c=27.5\text{mm}$

(b) case of $c=35.0\text{mm}$

Fig.13 Comparison of stress taking bending moment into consideration

Fig.14 Effect of distance C from center of T-flanges to bolt axis on K'_c/K_{pt}

Table 2. Effect of the width of T-flanges on the force ratio (the case where c is 27.5mm)

	Width	24	48
	Two-Dim	0.093 6	0.039 0
ϕ	Beam	0.111	0.046 4
	EXP	0.062	0.038

Fig.15 Assumption of load distribution (two dimensional model)

width 24mm. In the calculation, the effect of the width appears in the calculation of K_s in $K_t/(K_t+K_s)$ of Eq.(1). In the calculation of K_s , taking the outer diameter of the finite hollow cylinder as the width of T-flanges the same as in the section 4.1, the value of K_s is obtained as 2.48 kN/ μ m for the width 48mm, and 0.773kN/ μ m for the width 24mm. By using the correction factor K_t'/K_{pt} in the case of $C=27.5$ in Table.1, the force ratio is obtained and shown in Table 2. Comparing the numerical values with the experimental ones, they are in a fairly good agreement. This fact implies that the effect of the width of T-flanges does not depend on the correction factor but on the calculation of K_s .

In the analysis of K_{pt} by the two dimensional model, the load is assumed to be uniform distributed force shown in Fig.4(a), but in T-flanges {Fig.8(a)}, there are holes. In order to examine the effect of the holes, replacing the load distribution by a linear distribution shown in Fig.15 and analysing the value of K_{pt} , the correction factor K_t'/K_{pt} is obtained as 0.2775 for the case where C is 27.5mm. The value of K_t'/K_{pt} is 0.2566 for the case of the uniform distribution, which is slightly larger than that for the case of the linear. As a result, the force ratio is obtained as 0.0662 (for the linear) and 0.0612 (for the uniform).

5. Conclusions

In this paper, as a basic investigation to establish a more precise and simpler calculation method for the force ratio of bolted joints, discussion on the force ratio is made in the case where clamped parts are T-flanges joined by two row bolts with the following results obtained.

(1) Replacing T-flanges by a finite strip (two dimensional model) and analysing the ratio of the compressive spring constant K_t' to the tensile spring constant K_{pt} , the method to obtain the force ratio is demonstrated.

(2) In order to establish a simple method to obtain the force ratio, replacing T-flanges by a beam on an elastic foundation (beam model) and analysing the correction factor K_t'/K_{pt} based on the theory of the beam on the elastic foundation, the method is also demonstrated.

(3) In order to analyse the increment of the stress produced in bolts by the bending moment with the application of the load, defining newly the bending compliance for bolts and for clamped parts, a calculation method is proposed. Moreover, based on this method, the methods of analyses by the two dimensional model and by the beam model are shown.

(4) In order to verify the results obtained by (1), (2) and (3), experiments are carried out. Comparing these results, it is shown that the numerical results are in a fairly good agreement with the experimental ones. As a result, it is shown that the force ratio and the stress produced in bolts can be fairly well estimated by the calculation method of the beam model.

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