532.582:533.69.048

Paper No. 215-10

The Aerodynamic Characteristics at the Mid-span of a Circular Cylinder with Tangential Blowing*

By Ryoji WAKA**, Fumio YOSHINO***,
Tsutomu HAYASHI****and Tadashi IWASA*****

Three different methods were devised to determine the induced angle of attack as a function of lift coefficient at the mid-span of a circular cylinder with tangential blowing immersed in a uniform flow. The characteristic values such as aerodynamic coefficients of the cylinder were corrected to give two-dimensional ones by making use of the induced angle of attack thus obtained. The circumferential static pressure distribution on the surface of the cylinder was also calculated by modified Parkinson-Jandali's method to apply to an asymmetric flow around the cylinder with blowing.

Key Words : Fluid Mechanics, Boundary Layer Control, Induced Angle of Attack, Wing Characteristics, Circular Cylinder, Tangential Blowing, Bluff Body, Potential Flow.

1. Introduction

It is important to investigate a two-dimensional circular cylinder with tangential blowing in a uniform flow, because it offers the basic data on the industrial problems such as a high lift device, a film cooling, improvement of efficiency of a diffuser and so on. In case of a circular cylinder, there are also many features that the model is easily made, the angular location of the slot can be varied at will. Therefore a considerable number of investigations on the cylinder with tangential blowing have been carried out⁽¹⁾⁻⁽⁴⁾. On the other hand, according to author's experience, it is difficult to make an aspect ratio of the cylinder large enough, while a roll up of a strong trailing vortex takes place near the side-wall. As a result of this, it was found that the induced velocity became large unexpectedly at the mid-span.

Therefore the investigations on the whole flow field including not only the spanwise distributions of the characteristic values but also the flow itself in the test section are necessary to know the two-dimensional characteristic values at the mid-span. And the spanwise distributions of various characteristic values, the spanwise range of effects of the side-wall, the relation between this range and the angular location of the slot and the vortex system near the side-wall were already published in our reports⁽⁵⁾⁽⁶⁾. The authors stated

in these papers that there were two induced velocities caused by two kinds of vortices, one of which was a trailing shed-vortex due to the nonuniformity of the spanwise circulation distribution, the other being a trailing filament-vortex due to the boundary layer on the side-wall. If the boundary layer was thinner enough than the span of the cylinder, the induced velocity due to the latter was negligibly small in comparison with the one due to the former at the mid-span. Hence the induced velocity only due to the trailing shed-vortex was exclusively considered at the mid-span.

However, any discussion on the induced angle of attack and the method of determination of it has not been made sufficiently. As a broad wake in general accompanies a flow around a circular cylinder, there are neither the known "true" two-dimensional characteristics nor the Kutta condition in case of the cylinder. Hence it is difficult to determine the induced angle of attack simply. Moreover there are few investigations, as far as the authors know, on the two-dimensional characteristic values corrected for the effect of the induced angle of attack for the cylinder with blowing.

From the viewpoints mentioned above, experiments on the characteristic values of the cylinder were carried out at the mid-span for variables like the aspect ratio, the location of the slot and the jet intensity. Three simpler methods based on the experimental results were devised to determine the induced angle of attack, and some examples of the two-dimensional characteristic values corrected for the effect of it were given at the mid-span. Moreover the circumferential static pressure distribution was calculated by a modified Parkinson-Jandali's method which was extended to the asymmetric flow around the cylinder.

2. Nomenclature

C_d : section drag coefficient

* Received 5th January, 1981.

** Research Assistant, Faculty of Engineering, Tottori Univ., Koyama, Tottori.

*** Professor, Faculty of Engineering, Tottori Univ., Koyama, Tottori.

**** Associate Professor, Faculty of Engineering, Tottori Univ., Koyama, Tottori.

***** Isuzu Electric Co., Ltd., Amagasaki.

C_{di} : section induced drag coefficient
 C_{d0} : section profile drag coefficient
 C_l : section lift coefficient
 C_μ : (momentum of the jet per unit span)/
 $\{(1/2)\rho U_\infty^2 D\}$
 D : diameter of the circular cylinder
 Re : Reynolds number ($= U_\infty D/\nu$)
 U_∞ : velocity of the uniform flow
 y : coordinate taken in the starboard di-
 rection from the mid-span
 α_i : induced angle of attack
 θ : angle measured clockwise from the
 leading edge of the cylinder
 θ_j : angular location of the slot
 θ_m : angular position of the minimum
 pressure
 θ_{sb} : angular width of the dead water re-
 gion
 θ_u, θ_l : angular positions of the separation
 points on the upper and the lower
 surfaces of the cylinder, respec-
 tively
 λ : aspect ratio of the cylinder
 Suffix ∞ to the characteristic values of
 the cylinder indicates a two-dimensional
 value.

3. Experimental Apparatus and Method

3.1 Experimental apparatus

The experimental apparatus is the same one as illustrated in the reference (5).

Fig.1 shows a cross section of the model cylinder. A hollow cylinder with an outer diameter of 100mm and a small cylinder with a diameter of 25mm form a slot with a constant height of 0.55mm across the whole span. The outer surface of the hollow cylinder is chromium-plated and the static pressure holes totaling 272 are circumferentially distributed at ten spanwise locations. There are a large number of holes

Fig.1 Cross section of the model.

at the mid-span of all the spanwise locations, that is, 56 holes are circumferentially distributed at intervals of 6° from the first one which is located at the position of $\theta = \theta_j + 6.6^\circ$.

The test section of the wind tunnel has the dimensions of 700 x 1000mm in cross section and 1500mm in length. The model cylinder is mounted at the center of the test section and passes through two partition-plates (hereinafter referred to as "side-wall") and the tunnel wall. The desired aspect ratio λ of the cylinder can be ob-

tained by adjusting the spanwise location of this side-wall. The structure of a cylinder-side-wall juncture is the same one that is shown in Fig.4(a) of the reference (5). The blockage ratio in the test section is 0.143.

The maximum velocity in the test section is about 52m/s and the turbulent intensity is about 0.4% at the Reynolds number of 2.1×10^5 . The maximum thickness of the boundary layer y_δ on the side-wall is about 7mm ($y_\delta/D \approx 0.07$), and the shape factors H_1 of the boundary layer are 1.3 and 1.4 at $x/D = -1.6$ and -0.8 on the side-wall respectively, where x is a coordinate taken in streamwise direction from an origin of the cylinder.

3.2 Experimental method

The experiments were carried out by varying λ , θ_j and C_μ under the condition of constant Re of 2.1×10^5 . When $\lambda=4$ and 6, θ_j was fixed at $\theta_j=90^\circ$, and when $\lambda=8$, three different θ_j 's were used, that is, $\theta_j=50^\circ, 90^\circ$ and 120° . C_μ was varied from 0 to nearly 0.35 in all the cases mentioned above.

The circumferential static pressure distributions at the mid-span were measured with a multitube manometer, and the pressure distributions at the other spanwise locations were also measured according to circumstances. The measured pressure distributions may be affected by the blockage ratio, but they are not corrected at all, because it is difficult to correct the pressure distributions of the cylinder accurately and also the results corrected by the usual method were within the accuracy of the measurement.

4. Calculation of the Circumferential Static Pressure Distribution

The calculation of a circumferential static pressure distribution in the present investigation is based on Parkinson-Jandali's method⁽⁷⁾ which can be applied to a two-dimensional flow around a symmetric bluff body with a broad wake. The pressure distribution of the cylinder with tangential blowing is calculated by extending Parkinson-Jandali's method to a two-dimensional asymmetric flow.

Fig.2 shows the basic plane (T-plane) and the physical plane (t-plane). A point in the T-plane, $T=X+iZ$ (i is imaginary unit), is transferred to the point in the t-plane, $t=x+iz$, by the transformation,

$$t = r(T) = e^{-iY} \left(T - \cot\delta - \frac{1}{T - \cot\delta} \right) \quad (1),$$

where Y and δ are the angles between the incident flow and the X-axis and between the stagnation point and the X-axis in the T-plane respectively (Fig.2). Then a complete circle with radius R the center of which is at the origin, AS_1BS_2A , in the T-plane is conformally mapped to a circular arc slit, $AS'_1BS'_2A'$, in the t-plane by Eq.(1). This slit mapped from the circle is not symmetric but asymmetric with respect to the x-axis. The order of AS_1BS_2A corre-

Fig.2 Basic and physical planes.

sponds to that of AS_1BS_2A' . The stagnation points S_1 and S_2 in the T-plane are mapped to the separation points S'_1 and S'_2 in the t-plane respectively, and the stagnation streamlines in the T-plane are transformed into separation streamlines which flow away tangentially in the t-plane.

Now, consider the complex potential of a flow around a circle with radius R immersed in a uniform flow U_∞ the direction of which is inclined by the angle γ with respect to the X-axis in the T-plane. In addition to this, consider two sources of different strengths $2Q_1$ and $2Q_2$ at symmetric angular positions ϵ and $-\epsilon$ on the circumference of the circle respectively, a sink of strength $(Q_1 + Q_2)$ at the origin and a circulation of strength Γ around the circle (see Fig.2).

Then the total complex potential $W(T)$ of the asymmetric flow in the T-plane is expressed as follows;

$$W(T) = U_\infty \left(T e^{-i\gamma} + \frac{R^2}{T} e^{i\gamma} \right) + \frac{Q_1}{\pi} \ln(T - R e^{i\epsilon}) + \frac{Q_2}{\pi} \ln(T - R e^{-i\epsilon}) - \frac{(Q_1 + Q_2)}{2\pi} \ln T + \frac{i\Gamma}{2\pi} \ln T e^{-i\gamma} \quad (2).$$

Substituting $R e^{i\beta}$ for T in Eq.(2) after differentiation, the absolute velocity $|w(\beta)|$ on the circumference of the circle in the T-plane is given by

$$|w(\beta)| = 2U_\infty \sin(\beta - \gamma) + \frac{1}{\pi R} \left\{ \frac{Q_1(\sin\beta + \sin\epsilon)}{2(\cos\beta - \cos\epsilon)} + \frac{Q_2(\sin\beta - \sin\epsilon)}{2(\cos\beta - \cos\epsilon)} + \frac{\Gamma}{2} \right\} \quad (3),$$

where β is an angle in the T-plane (Fig.2). Q_1 and Q_2 are determined from Eq.(3) by setting $|w(\beta)| = 0$ at $\beta = \pm\delta$ on the circumference of the circle. Substituting these values thus obtained for Q_1 and Q_2 into Eq.(3), the absolute velocity is given as

$$|w(\beta)| = \frac{\cos\delta - \cos\beta}{\cos\epsilon - \cos\beta} \left[2U_\infty \{ \sin(\beta - \gamma) + \sin\gamma(\cos\epsilon - \cos\delta) \} + \frac{\Gamma}{2\pi R} \right] \quad (4).$$

Therefore the absolute velocity $|w(\beta)|$ on the circular arc slit in the t-plane is obtained from Eqs.(1) and (4) as follows;

$$\begin{aligned} \frac{|w(\theta)|}{U_\infty} &= \frac{1}{|f'(T)|} \frac{|w(\beta)|}{U_\infty} \\ &= \frac{1 - 2\cos\beta\cos\delta + \cos^2\delta}{2U_\infty(\cos\epsilon - \cos\beta)} \left[2U_\infty \{ \sin(\beta - \gamma) + \sin\gamma(\cos\epsilon - \cos\delta) \} + \frac{\Gamma}{2\pi R} \right] \quad (5). \end{aligned}$$

Here the separation velocities K and $-K$ at the separation points S'_1 and S'_2 respectively are related to the base pressure coefficient C_{pb} obtained empirically by

$$|\pm K| = (1 - C_{pb})^{\frac{1}{2}} \quad (6),$$

where C_{pb} is defined as the mean value of C_p 's at the upper and the lower separation points.

Introducing the radius R which is taken to be $R = 1/\sin|\delta|$ and Eq.(6) into Eq.(5), the absolute velocity $|w(\theta)|$ is given by

$$\frac{|w(\theta)|}{U_\infty} = \frac{1 - 2\cos\beta\cos\delta + \cos^2\delta}{\cos\epsilon - \cos\beta} \left[\sin(\beta - \gamma) + \sin\gamma\cos\epsilon - \frac{\sin^3\delta\sin\gamma\cos\gamma}{K} \right] \quad (7).$$

The circulation Γ and the separation velocity K are related with each other as follows;

$$\frac{\Gamma}{\pi R} = 4U_\infty \sin\gamma \left(\cos\delta - \frac{\sin^3\delta\cos\gamma}{K} \right),$$

and the angular locations $+\epsilon$ and $-\epsilon$ of each source are given as

$$\begin{aligned} \cos(\pm\epsilon) &= \frac{1}{K \mp \sin^2\delta \sin\gamma} \left[K \cos\delta + \sin^2\delta \{ \sin(\delta \mp \gamma) + \frac{\sin^3\delta \sin\gamma \cos\gamma}{K} \} \right] \quad (8). \\ &\text{(double signs are in the same order)} \end{aligned}$$

Hence the pressure distribution of the circular arc slit $S'_1AS'_2$ is given by

$$C_p = 1 - \left\{ \frac{|w(\theta)|}{U_\infty} \right\}^2 \quad (9).$$

On the other hand, the relation between β and θ is expressed as follows;

$$\begin{aligned} \sin(\pi - \theta) &= \sin(\beta - \gamma) \cos\delta + \frac{1}{1 - 2\cos\delta\cos\beta + \cos^2\delta} \\ &\quad \times \left[\sin^2\delta \{ \sin\gamma - 2\cos\delta\cos\beta\sin\gamma + \cos\delta\sin(\beta + \gamma) \} \right] \quad (10). \end{aligned}$$

Now setting $\theta = \theta_u$ at $\beta = \delta$ and $\theta = \theta_l$ at $\beta = -\delta$ in Eq.(10), δ and γ are related to θ_u and θ_l as follows;

$$\delta = \frac{\theta_l - \theta_u}{4} \quad \text{and} \quad \gamma = \frac{\theta_u + \theta_l}{2} - 180 \quad (11).$$

Consequently, the pressure distribution along $S'_1AS'_2$ can be calculated by making use

of Eqs.(6) to (10), if three empirical values of θ_u , θ_l and C_{pb} are prescribed. However, when the calculated distribution agrees fairly well with the experimental one, the condition that $\partial C_p / \partial \theta = 0$ is not satisfied at the separation points S'_1 and S'_2 . This is described later.

5. Calculated and Experimental Results

5.1 The calculated result of the circumferential static pressure distribution

Fig.3 shows the calculated pressure distributions for a symmetric ($C_\mu=0$) and an asymmetric ($C_\mu>0$) flow with respect to the x-axis. This figure includes also the pressure distributions calculated by Dunham's

Fig.3 Static pressure distribution on the circular cylinder.

equation⁽³⁾ together with the experimental results. When $C_\mu>0$, in spite of asymmetry of the flow, the calculated distribution is consistent fairly well with the experimental result over a wide range of θ . The present calculated result is improved for the range of θ between θ_m and θ_u in comparison with the result calculated by Dunham's equation. When $C_\mu=0$, the flow around the cylinder is symmetric with respect to the x-axis, so the present calculated result is consistent with that calculated by Parkinson-Jandali's method and also agrees fairly well with the experimental values. As a whole, the degree of consistency of the calculated result with the experimental one, however, becomes worse a little near the separation points. This is caused by the fall of the calculation accuracy there, because the effects of the boundary layer on the cylinder and of the vortices in the wake and so on are ignored in this theory.

Now, the separation condition is described here. If the condition of "smooth separation" by Woods⁽⁸⁾ is applied to the separation point S'_1 , the separation velocity $K(\neq 0)$ at that point is determined uniquely as a function of the angles δ and γ in the T-plane. In this case, the curvature of the separation streamline is finite and C_p is continuous at S'_1 . However, the static pressure distribution and the value of C_{pb} calculated by making use of this condition were not so consistent with the experimental results. Then, ignoring Woods's condition, the pressure distributions were calculated for many given values of K , different a lit-

tle from each other, in Eq.(6). The pressure gradient $\partial C_p / \partial \theta$ and the curvature of the separation streamline became infinite and C_p was discontinuous at S'_1 for the value of K which gives the best fitted curve with measured values. Then θ and C_p at S'_1 were replaced by the experimental ones θ_u and C_{pb} to avoid a singularity of the calculated pressure curve. The same manipulation mentioned above must be made at the other separation point S'_2 . Now, the characteristic values of the cylinder were determined from the pressure distribution obtained in the manner described above, and they are shown in Fig.4(a) to 14 as the theoretical calculated results together with the experimental values.

5.2 The characteristic values at the mid-span

Fig.4(a) and 4(b) show the relations between C_l , C_d and C_μ respectively. As C_μ increases, C_l increases monotonically. When $C_\mu<0.15$, the influence of λ on C_l is scarcely seen, but when $C_\mu\geq 0.15$, the difference of C_l due to λ begins to appear a little. On the other hand, as C_μ increases, C_d decreases once and takes a minimum value at $C_\mu \approx 0.1$, after that C_d increases again. The rate of increase of C_d becomes larger in order of $\lambda=8, 6$ and 4 , and the values of C_d at the same C_μ become also larger in the same order. This suggests that a large induced drag acts on the cylinder and the magnitude of it increases, as λ decreases. When C_μ and λ are fixed at constant values and θ_j is varied near the location of $\theta_j=90^\circ$, the larger θ_j is, the larger C_l is (not shown here).

Fig.4(a) Relation between C_l and C_μ .

Fig.4(b) Relation between C_d and C_μ .

Now the induced drag, that is, the induced angle of attack α_i depends only on the value of C_l , so hereinafter the experimental results are shown for C_l .

Fig.5 shows the relation between θ_m and C_l (θ_m - C_l relation). θ_m increases gradually with an increase of C_l . As C_l increases, the minimum pressure point moves farther downstream and the difference of θ_m due to λ begins to appear.

Fig.5 Relation between θ_m and C_l .

Figs.6 and 7 show the relations between θ_u and C_l (θ_u - C_l relation) and between θ_{sb} and C_l (θ_{sb} - C_l relation), respectively. θ_u increases monotonically with an increase of C_l except when C_l is considerably small. The difference of θ_u due to λ begins to appear between $\lambda=4$ and $\lambda=6,8$, as C_l increases. On the other hand, θ_{sb} decreases almost monotonically with an increase of C_l , so that the angular separation region becomes narrower with an increase of C_l . However, the

Fig.6 Relation between θ_u and C_l .

Fig.7 Relation between θ_{sb} and C_l .

difference of θ_{sb} due to λ is not seen, because the effects of the induced velocity on θ_l and θ_u cancel out each other from the definition of θ_{sb} ($\theta_{sb}=\theta_l-\theta_u$).

Fig.8 shows the relation between θ_{sb} and C_{pb} (θ_{sb} - C_{pb} relation). It is found that the narrower the separation region is, the higher the base pressure is. The influence of λ on θ_{sb} - C_{pb} relation is not seen, unlike Figs.5 and 6, because θ_{sb} and C_{pb} are the quantities which include little or no effect of the induced velocity. Therefore the range between two solid curves in this figure, which are drawn excluding extremely detached values, indicates the scattering range of the two-dimensional experimental values which scarcely include the effect of the induced velocity.

Fig.9 shows the relation between C_l and C_d (C_l - C_d relation). As C_l increases, C_d decreases once and takes a minimum value, after that C_d increases again. C_d is mainly given by the sum of the profile drag coefficient C_{d0} and the induced drag coefficient C_{di} for the present Reynolds number. As C_l increases, C_{d0} decreases and C_{di} increases. When C_l is small, the decrement of C_{d0} is larger than the increment of C_{di} , so that as a whole, C_d decreases. When C_l becomes larger and larger, however, the increment of C_{di} comes to exceed the decrement of C_{d0} , so that C_d begins to increase reversely.

Fig.8 Relation between C_{pb} and θ_{sb} .

Fig.9 Relation between C_l and C_d .

The rate of the increase of C_d to that of C_l is large in order of $\lambda=4, 6$ and 8 , and the value of C_d is also large in the same order. Hence the influence of λ on C_l - C_d relation appears clearly with an increase of C_l .

6. Discussion

6.1 The method of determination of the induced angle of attack

Two effects must be sufficiently considered to obtain the two-dimensional characteristic values of the cylinder with the side-wall and tangential blowing. One of them is the effect of the side-wall and the other is that of the induced angle of attack α_i . The range affected by the side-wall has been already evidenced experimentally, and it was about $1.8D$, at the maximum, from the side-wall toward the mid-span. Therefore when $\lambda \geq 4$, the two-dimensional characteristic values at the mid-span can be obtained by considering the effect of α_i only.

In this report, three different simpler methods are devised to determine α_i by considering the following two experimental facts. First, if C_l becomes larger and larger, the flow around the cylinder approaches a potential flow with a circulation around it. Second, the spanwise distributions of lift coefficients C_{ln} , which is normalized by C_l at the mid-span, are congruent and independent of C_u , and they are exclusively dependent on λ only (Fig.10). The influence of θ_j on the spanwise distribution of C_{ln} is minor in comparison with that of λ (not shown here). The three methods of determination of α_i are as follows; (1) The method by the theoretical calculation (method (1)), (2) the method using θ_m - C_l relation (method (2)) and (3) the method using C_l - C_d relation (method (3)).

Fig.10 Spanwise distributions of lift coefficients C_{ln} normalized by C_l at the mid-span.

(1) The method (1) The spanwise distributions of α_i due to the trailing filament-vortex and the trailing shed-vortex⁽⁹⁾ have been already calculated under some assumptions by authors⁽⁵⁾. As a result, it was found that α_i due to the former was very small in comparison with that due to the latter at the mid-span. Then only the induced angle of attack α_i due to the trailing shed-vortex is considered here and the spanwise distribution of α_i along the cylinder axis is given as follows;

$$\alpha_i = -\frac{\pi}{4} \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{B} y \tanh \frac{n\pi}{B} H \quad (12),$$

where B is the semi-span of the cylinder, $-B \leq y \leq B$, and H is half the height of the test section⁽⁹⁾. Now, a_n in Eq.(12) indicates the Fourier coefficient, when the spanwise distribution of a circulation $\Gamma(y)$ is expressed in the Fourier series as

$$\Gamma(y) = BU_{\infty} \left(\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{B} y \right) \quad (13).$$

Moreover $\Gamma(y)$ and the measured lift coefficient are related with each other by

$$\Gamma(y) \doteq \frac{1}{2} U_{\infty} D C_l(y) \quad (14).$$

Therefore the spanwise distribution of α_i can be calculated by making use of the truncated forms of Eqs.(12) and (13), and Eq.(14).

(2) The method (2) In general the angular position of the minimum pressure θ_m ought to be determined by the wake width and the induced velocity. As the angular width of the dead water region θ_{sb} increases, θ_m decreases, and as α_i increases, θ_m increases. Then, considering the two-dimensional potential flow ($\alpha_i=0$) around the cylinder, θ_m ought to approach asymptotically a constant value of 90° with a decrease of θ_{sb} . In case of the three-dimensional flow ($\alpha_i \neq 0$), if C_l is large enough, θ_m is scarcely affected by the wake width but only by the induced velocity because θ_{sb} decreases with an increase of C_l . This means that θ_m approaches asymptotically a value of $90^\circ + \alpha_i$ with an increase of C_l . And when λ is constant, α_i must be proportional to C_l because the spanwise distributions of C_l are similar and independent of C_u .

Therefore if an asymptote is drawn for each λ in Fig.5 so as to pass the point of $\theta_m=90^\circ$ when $C_l=0$ and to approach asymptotically θ_m with an increase of C_l , the relation between α_i and θ_m is determined for each λ from the gradient of the asymptote.

(3) The method (3) Since the spanwise distribution of $C_{ln}(y)$ is congruent and independent of C_u ($\neq 0$) and depends only on λ , it can be expressed in the Fourier series as

$$\frac{C_l(y)}{C_l} = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi}{B} y,$$

Where A_n is a function of λ . A_n in the above equation and a_n in Eq.(13) are related with each other by

$$A_0 = \frac{\lambda}{C_l} a_0 \quad \text{and} \quad A_n = \frac{\lambda}{C_l} a_n.$$

Introducing the above relations into Eq.(12), $\alpha_i(y)$ is given as

$$\alpha_i(y) = -\left(\frac{\pi}{4} \sum_{n=1}^{\infty} n A_n \cos \frac{n\pi y}{B} \tanh \frac{n\pi H}{B}\right) \frac{C_1}{\lambda}.$$

Hence α_i at the mid-span ($y=0$) is determined by

$$\alpha_i(0) = -\left(\frac{\pi}{4} \sum_{n=1}^{\infty} n A_n \tanh \frac{n\pi H}{B}\right) \frac{C_1}{\lambda} = \frac{C_1}{\pi \lambda} \Delta(\lambda) \quad (15),$$

where $\Delta(\lambda)$ is a function of λ . Therefore the relation between C_1 and the induced drag coefficient C_{di} is given as follows;

$$C_{di} = \frac{C_1^2}{\pi \lambda} \Delta(\lambda) = C_1 \alpha_i \quad (16).$$

Now if the relation between C_1 and C_{di} at the mid-span is known for any λ , $\Delta(\lambda)$ is determined from Eq.(16) and then α_i from Eq.(15). It is difficult, however, to know the relation between C_1 and C_{di} empirically because the profile drag coefficient C_{d0} varies according to C_1 . In this investigation, regarding the value of C_{d0} as approximately constant, two extreme values of C_{d0} are assumed for convenience to determine C_{di} . One of them is a case of $C_{d0}=0$ and the

other is a case of $C_{d0}=0.35$ which is obtained from the smooth connecting curve of the experimental values for $\lambda=4$ in Fig.9.

For a practical calculation, several values of C_d which corresponded to enough large C_1 's were read from the smooth connecting curve for each λ in Fig.9. And C_{di} 's were determined by subtracting the assumed C_{d0} from C_d 's, thus obtained for each λ . Then several $\Delta(\lambda)$'s were given from these several pairs of C_1 and C_{di} , and Eq.(16). The mean value of these $\Delta(\lambda)$'s was used to determine the relation between α_i and C_1 for each λ .

Fig. 11(a) to (d) show $C_{1\infty} - C_{d\infty}$ relations. These are typical examples of the characteristic values corrected by making use of α_i 's determined by the three methods described above. Judging from various viewpoints, the most recommendable method of them is the case of $C_{d0}=0$ in the method (3).

The method (1) requires the spanwise distribution of C_1 to determine α_i and in spite of laborious calculation, a larger scatter is included in the corrected result. The method (2) is simpler and easier than the others, but the degree of error included in α_i depends exclusively on how to draw the asymptote in Fig.5. This means that there is a possibility to include a large error in α_i . Finally, in the case of $C_{d0}=0.35$ in method (3), though the scatter of the corrected result is smaller, $C_{d\infty}$ has a tendency to increase again with an increase of $C_{1\infty}$, after it takes the minimum value, and this should not occur. This method seems to underestimate C_{di} .

As mentioned above, the method which gives a better two-dimensional approximation to the characteristic values is the method (2) or the case of $C_{d0}=0$ in the method (3). However, it was concluded by judging synthetically the accuracy of the experimental values, the scatter of the corrected result and so on that the most recommendable method was the latter one.

6.2 Two-dimensional aerodynamic characteristics

Fig.11(c) shows $C_{1\infty} - C_{d\infty}$ relation. The results corrected for the experimental values are independent of λ and are approximately expressed by a single curve within the limit of the experimental scatter. As $C_{1\infty}$ increases, $C_{d\infty}$ decreases, and when $C_{1\infty} > 5$, $C_{d\infty}$ becomes nearly equal to zero. This result suggests that when $C_{1\infty} > 5$, the flow around the cylinder can be assumed to be approximately a two-dimensional potential flow with a circulation around it, ignoring the wake of the cylinder.

Figs.12, 13 and 14 show the relations between $\theta_{m\infty}$, $\theta_{u\infty}$, $\theta_{sb\infty}$ and $C_{1\infty}$ respectively. θ_m approaches asymptotically a value of $\theta_m=90^\circ + \alpha_i$ with an increase of C_1 in Fig.5, while $\theta_{m\infty}$ approaches a value of $\theta_{m\infty}=90^\circ$ with an increase of $C_{1\infty}$. Taking the scatter of θ_m into account, $\theta_{m\infty}$ is nearly equal to 90° , when $C_{1\infty} > 5$. This result corresponds to that obtained from Fig. 11(c). It is confirmed from this that the results corrected for the experimental values give the two-dimensional characteristics of the cylinder. The influence of λ on $\theta_{u\infty}$ is not seen in Fig.13, even

Fig.11 Relations between $C_{1\infty}$ and $C_{d\infty}$.

when $C_{l\infty}$ is large enough. And the scatter in Fig.13 becomes less than that in Fig.6. The difference between the result in Fig.14 and that in Fig.7 does not appear so clearly, because θ_{sb} and C_l are hardly influenced by correction of α_i . However, the scatter in Fig.14 becomes less than that in Fig.7.

Fig.12 Relation between $\theta_{i\infty}$ and $C_{l\infty}$.

Fig.13 Relation between $\theta_{u\infty}$ and $C_{l\infty}$.

Fig.14 Relation between $\theta_{sb\infty}$ and $C_{l\infty}$.

7. Conclusions

Experiments were carried out on the aerodynamic characteristics at the mid-span of a circular cylinder with tangential blowing, and also a calculation of the circumferential static pressure distribution was made by a modified Parkinson-Jandali's method. The conclusions from all of these are as follows;

(1) Three different methods were devised to determine the induced angle of attack at the mid-span of a circular cylinder with tangential blowing, when $\lambda \geq 4$.

(2) The most recommendable method of them is the case of $C_{d0} = 0$ in the method using the relation between C_l and C_d at the mid-span.

(3) Two-dimensional characteristic values of the cylinder with tangential blowing were given by correcting the experimental values at the mid-span when $\lambda \geq 4$ for the effect of the induced angle of attack.

(4) The circumferential static pressure distribution on the cylinder was calculated by a modified Parkinson-Jandali's method extended to an asymmetric flow.

Acknowledgment

The authors are grateful to Emeritus Professor of Nagoya University Dr. Yoshimasa Furuya for his kind suggestions on the experiment and to Professor of Nagoya University Dr. Shintaro Otsuka for the calculation. They also wish to thank Mr. Tatsuo Hayashi for his excellent technical contribution.

References

- (1) Sato, M. and Matsuoka, K., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.36, No.289 (1970-9), p.1493.
- (2) Furuya, Y. and Yoshino, F., Bulletin of the JSME, Vol.18, No.123 (1975-9), p.1002.
- (3) Dunham, J., J.Fluid Mech., Vol.33, part 3 (1968-9), p.495.
- (4) Ueda, S. and Tanaka, H., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol. 41, No.350 (1975-10), p.2853.
- (5) Yoshino, F., et al., Bulletin of the JSME, Vol.24, No.192 (1981-6), p.926.
- (6) Waka, R., et al., Proceedings of the Second International Symposium on Flow Visualization, Bochum, (1980-9), p.367.
- (7) Parkinson, G.V. and Jandali, T., J.Fluid Mech., Vol.40, part 3 (1970), p.577.
- (8) Woods, L.C., The Theory of Subsonic Plane Flow, (1961), Cambridge Univ. Press.
- (9) Otsuka, S., Kikai No Kenkyu (in Japanese), Vol.27, No.12 (1975-12), p.91.