Behavior of Air-liquid Surface Moving Downward
In a Straight Pipe

By Kouichi MURAKAMI, Naotsugu ISSHIKI, Hiroshi YAMAUCHI

In this paper an experimental study on the behavior of the air-liquid surface in various vertical and circular pipes is described. The case when a liquid piston is suddenly started from a vertical pipe which is previously filled up with a liquid and pressurized air on the top, is examined.

It is known that the measured velocity of the tip of the surface agrees approximately with the simplest one dimensional model, and the shape of the surface is divided into two areas; one is flat shape for the case of nearly constant velocity, and the other is spherical shape accompanied with an accelerating velocity. And further, the effects of an obstacle on the wall on the air-liquid surface behavior are observed, and it is found that sometimes a turbulence appears on the surface and grows up sharply, and the condition for the growth of the disturbance depends on Reynolds number and stability number.

Key Words: Unsteady Flow, Numerical Analysis, Free Surface, Orifice, Pipe, Gas Jet, Taylor Instability

1. Introduction

There are flows in which a pressurized air-liquid surface is suddenly accelerated down through a channel in many engineering applications such as, the fast breeder nuclear reactor which may develop an accident under sodium-water reactions, the chemical apparatus which may develop a sudden gas bubble explosion, and the once-through heat exchanger which may develop a strong pulsating flow. In analysing these cases, to know the velocity of the tip surface is one of the important subjects from a viewpoint of safety. So the behavior of liquid piston moving down through a straight pipe is taken up here.

Moreover, the effects of an obstacle attached on the surface of the channel are studied here to know the effect of a spacer or an orifice in these channels. Also in these phenomena the tip surface moves down leaving a part of the liquid layer on the pipe wall, and, according to know the volume of the residual liquid is very important.

As the first representative model for this study of pressurized surface behavior in the pipe, we picked up the following case of a vertical and circular pipe being filled up with a pressurized liquid, and then suddenly a movement of liquid piston being started with the bottom gate removed. And the characteristic behavior of the surface, specially the shape and the velocity of the air tip are examined experimentally and numerically, and the following are made clear.

First, regardless of the liquid volume left on the pipe wall, the velocity of the air tip agrees almost with a one dimensional flow model. And the shape of the tip surface at the constant velocity region is different from the one at the acceleration region.

Further, for the case of a small obstacle being attached on the pipe wall, the effects of the obstacle on the shape of the tip are examined experimentally, revealing the fact that, when the tip surface passes through the obstacle, the surface is wrinkled or disturbed suddenly, which is named a turbulent core here and this turbulent core sometimes grows up sharply to become smoky. The factors related to the generation and the growth of the turbulent core are examined experimentally. Further a measurement of the liquid film left on the pipe wall shows that in the case of a disturbed tip surface more liquid is left on the pipe wall than in the case of a smooth tip.

2. Experimental Apparatus and Procedure

Fig.1 shows a schematic diagram of the experimental apparatus which is composed of an air tank, a smooth lucid acrylic pipe of the inner diameter, an obstacle attached on the wall of the pipe at the position, a nozzle with the hole

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diameter \( \varnothing_n \) and with the area contraction ratio \( \varepsilon = (\varnothing_n/\varnothing)^2 \) at the bottom end of the pipe, and a movable plug. When the experiment is fulfilled, in the first place the exit of the nozzle is stopped by the movable plug set by a stopper, the liquid is introduced into the pipe from the bottom nozzle to fill it to an initial height \( l_0 \), the air tank is pressurized to a prescribed pressure \( P_0 \) (gage pressure), and then the stopper is removed and the liquid in the pipe is allowed to flow down through the nozzle. At this time, the change of the gas-liquid surface moving downward in the pipe is photographed by a streak camera, and the velocity of the tip surface and the shape of the surface are measured.

Moreover, by the hydrogen bubble method bubble lines are generated from straight electrodes stretched along the diameter of the pipe at a depth from the initial surface \( k_b \), immediately before the liquid starts to move, and the transient changes of the bubble lines are observed. The inner diameters \( d \) of the pipe used in this study are 46, 24, 17, 15, and 11 mm, and the experimental conditions of each pipe are shown in Table 1. As shown in this table, for two pipes of the inner diameters 24 and 17 mm the sudden contractions of the cross-sectional area ratios 0.95 and 0.88 are used respectively as the obstacles, and for the other pipes, a sharp edged orifice of thickness 5 mm is used. The pressure \( P_0 \) is set in the range of 0-392 kPa.

The liquid used in this experiment is mainly tap water. But for two pipes of the inner diameters 15 and 11 mm, the liquids shown in Table 2 are used to examine the properties of the liquids on the behavior of the gas-liquid surface. And the kinematic viscosity \( \nu \) and the surface tension \( \sigma \) are determined respectively by a Ubbelohde viscometer and a du Nouy tensiometer.

3. One-dimensional Average Flow Model

For the one dimensional average flow model, it is assumed that there are no velocity and pressure gradients in the direction, and the liquid flows uniformly in the pipe as if it was a rigid body. Now, if \( P_{\text{ave}} \) is equal to the pressure at the entrance of the nozzle, the equation of motion for the one dimensional liquid piston of the height \( l \) is

\[
\frac{1}{2} \rho_b i \frac{d}{d t} \left( \frac{d l}{d t} \right)^2 + \frac{1}{2} \rho_i \frac{d l}{d t}^2 - \rho_i \frac{d^2 l}{d t^2} = P_0 - P_{\text{ave}} + \sigma \frac{d l}{d t} - (1)
\]

where \( \rho_b \) is the density of liquid, \( g \) is the acceleration of gravity, \( t \) is time, \( i \) is the friction factor of the pipe, \( k_b \) is the friction loss factor of the obstacle, and the value of \( k_b \) is the one computed by Osborne, and when there is liquid at the obstacle, \( t \) is taken as unity, and after the air passes through the orifice, it is assumed that there is no pressure loss at the orifice and \( t \) is equal to 0. And if it is assumed that the pressure drop at the nozzle cannot be recovered, \( P_{\text{ave}} \) is

\[
P_{\text{ave}} = \frac{1}{2} C_f \rho_b \frac{d l}{d t}^2 - - - - - - (2)
\]

and if \( C_f \) is the contraction coefficient of nozzle, \( C_f \) is equal to \((1 - C_s C_t^2)/C_s^2 \), and \( l \) is given by \( l = l_0 + \int_0^t (d l/d t) dt \). Next, we select a characteristic velocity \( u = \sqrt{2P_0/\rho_b} \) and a characteristic length \( l_0 \), and define the following dimensionless variables:

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**Figure 1: Schematic illustration of experimental apparatus**

**Table 1: Experimental condition**

<table>
<thead>
<tr>
<th>( d ) (mm)</th>
<th>46</th>
<th>24</th>
<th>17</th>
<th>15</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_m )</td>
<td>1.95</td>
<td>1.7</td>
<td>1.7</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>( k_b ) (m)</td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Obstacle**

- S: Sudden contraction
- 0: Orifice

**Liquid**

- Water
- Water
- Water
- All
- Spindle
- Kerosine

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**Table 2: Properties of liquids**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Temp (°C)</th>
<th>( \rho_b ) (kg/m³)</th>
<th>( \nu ) (m²/s)</th>
<th>( \sigma ) (MN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olive Oil</td>
<td>18</td>
<td>915.0 ( \times 10^{-6} )</td>
<td>41.8</td>
<td></td>
</tr>
<tr>
<td>Olive:Spin 1:1</td>
<td>16</td>
<td>890.0</td>
<td>40.1</td>
<td>38.4</td>
</tr>
<tr>
<td>Olive:Spin 1:3</td>
<td>16</td>
<td>878.0</td>
<td>28.2</td>
<td>35.1</td>
</tr>
<tr>
<td>Spindle Oil</td>
<td>16</td>
<td>866.0</td>
<td>17.2</td>
<td>33.4</td>
</tr>
<tr>
<td>Aniline</td>
<td>16</td>
<td>1024.0</td>
<td>4.46</td>
<td>55.5</td>
</tr>
<tr>
<td>Kerosine</td>
<td>16</td>
<td>790.0</td>
<td>1.77</td>
<td>29.7</td>
</tr>
<tr>
<td>Ethyl Alcohol</td>
<td>14</td>
<td>790.0</td>
<td>1.69</td>
<td>24.2</td>
</tr>
<tr>
<td>Water</td>
<td>16</td>
<td>799.0</td>
<td>1.07</td>
<td>74.7</td>
</tr>
<tr>
<td>Methyl Alcohol</td>
<td>14</td>
<td>795.0</td>
<td>0.80</td>
<td>24.6</td>
</tr>
<tr>
<td>Ethyl Ether</td>
<td>13</td>
<td>711.0</td>
<td>0.35</td>
<td>19.3</td>
</tr>
</tbody>
</table>

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\[ \frac{d z^*}{dt^*} = \frac{d x}{d t} / u_1, \quad \frac{dz^*}{dt} = \frac{x}{t}, \quad t^* = \frac{t}{u_1}, \quad d^* = \frac{d}{dt} \]

We rewrite the equation (1), in terms of these dimensionless variables, and eliminate \( F_{09} \) to get

\[ \frac{d^2 z^*}{d t^*} + \frac{1}{2} \left( k b^* + C \right) \frac{dz^*}{dt^*} \left( \frac{d^2 z^*}{d t^*} \right)^2 - g^* = - \frac{1}{2} \frac{dz^*}{dt^*} \]

From eq. (3), the following are known: if \( e \) comes close to unity, the first term of the left hand of eq. (3) has a large value, consequently the velocity of the tip of gas-liquid surface is accelerated always till the tip surface arrives at the nozzle, and when \( e \) has a smaller value than 0.6, the first term of the left hand becomes smaller and the second becomes larger, and finally, the velocity of the tip surface near the nozzle is determined by the value \( C \) and the velocity becomes nearly constant. Moreover, the effect of friction loss at the pipe wall on the velocity is neglected here, and \( C \) is assumed to be equal to 0. And then the velocity of the tip surface is calculated numerically by Runge-Kutta method.

Fig. 2 shows the analytical relation between the position of the tip surface \( z^* \) (\( z^* = z / t \)) and the velocity of the tip surface \( (-d z^*/dt) \) in the case when the same orifice is used as the obstacle for different values of \( e \) in the range of 0.15-1, where the value of \( kb \) used is 0.11, and dimensionless acceleration of gravity \( g^* \) is 0.05. The analytical solutions of two cases of contraction coefficient \( C_0 \) being 1.0 and 0.87 are shown in Fig.2 by the solid lines and the dotted lines, respectively. And from this the measured velocity of the tip surface is plotted. And then the solid lines \((C_0=1.0)\) of the analytical solution agree well with the experimental data in the acceleration region when \( e \) is greater than 0.6, namely at which the tip surface is accelerated near the nozzle. But in the constant velocity region, namely in which the velocity of the tip surface becomes nearly constant near the nozzle, the dashed line \((C_0=0.87)\) of the analytical solution agree well with the experimental data.

As mentioned above, whether the residual liquid volume on the pipe wall is much or less, the velocity of the tip surface can be expressed by the one dimensional average flow model.

4. Behavior of Gas-Liquid Surface in Acceleration Process

When the velocity of the tip surface increases downward, a finger shape tip surface appears and grows sharply in the neighborhood of the center of the pipe, as shown in Fig. 4. Considering a co-ordinate attached to the moving liquid in the pipe, the acceleration of the system \( g^* \) is \( -d^2 z^*/dt^2 \), and the phenomenon becomes one under the relative acceleration \( g^*=C_0-g \), as shown in Fig.3. If \( g^* \) is greater than \( g \), the phenomenon becomes one of Rayleigh-Taylor instability. This instability often appears when the system is accelerated from heavy liquid towards light gas, consequently, disturbances grow up sharply, causing a finger shape to appear.

In practice, relative acceleration of the system \( g^* \) changes as time passes, but in this analysis, \( g^* \) is assumed to be constant. And then the basic equations of the motion and the continuity are almost similar to those of the previous report[9], but the characteristic length \( d \) and the characteristic velocity \( u_0 (=D/D) \) are selected for this case. Further, new dimensionless variables \( t^* = tu_0/d, \quad g^*=gD/u_0^2 \) are defined here. Because the shape of the gas-liquid surface is generally able to be expressed in the Fourier-Bessel series, the following equation of unit element is used for the initial shape \( h^* \) of the surface in this analysis.

\[ h^* = h_\infty \left( 2 \sin r^* \right) \quad (2 \sin r^*) \quad (3) \]

where \( \sin r^* \) are roots of \( J_1(\sin)=0 \). For the calculation by SMAC method, it is assumed that the liquid slips on the wall, and surface tension is neglected, which \( g^* \) has a constant value of 0.5.

Fig.4 shows graphically the growth of an initial disturbance solved numerically.
for the case of \( n=1, \frac{h_1}{\lambda}=0.1, \) and \( \text{Re}_2 = (\mu_2 \rho / \nu) = 100. \) At the center of the liquid surface a projection of finger shape grows up sharply, and the bottom part of the surface in the neighborhood of the wall moves downward with a nearly constant velocity of 0.15. In the same figure, the movement of lines originally set in the liquid is shown. If \( h_{b}^* (= h_{b}/\lambda) \) is less than 0.3, the line moves similarly to the movement of the surface. On the other hand, if \( h_{b}^* \) is greater than 0.4, the line remains almost straight. And these results agree well with the analysis of Rajagopal. And the observation picture shown in the same figure is almost similar to the result of numerical solution. The movement near the wall of the bubble line of the initial depth \( h_{b}^* = 2.2 \) is taken as the basis. The movement of the contour of each bubble is shown in Fig.5, where nondimensional movement length \( l_{b}^* (= l_{b}/\lambda) \) of the measured point in \( z \)-direction is taken on the abscissa, and the \( z \)-direction motion of the bubble contour at the center and near the wall are taken on the ordinate. These motions are indicated by the dotted lines and the solid lines, respectively. From this figure, the following are known: the center part of the surface juts out and the surrounding part falls slowly. But the bubble contour, of which the depth \( h_{b}^* \) is greater than 0.5, is independent of the disturbance of the surface and remains almost straight.

If the acceleration process continues further, the projection of the finger shape is torn off and drifts in the gas flow. And then, as shown in the right-hand end of the photograph of Fig.4, the tip surface changes to a hemispherical shape, and moves downward keeping the same shape.

5. Shapes of Tip Surface

5.1 Classification of shapes of tip surface

The photographs on the upper side of Fig.6 show the shapes of the tip surface near the nozzle, when the gas-liquid surface moves downward under a pressure \( p_0 \) of 392 kPa in a pipe of inner diameter \( d = 24 \text{ mm} \) without an obstacle.

When \( \varepsilon \) has a large value, the process of acceleration continues until the tip surface arrives at the nozzle. And then, the projection of the finger shape grows up due to Rayleigh-Taylor instability, the projection drifts in the gas flow, the tip surface changes to a hemispherical shape and the liquid film on the wall becomes smooth, such as shown in the photograph of \( \varepsilon = 1 \) in this figure. On the other hand, when \( \varepsilon \) is small, the tip moves downward with constant velocity keeping the flat shape, such as shown in the photographs of \( \varepsilon = 0.44 \) and 0.06.

The photographs on the lower side of Fig.6 show the shape of the tip near the nozzle, where an obstacle of the friction loss factor \( h_{b}^* = 0.02 \) is set halfway in the pipe.

When the process of acceleration takes place over the whole length of the pipe, the tip keeps a quasi-stable hemispherical shape in spite of the existence of an obstacle, but the film on the wall
is disturbed. The area of this phenomenon is named "acceleration region" here.

When $\varepsilon$ is taken smaller and the velocity of the tip is kept almost constant near the nozzle, the disturbance on the surface grows up. For example, the case of $\varepsilon=0.04$ is shown here. The shape of surfaces is entirely different from the flat shape, and it is a snake shape which moves downward bending and rotating. The area of this phenomenon is named "disturbed region", where it is known from experimental observation that a large quantity of liquid is left on the pipe wall. Further, when $\varepsilon$ becomes smaller, the tip has a flat shape, such as shown in the photograph of $\varepsilon=0.66$, and the liquid film left on the wall becomes very thin in spite of the existence of the obstacle. The area of this phenomenon is named "stable region" here.

In this way, for the case where an obstacle is attached on the pipe wall, the constant velocity region is divided into three regions, namely "disturbed", "stable", and "transitional" regions. The last one being defined here as the middle region between disturbed and stable regions.

Experiments for observation of the tip surface shape of the case of the pipe of $d=24\text{mm}$ with an obstacle of the friction loss factor $k_b=0.02$ were performed, using tap water and changing $\varepsilon$ and $P_0$; and as a result, a map of the classification of above mentioned regions is attained as Fig. 7. From this map, the following are known: there are "acceleration regions" at the right-hand side of the line $a_1$, and "constant velocity region" at the left side of this line. This is true from the analytical result showing that the tip moves downward always acceleratedly when $\varepsilon$ is greater than 0.6. And the constant velocity region is divided into two regions, namely the disturbed region shown in the upper part of line $a_2$, where the velocity of the tip is about $2.7\text{m/s}$, and the stable region shown in the lower part of $a_2$. After all, if the inner diameter of the pipe and the kind of the liquid are known, the classification of the disturbed and the stable regions depends on the velocity of the tip surface.

The effects of the kind of the liquid on the classification of the disturbed and the stable regions are examined on the case when the inner diameter $d$ of the pipe is $15\text{mm}$, $k_b$ of the obstacle is 1.0, and $\varepsilon$ of the nozzle is less than 0.6, and as a consequence Fig.8 is obtained. $Re(=\nu d/\nu)$ and stability number $S(=\nu d/\nu g)$ are used as nondimensional numbers in order to classify these regions, in the same manner as Ohnesorge used them in order to classify the state of the liquid jet into the air from the nozzle, where $\sigma$ is surface tension and the characteristic velocity $\nu_0$ is taken as the stationary velocity of the tip surface after passing through the obstacle. From this figure, we can distinguish three regions, namely the disturbed region of the symbol $*$ which is shown in the upper part of line $a_3$, the stable region of the symbol $\circ$ which is shown in the lower part of line $a_4$, and the transitional region of the symbol $\cdot$ which occupies the part between line $a_3$ and line $a_4$. But at the left side of line $a_5$, there is symbol $\circ$ showing a stable region of, say, olive oil in spite of the disturbed region. The reason is thought to be that no disturbance does appear on the surface when the tip passes through the obstacle. In a liquid of high viscosity.

5.2 Generation of turbulent core by obstacle

First, the above mentioned classification by line $a_3$ in Fig.8 is explained. When the tip surface passes through the obstacle, the surface near the obstacle becomes smooth near the tip but wrinkly on the backward liquid film, as shown in the left of Fig.9(l), or the surface sometimes comes to be disturbed, as shown in the left of (2). Since these little wrinkles
or disturbances on the gas-liquid surface often grow up sharply to become wavy near the nozzle, as shown in this figure, these little wrinkles or disturbances are named "turbulent core" here.

In the case of a pipe of same inner diameter with same obstacle, a turbulent core appears in the region over the same Re for any liquid, and then line $a_2$ in Fig. 8 is almost perpendicular to Re-axis. Therefore, for various pipes, various liquids, and various obstacles, the generation of turbulent core is examined experimentally on a Re $\sim$ friction loss factor of obstacle $k_b$ logarithms map, as shown in Fig.10. In this figure, a wrinkly turbulent core such as (1) of Fig.9, a disturbed core such as (2), and no turbulent core are indicated by the center line symbol, black symbol, and white symbol, respectively. The center line and black symbols, which indicate possible generation of turbulent core on the surface, are located almost at the right-hand side of line $a_2$, and the white symbols, which indicate no possibility of turbulent core generation, are located almost at the left side of $a_2$. Namely, the following equation about the generation of turbulent core can be given:

- Generation of turbulent core
  \[ \text{Re} \cdot k_b > 5 \times 10^3 \]
  \[ \text{No generation of turbulent core} \]
  \[ \text{Re} \cdot k_b < 5 \times 10^3 \]

5.3 Growth of turbulent core

Fig.11 is a map of the classification of the regions obtained in the same manner as Fig.8, using pipes of various inner diameters and various liquids under two conditions, namely the condition of the generation of the turbulent core (eq.(6-1) ), and the condition of constant velocity region ($c$<0.6). From this figure, the following are revealed: the region in which the following equation is satisfied is called "disturbed region" here, and in this region a turbulent core grows up sharply.

- $S > 60/Re$

This rapid growth of a turbulent core can be thought to be due to the phenomena analyzed by Squire of growth of waves on gas-liquid surface when the liquid from a nozzle is ejected into the gas. In other words, it is likely that the liquid film on the pipe wall near the tip surface is disturbed slightly by the obstacle, and at convex part of the disturbance the relative velocity of air increases, the static pressure at this part decreases, the convex part drawn out gradually into the gas, and consequently, the surface grows sharply uneven.

But even if a turbulent core generates, under the condition of the following equation the disturbance on the surface will disappear under the effects of viscosity, surface tension, and so on, and the tip surface will become stable.

\[ S < 30/Re \]

Eqs.(7),(8) apply only for the case where the gas is air.

6. Measurement of Liquid Film on Pipe Wall and Discussion

6.1 Experimental apparatus and procedure

When the tip surface passes through an obstacle, a turbulent core generates, grows up sharply, and then a large quantity of liquid is left on the pipe wall. Now, time change of thickness of liquid film falling down on the pipe wall is measured by Fukano's conductance method. According to this method, output voltage between measured electrodes $S$ is equal to $A \cdot S_{_n}$, where $S_{_n}=(\pi^2/d^2-(d-2c)^2)$ is reciprocal ratio of the liquid film to cross sectional area of the pipe, $d$ is thickness of the liquid film, and $A$ is constant in this measured system. Following this equation, when $S_{_n}$ is taken as unity, the liquid fills up entirely the space between the measured electrodes, and as the value of $S_{_n}$ becomes larger, the value of $d$ becomes smaller.
A schematic diagram of the experimental apparatus is almost similar to Fig.1, and a circular pipe of inner diameter $d$ of 25mm and total length of 1.8m with an obstacle attached on the pipe wall at the position $x=90cm$ is used. This obstacle is an orifice of hole diameter $D=24.5mm$ and the thickness 5mm, and the measured part of the thickness of the liquid film is a pair of annular and copper electrodes of the thickness 1mm, which are buried along inner circle of the pipe with $x$-direction distance of 5mm between these electrodes. These pairs of electrodes are arranged as shown in Fig.12, and these pairs are located directly in front and at back of the orifice, where the distance from the orifice is 3cm.

Liquid film measurements are performed in the case of initial liquid height $L$ of 1.75m and pressure $P_0$ of 69kPa, using tap water. According to Fig.7, we examined four regions, namely, stable region of $\varepsilon=0.1$, transitional region of $\varepsilon=0.18$, disturbed region of $\varepsilon=0.3$ and 0.45, and accelerated region of $\varepsilon=0.8$ and 1.0. The output voltage $E$ of each electrode is recorded on a data recorder through an isolation amplifier, and played back on a visigraph. Further in order to make the electric current density uniform, the distance between the electrode of power supply and a pair of measured electrodes is kept larger than 15cm, and a constant direct current of 0.5mA is supplied. Moreover, before every experiment of liquid film measurements, acrylic test poles of known outer diameter are immersed in tap water of the pipe, and the relations between $E$ and $S_n$ of every pair of electrodes are examined.

6.2 Effect of obstacle on thickness of liquid film

Fig.13 shows the time change of $S_n$ in each measured electrode for the disturbed region of $\varepsilon=0.45$. According to Fig.12, if the position of electrode $la$ is at larger than 90cm (hereafter unit "cm" is omitted), the position $za$ is upward of the obstacle, and if $la$ is at less than 90, the position is downward of the obstacle. From Fig.13, it is known that, after the tip surface passes through the obstacle, in the first place at the position of $la=75$ the tip surface begins to be disturbed slightly, and as $a$ decreases, the gradient of the curve $S_n$ immediately after the beginning of a rising of the curve decreases, and then, the liquid film on the pipe wall becomes very thick. Namely, the thickness $\delta$ of the liquid film on the pipe wall changes largely, and the turbulent core grows up sharply.

Fig.14 shows the $x$-direction distribution of thicknesses $\delta$ of the film, in the case that the tip surface arrives at the position of $la=15$, and the following are revealed. In the stable region of $\varepsilon<0.1$, the thickness $\delta$ of the liquid film is small and the disturbance of the surface

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Fig.12 Schematic illustration of apparatus for measurement of thickness of liquid film

Fig.13 Relations between $\varepsilon$ and $S_n$

Fig.14 Distribution of $\delta$ in $x$-direction

Fig.15 Relations between $\varepsilon$ and $\delta a^*$
is slight. But in the disturbed region of $c=0.45$, $\delta$ is greater than 8mm and the film is very thick within the limits of 10cm from the tip surface, and the liquid volume left on the pipe wall near the tip surface is large. Because the shape of the tip surface is stably hemispherical in the acceleration region, the liquid film of the acceleration region of $c=1.1$ is thinner near the tip surface than the liquid film of the disturbed region of $c=0.45$.

Next, liquid volume left on the pipe wall backward of the tip surface at the time when the tip surface arrives at $z_c$, is divided by $\pi d^2/4$, and the result is called $Q_t^a$ here. Fig.15 shows a rough estimate of $Q_t^a$ for $c$ in the case of $z_c$ being 15 and 65. From this figure it is known that the difference between $Q_t^a$ of $z_c=15$ and $Q_t^a$ of $z_c=65$ is equal to about 0.4 in the stable region of $c=0.1,4,1$ in the disturbed region of $c=0.45$, and 1.6 in the acceleration region of $c=1$. In the disturbed region of $c=0.45$, $Q_t^a$ shows an increase of 4.1, while the tip surface moves 20%. Namely, the tip surface moves downward leaving on the pipe wall about 25% of efflux liquid volume from the bottom nozzle.

6.3 Velocity of tip in disturbed region
Now the difference in the velocity of the tip between the disturbed and stable regions is examined experimentally in the constant velocity region. Fig.16 shows the velocity of the tip at the position of the tip in the two regions, namely the disturbed region, where initial liquid height $z_b$ is higher than the position of the obstacle $z_b$ and a turbulent core generates, and the stable region, where $z_b$ is less than $z_b$ and a turbulent core does not appear. The velocity of the tip in the disturbed region (black symbol) increases approximately 20% near the nozzle in comparison with the velocity in the stable region (white symbol), and this agrees nearly perfectly with the above mentioned fact that, in the disturbed region of $c=0.45$ the tip surface moves downward leaving on the pipe wall about 25% of the efflux liquid volume from the nozzle.

7. Conclusions
The behaviors of a pressurized air-liquid surface in vertical and circular pipes with a nozzle at the bottom end are studied experimentally, and the following are revealed.

(1) The progress of the tip surface is divided in two regions, namely acceleration region where $c$ is greater than 0.6, and constant velocity region where $c$ is less than 0.6. In the acceleration region, the velocity of the tip agrees well with a one dimensional average flow model based on eqs.(3),(4), and in the constant velocity region, the velocity of the tip agrees well with this model when the contraction coefficient of the nozzle is taken into consideration.

(2) When the velocity of the tip increases downward, the behavior of the liquid near the surface becomes a problem of Rayleigh-Taylor instability, and the surface is disturbed sharply, but this disturbance affects only the liquid movement near the surface.

(3) In the acceleration region, the shape of the tip surface is hemispherical regardless of an obstacle on pipe wall. And in the constant velocity region without an obstacle, the shape of the tip is flat.

(4) When the tip surface passes through an obstacle, the generation of a turbulent core, which is wrinkles or disturbances on air-liquid surface, depends on eqs.(6-1), (6-2), $z_b$, $x$.

(5) In the constant velocity region with air as the gas, when eq.(7) is satisfied, the turbulent core grows up sharply, and a disturbed region, where the surface is disturbed largely and becomes snaky, appears. But when eq.(8) is satisfied, the turbulent core disappears, and a stable region appears.

(6) In the disturbed region, a great deal of liquid is left on the pipe wall. The remaining liquid volume amounts to almost 25% of the efflux liquid volume from the nozzle, and the velocity of the tip of the disturbed surface is faster than the velocity of the stable surface.

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