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Fundamental Investigation of an Oil Damper*
2nd Report Analysis Based on the Unsteady Flow
Assumption

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This paper deals with dynamic characteristics of an oil damper, whose piston is vibrating axially in a viscous fluid filling a fixed circular cylinder. Assuming a one-dimensional flow past the annular cross-sectional clearance between the piston and the cylinder, we evaluated a drag force acting on the piston. And the analytical results were compared with experimental observations. The following results were obtained.

- (1) The velocity profile in the clearance depends on the dimensionless factor $\lambda = h\sqrt{\omega/(2\nu)}$, where h , ω , ν are radial clearance, circular frequency and kinematic viscosity, respectively. Being a parabolic distribution within $\lambda < 2$ as is the case in a steady flow, for larger values of λ the velocity profile approaches uniformity over the clearance.
- (2) Pressure drag and friction drag exerted on the piston are both out of phase with the velocity of the piston. Therefore "damping coefficient" and "added mass" must be deemed the representative characteristics of the oil damper.

Key Words: Vibration, Vibration Isolation, Oil Damper, Unsteady Flow, Damping Coefficient, Added Mass, Dynamic Characteristics

1. Introduction

Recently, vibration isolation has been discussed from many different view points. However the developments of isolators and devices are retarded in spite of the advance in their theories.⁽¹⁾ One of the reasons is due to the difficulty to produce an isolator with optimum damping, so it may be desirable to add suitable damping using another damper to the main isolator which has small amount of damping.

Though there are many types of dampers, they have both merits and demerits. The magnetic damper is frequently reported recently, and it may come to be applied practically before long.^{(2),(3)} Meanwhile on the oil damper theoretical study is not yet developed enough. The reason may be considered as follows: The structures of oil dampers are not so simple that the actual phenomena can be analyzed accurately.⁽⁴⁾ Moreover behaviors of oil dampers are often dependent on environmental conditions such as temperature, mounting orientation, and so on, and have nonlinear characteristics essentially. But these demerits will be overcome in future by research. On the other hand, the oil damper has a large merit in that it produces a large damping force for its small size. So it can be

expected that the oil damper will play a large part in development and improvement of the vibration isolators.

The authors are studying how to design oil dampers which are able to change the damping coefficient according to working states without stopping the machine. To achieve this purpose, they began to study a simple shaped oil damper which had an annular clearance between piston and cylinder as a flow path of oil. The equations which have been used for this type of oil damper are gross approximation by assumptions that the oil flow is steady, etc.⁽⁵⁾⁻⁽⁷⁾ A new equation derived in the 1st report⁽⁸⁾ of this work is improved and can be used more widely than the conventional ones. But the limit of application under the assumption of steady flow still remains in the new equation as well.

In this report, dynamic characteristics of the oil damper are much clear by an analysis assuming a one-dimensional unsteady flow through the clearance. The theoretical analysis derived in this report agrees well with the experimental values over a wider range than the equation derived in the 1st report. And it is confirmed that the equation based on the analysis for an unsteady flow has great availability.

2. Theoretical Analysis

In order to simplify the analysis, the following assumptions are made:

- (1) The cylinder is at rest and the piston vibrates at steady amplitude and frequency.
- (2) The annular cross-sectional clearance between the piston and the cylinder is replaced approximately by a path with two

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parallel flat walls.

(3) The oil flow through the path is one-dimensional, and all fluid particles move in parallel to direction of the central axis of the cylinder.

(4) The oil is an incompressible Newtonian fluid.

(5) Pressure gradient in the direction of flow is constant over the whole length of the piston. Pressure on the front or rear face of the piston is uniformly distributed and it is equal to the pressure at inlet or outlet of the path, respectively.

(6) The effects of the gravitational force and the piston rod are negligible.

Strictly speaking, there is no stationary foundation. So the cylinder and the piston of oil damper are both vibrating, as shown in Fig.1(a). In this report, however, it is assumed that the oil damper is mounted on a fixed foundation, as shown in Fig.1(b), in order to study the fundamental behaviors of the damper. If the relative displacement between the mass and the foundation shown in Fig.1(a) is equal to the absolute displacement of the mass shown in Fig.1(b), they are regarded as equivalent systems.

The oil damper is represented schematically in Fig.1(c) according to the assumption (2), and the flow through the clearance is described in Cartesian coordinates x - y . When the annular clearance is approximated by the straight path mentioned above, it is desirable that there be no change in cross-sectional area $A_0 = bh$ of the clearance. For that reason, breadth b of the path must be equal to average value of inner perimeter of the cylinder and perimeter of the piston, that is, $b = \pi(dc + dp)/2$, because radial clearance $h = (dc - dp)/2$ should not be changed, where dc and dp are diameters of the cylinder and the piston, respectively. As an inevitable consequence the side area of the piston becomes greater than the actual side area $\pi dp l$ and its value is $b l = \pi(dc + dp)l/2$.

From the assumption (3), the equation of motion can be written as

$$\frac{\partial v}{\partial t} - v \frac{\partial^2 v}{\partial x^2} = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (1)$$

where ρ and ν are density and kinematic viscosity of the fluid, respectively. And the assumption (1) yields two boundary conditions

$$v(0, t) = v_p \cos \omega t, \quad v(h, t) = 0 \quad (2)$$

By the substitution

$$\partial p / \partial y = (\Delta P_f / l) \cos(\omega t + \xi) \quad (3)$$

Fig.1 Single-degree-of-freedom system equipped with an oil damper

the partial differential equation (1) can be reduced to an ordinary differential equation. In Eq.(3) ΔP_f is a pressure difference between front and rear faces of the piston caused by the pressure loss within the path. It is further assumed that solutions for the velocity v exist as products of a function of x alone and a function t alone. Moreover it is clear that the solutions are periodic in time. Therefore it is initially assumed that

$$v(x, t) = X(x) e^{i\omega t}, \quad (i = \sqrt{-1}) \quad (4)$$

From the relation between Eq.(2) and Eq.(4) only the real part of the complex quantity in solution v has physical meaning. The complex notation is applied to the equation (3) too. Then, substituting these equations into Eq.(1) and rearranging, we obtain the following ordinary differential equation of motion for the function $X(x)$:

$$\frac{d^2 X}{dx^2} - i \frac{\omega}{\nu} X = \frac{\Delta P_f}{\mu l} e^{i\xi} \quad (5)$$

where $\mu (= \rho \nu)$ is viscosity of fluid. The general solution of this equation is

$$X(x) = (A_1 \cosh \lambda \frac{x}{h} + A_2 \sinh \lambda \frac{x}{h}) \cos \lambda \frac{x}{h} + i(A_2 \cosh \lambda \frac{x}{h} + A_1 \sinh \lambda \frac{x}{h}) \sin \lambda \frac{x}{h} + P v_p e^{i(\xi + \pi/2)} \quad (6)$$

Here A_1 and A_2 are the integral constants, and

$$\lambda = h \sqrt{\omega / (2\nu)} \quad (7)$$

$$P = \Delta P_f / (\omega \rho l v_p) \quad (8)$$

represent the dimensionless frequency and pressure gradient. The constants A_1 and A_2 are determined by boundary conditions $X(0) = v_p$ and $X(h) = 0$, which are derived from Eq.(2). Substituting $X(x)$ into Eq.(4) and returning to the real notation, the solution of the equation (1) can be written as

$$\begin{aligned} \frac{v}{v_p} = & \frac{1}{\cosh 2\lambda - \cos 2\lambda} \{ \cos \omega t \{ \cosh \lambda (2 - \frac{x}{h}) \cos \lambda \frac{x}{h} - \cosh \lambda \frac{x}{h} \cos \lambda (2 - \frac{x}{h}) \} \\ & + \sin \omega t \{ \sinh \lambda (2 - \frac{x}{h}) \sin \lambda \frac{x}{h} - \sinh \lambda \frac{x}{h} \sin \lambda (2 - \frac{x}{h}) \} \} \\ & + \frac{P}{\cosh \lambda + \cos \lambda} \{ \sin(\omega t + \xi) \{ \cosh \lambda (1 - \frac{x}{h}) \cos \lambda \frac{x}{h} + \cosh \lambda \frac{x}{h} \cos \lambda (1 - \frac{x}{h}) \} \\ & - \cos(\omega t + \xi) \{ \sinh \lambda (1 - \frac{x}{h}) \sin \lambda \frac{x}{h} + \sinh \lambda \frac{x}{h} \sin \lambda (1 - \frac{x}{h}) \} \} \\ & - P \sin(\omega t + \xi) \end{aligned} \quad (9)$$

Above solution contains two unknown quantities P and ξ , and these can be determined by the equation of continuity

$$A_p v_p \cos \omega t = -b \int_0^h v(x, t) dx \quad (10)$$

where $A_p = \pi d_p^2/4$ is cross-sectional area of the piston. The left-hand side of Eq.(10) means the volume of oil excluded by the piston, and the right means the volume of flow passing the clearance. Substituting Eq.(9) into Eq.(10), integrating, and simplifying, we thus have the equation

$$\{(\lambda c_1 - s_1)P \sin \xi + s_2 P \cos \xi - s_1/2 - \lambda A c_1\} \cos \omega t + \{(\lambda c_1 - s_1)P \cos \xi - s_2 P \sin \xi - s_2/2\} \sin \omega t = 0 \quad (11)$$

where c_1, c_2, s_1 and s_2 are addition or subtraction of a hyperbolic and a trigonometric function, and they are written as

$$\left. \begin{aligned} c_1 &= \cosh \lambda + \cos \lambda, & c_2 &= \cosh \lambda - \cos \lambda \\ s_1 &= \sinh \lambda + \sin \lambda, & s_2 &= \sinh \lambda - \sin \lambda \end{aligned} \right\} \quad (12)$$

In order to satisfy the equation (11) for all values of time t , both coefficients $\cos \omega t$ and $\sin \omega t$ must be zero. Thus the following two equations are obtained:

$$P \cos \xi = \frac{1}{2} \frac{\lambda(2A+1)c_1 s_2}{(\lambda c_1 - s_1)^2 + s_2^2} \quad (13)$$

$$P \sin \xi = \frac{1}{2} \frac{(\lambda c_1 - s_1)(2\lambda A c_1 + s_1) - s_2^2}{(\lambda c_1 - s_1)^2 + s_2^2} \quad (14)$$

where $A = A_p/bh$ represents the ratio of cross-sectional area of the piston to that of the clearance. The quantities P and ξ , as shown in Eqs.(13) and (14), are expressed in terms of A and λ . And the pressure drag $D_p(t)$ which being caused by a pressure difference between front and rear walls of the piston can be calculated by the equation

$$\begin{aligned} D_p(t) &= A_p \Delta P_f \cos(\omega t + \xi) \\ &= 2\mu l(b/h) A v_p \lambda^2 P \cos(\omega t + \xi) \end{aligned} \quad (15)$$

Besides, the piston receives the friction drag $D_f(t)$ due to a shearing force working on the side wall, and the drag force can be expressed as

$$D_f(t) = -\mu b l \left. \frac{\partial v}{\partial x} \right|_{x=0} \quad (16)$$

The minus sign of the right-hand term indicates that the direction of the drag force is always opposite to that of the motion. Using the equation (9), velocity gradient on the piston wall $x=0$ is derived as follows:

$$\begin{aligned} -\frac{h}{v_p} \left. \frac{\partial v}{\partial x} \right|_{x=0} &= \frac{\lambda}{2} \left\{ \frac{\lambda^2(2A+1)c_1 s_2 + s_1}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_1}{c_2} \right\} \cos \omega t \\ &\quad - \frac{\lambda}{2} \left\{ \frac{\lambda(2A+1)c_1(\lambda s_1 - 2c_2) + s_2}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_2}{c_2} \right\} \sin \omega t \end{aligned} \quad (17)$$

The left-hand term represents a dimensionless velocity gradient, which is designated as $F \cos(\omega t + \delta)$. Comparing left-term with right-term, we obtain

$$\begin{aligned} F \cos \delta &= \frac{\lambda}{2} \left\{ \frac{\lambda^2(2A+1)c_1 s_2 + s_1}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_1}{c_2} \right\} \\ &= \lambda^2 P \cos \xi + \lambda s_1 / (2c_2) \end{aligned} \quad (18)$$

$$\begin{aligned} F \sin \delta &= \frac{\lambda}{2} \left\{ \frac{\lambda(2A+1)c_1(\lambda s_1 - 2c_2) + s_2}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_2}{c_2} \right\} \\ &= \lambda^2 (P \sin \xi - A) + \lambda s_2 / (2c_2) \end{aligned} \quad (19)$$

Using the quantities F and δ , the friction drag $D_f(t)$ can be written as

$$D_f(t) = \mu l(b/h) v_p F \cos(\omega t + \delta) \quad (20)$$

The results of discussion are summarized in Table 1. The central column of Table 1 shows the analysis based on steady flow assumption, in this case the piston moves at constant velocity $v_{prms} = v_p/\sqrt{2}$. The drag forces obtained by the analysis for steady flow will carry the subscript s in order to distinguish them from the forces due to the unsteady flow analysis.

The shape of the velocity distribution in the annulus between piston and cylinder is determined by the dimensionless factors A and λ , and examples are shown in Fig.2. The abscissa indicates a dimensionless ratio of the flow rate v to the maximum velocity v_p of the piston, and the ordinate the ratio of a distance x from the piston to the radial clearance h . Fig.2(a) shows the velocity distribution in the case of steady flow ($\lambda=0$) evaluated by Eq.(1.7) in Table 1. The state of $A = A_p/bh = d_p^2/(d_c^2 - d_p^2) = 0.5$ means that the diameter of the piston is $1/\sqrt{3}$ times as long as the inner diameter of the cylinder.

Fig.2(b)-(d) show the velocity distribution for various values of λ , i.e., 2, 5 and 10, when ω increases, whereas A remains a constant value of 0.5. Each of them contains seven stages of velocity distribution in the half-cycle of oscillation $\omega t = 0 \sim \pi$. The flow pattern in the remainder of a cycle, $\omega t = \pi \sim 2\pi$, are merely reversed in the direction of flow from the figure. It is obvious from Fig.2 that a parabolic distribution is gradually converted into a rectangular one with an increasing frequency. On the contrary, when A increases, the velocity profile will not change so much if abscissa's scale is changed, as shown in Fig.2(e),(f).

Table 1. Comparison between analyses of steady flow and unsteady flow

Item	Steady flow	Unsteady flow
Equation of continuity	$A_p v_{prms} = -b \int_0^h v(x) dx$ [1.1]	$A_p v_p \cos \omega t = -b \int_0^h v(x, t) dx$ [1.2]
Equation of motion	$\frac{dp}{dy} = \mu \frac{d^2 v}{dx^2}$ [1.3]	$\frac{\partial v}{\partial t} - \nu \frac{\partial^2 v}{\partial x^2} = -\frac{1}{\rho} \frac{\partial p}{\partial y}$ [1.4]
Boundary condition	$v(0) = v_{prms}, v(h) = 0$ [1.5]	$v(0, t) = v_p \cos \omega t, v(h, t) = 0$ [1.6]
Solution	$\frac{v}{v_{prms}} = (1 - \frac{x}{h}) \times \{1 - 3(2A+1)\frac{x}{h}\} \quad [1.7]$	$\frac{v}{v_p} = \frac{1}{\cosh 2\lambda - \cos 2\lambda} [\cos \omega t \{ \cosh \lambda (2 - \frac{x}{h}) \cos \lambda \frac{x}{h} - \cosh \lambda \frac{x}{h} \cos \lambda (2 - \frac{x}{h}) \} + \sin \omega t \{ \sinh \lambda (2 - \frac{x}{h}) \sin \lambda \frac{x}{h} - \sinh \lambda \frac{x}{h} \sin \lambda (2 - \frac{x}{h}) \}] + \frac{P}{\cosh \lambda + \cos \lambda} [\sin(\omega t + \xi) \{ \cosh \lambda (1 - \frac{x}{h}) \cos \lambda \frac{x}{h} + \cosh \lambda \frac{x}{h} \cos \lambda (1 - \frac{x}{h}) \} - \cos(\omega t + \xi) \{ \sinh \lambda (1 - \frac{x}{h}) \sin \lambda \frac{x}{h} + \sinh \lambda \frac{x}{h} \sin \lambda (1 - \frac{x}{h}) \}] - P \sin(\omega t + \xi) \quad [1.8]$
Pressure drag	$D_{ps} = 6\mu l \frac{b}{h} A(2A+1) v_{prms}$ [1.9]	$D_p(t) = 2\mu l \frac{b}{h} A v_p \lambda^2 P \cos(\omega t + \xi)$ [1.10]
Friction drag	$D_{fs} = 2\mu l \frac{b}{h} (3A+2) v_{prms}$ [1.11]	$D_f(t) = \mu l \frac{b}{h} v_p F \cos(\omega t + \delta)$ [1.12]

Fig.2 Velocity distribution in the path between two parallel flat walls

3. Added Mass and Damping Coefficient

From the balance of forces acting on the body of mass m shown in Fig.1(b), the equation of motion is derived as

$$m\ddot{y} + D_p(t) + D_f(t) + ky = f(t) \quad (21)$$

Substituting Eqs.(15) and (20) obtained in the previous chapter into the pressure drag $D_p(t)$ and the friction drag $D_f(t)$, and using the equations $\dot{y} = v_p \cos \omega t$ and $\ddot{y} = -\omega v_p \sin \omega t$, the equation of motion is transformed into

$$\left\{ m + \frac{m_0}{2\lambda^2} (2A\lambda^2 P \sin \xi + F \sin \delta) \right\} \ddot{y} + \mu l \frac{b}{h} (2A\lambda^2 P \cos \xi + F \cos \delta) \dot{y} + ky = f(t) \quad (22)$$

where $m_0 = \rho b h l$ means the total mass of a fluid filling the flow path. The second term of the coefficient of $\ddot{y}$ represents an added mass m_A due to accelerated motion of the fluid and the coefficient of $\dot{y}$ means a damping coefficient. If the fluid velocity in the path is uniformly distributed over its width, h , the added mass will be given by the quantity $A^2 m_0$. Therefore the ratio

$m_A/(A^2 m_0)$ means a magnification factor of added mass due to non-uniformly distributed velocity, and it is designated by CA . We shall call CA an added mass coefficient. The added mass coefficient and the damping coefficient are tabulated in Table 2. Both coefficients consist of two sources; pressure drag and friction drag, which are identified by subscripts p and f , respectively.

The added mass and the damping coefficient can be estimated from energy considerations. Average value ED of the energy dissipation within the damper caused by the damping forces $D_p(t)$ and $D_f(t)$ is

$$\begin{aligned} E &= \frac{1}{\tau} \oint (\text{damping force}) dy \\ &= \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \{D_p(t) + D_f(t)\} \dot{y} dt \\ &= \mu l (b/h) (2A\lambda^2 P \cos \xi + F \cos \delta) v_{prms}^2 \quad (23) \end{aligned}$$

If the damping force is given in the form $c\dot{y}$, ED is expressed as $c v_p^2/2 = c v_{prms}^2$. Comparing these equations, it is easily confirmed that damping coefficient is equal to Eq.[2.6], which is the sum of Eqs.(2.2) and [2.4] in Table 2. The dissipation function is generally defined as half of the rate of

Table 2. Damping coefficient and added mass of oil damper

	Steady flow	Unsteady flow
Damping coefficient	$C_{ps} = 6\mu l \frac{b}{h} A(2A+1)$ [2.1]	$C_p = 2\mu l \frac{b}{h} A \lambda^2 P \cos \xi$ [2.2]
	$C_{fs} = 2\mu l \frac{b}{h} (3A+2)$ [2.3]	$C_f = \mu l \frac{b}{h} F \cos \delta$ [2.4]
	$C_s = 4\mu l \frac{b}{h} (3A^2+3A+1)$ [2.5]	$C = \mu l \frac{b}{h} \lambda \left[\frac{\lambda^2(2A+1)^2 c_1 s_2^2 + s_1}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_1}{c_2} \right]$ [2.6]
Added mass coefficient	$CA_{ps} = \frac{6}{5} + \frac{1}{10A}$ [2.7]	$CA_p = \frac{P}{A} \sin \xi$ [2.8]
	$CA_{fs} = \frac{1}{10A} + \frac{2}{15A^2}$ [2.9]	$CA_f = \frac{F}{2A^2 \lambda^2} \sin \delta$ [2.10]
	$CA_s = \frac{6}{5} + \frac{1}{5A} + \frac{2}{15A^2}$ [2.11]	$CA = \frac{1}{4A^2} \left[\frac{\lambda(2A+1)^2 c_1 (\lambda c_1 - s_1)}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_2}{\lambda c_2} - 4A - 1 \right]$ [2.12]

energy dissipation E_D .⁽⁹⁾ However in a few books the dissipation function is defined as E_D itself.⁽¹⁰⁾

The added mass can be obtained by Rayleigh's method which is one of the energy methods. Kinetic energy E_M of the body of mass m and the same energy E_A of the fluid within the path given by the piston motion are calculated as follows:

$$E_M = \frac{1}{2} m \frac{\omega}{2\pi} \int_0^{2\pi/\omega} (v_p \cos \omega t)^2 dt$$

$$= (1/2) m v_{p \text{ rms}}^2 \quad (24)$$

$$E_A = \frac{1}{2} \rho b l \int_0^h \frac{\omega}{2\pi} \int_0^{2\pi/\omega} v^2(x, t) dt dx$$

$$= (1/2) m_A v_{p \text{ rms}}^2 \quad (25)$$

where

$$m_A = A^2 C_{A0} \rho \quad (26)$$

$$CA = CA_0 + \frac{CA_1}{A} + \frac{CA_2}{A^2} \quad (27)$$

$$CA_0 = \frac{\lambda c_1 (\lambda c_1 - s_1)}{(\lambda c_1 - s_1)^2 + s_2^2} \quad (28)$$

$$CA_1 = \frac{c_1 (\lambda s_1 - 2c_2)}{(\lambda c_1 - s_1)^2 + s_2^2} = CA_0 - 1 \quad (29)$$

$$CA_2 = \frac{1}{4} \left\{ \frac{c_1 (\lambda s_1 - 2c_2)}{(\lambda c_1 - s_1)^2 + s_2^2} + \frac{s_2}{\lambda c_2} \right\}$$

$$= \frac{1}{4} \left(CA_1 + \frac{s_2}{\lambda c_2} \right) \quad (30)$$

Total kinetic energy of the system is expressed as $E_M + E_A = \frac{1}{2} (m + m_A) v_{p \text{ rms}}^2$, so that m_A is an added mass under consideration. The added mass coefficient CA obtained by the energy method seems to be different from the one in Table 2, but the brief calculation will demonstrate the relationships

$$CA_p = \frac{P}{A} \sin \xi = CA_0 + \frac{CA_1}{2A} \quad (31)$$

$$CA_f = \frac{F}{2A^2 \lambda^2} \sin \delta = \frac{CA_1}{2A} + \frac{CA_2}{A^2} \quad (32)$$

It is clear that Eq.(27) is the same as Eq. [2.12] in Table 2 because $CA = CA_p + CA_f$.

The added mass coefficient CA is a function of two quantities A and λ , but its subcoefficients CA_0 , CA_1 and CA_2 are functions of λ alone. In the case of steady flow, i.e., $\lambda \rightarrow 0$, as shown in Fig.3, these coefficients CA_0 , CA_1 and CA_2 asymptotically approach $6/5$, $1/5$ and $2/15$, respectively. Therefore if A increases and the velocity of side wall of piston becomes negligibly small comparing to the flow rate, the added mass coefficient CA will approach $6/5$. This is the magnification factor of kinetic

energy in parabolic velocity distribution of the flow between two parallel flat walls. And in the second case of $\lambda \rightarrow \infty$, CA becomes unity because the velocity profile tends to have uniform distribution.

Applying Eq.[1.7], kinetic energy E_A s for steady flow is evaluated as follows:

$$E_A = \frac{1}{2} \rho b l \int_0^h v^2(x) dx$$

$$= (1/2) A^2 C_{A0} m_0 v_{p \text{ rms}}^2 \quad (33)$$

$$CA_s = \frac{6}{5} + \frac{1}{5A} + \frac{2}{15A^2} \quad (34)$$

and it is confirmed that the added mass coefficient CA_s for steady flow is the same as $\lim_{\lambda \rightarrow 0} CA$.

It should be noted that the added mass and the damping coefficient mentioned previously are caused by fluid motion within the path, and under the assumption (5) the effects of the fluid out of the path, that is, the fluid before and after the piston are quite neglected. These effects will be considered later in comparing with the experimental values.

Using the equations in Table 1, we can write the damping coefficient and the added mass in unsteady flow relative to steady flow as follows:

$$\frac{C_p}{C_{ps}} = \frac{\lambda^2 P \cos \xi}{3(2A+1)} = \frac{\lambda^3 c_1 s_2}{6\{(\lambda c_1 - s_1)^2 + s_2^2\}} \quad (35)$$

$$\frac{C_f}{C_{fs}} = \frac{F \cos \delta}{2(3A+2)}$$

$$= \frac{\lambda}{4(3A+2)} \left\{ \frac{\lambda^2(2A+1)c_1 s_2^2 + s_1}{(\lambda c_1 - s_1)^2 + s_2^2} \right\} \quad (36)$$

$$\frac{CA_p}{CA_{ps}} = \frac{10P \sin \xi}{12A+1}$$

$$= \frac{5}{12A+1} \frac{(\lambda c_1 - s_1)(2\lambda A c_1 + s_1) - s_2^2}{(\lambda c_1 - s_1)^2 + s_2^2} \quad (37)$$

$$\frac{CA_f}{CA_{fs}} = \frac{15F \sin \delta}{\lambda^2(3A+4)}$$

$$= \frac{15}{2\lambda(3A+4)} \left\{ \frac{\lambda(2A+1)c_1(\lambda s_1 - 2c_2) + s_2}{(\lambda c_1 - s_1)^2 + s_2^2} \right\} \quad (38)$$

The values of above equations are plotted against λ for various dimensionless areas A in Fig.4 and Fig.5.⁽⁷⁾ The factor λ equals $h\sqrt{\omega/(2\nu)}$, so Fig.4 and Fig.5 exhibit the frequency characteristics of the oil damper for ω .

Fig.3 Subcoefficients of magnification factor CA of kinetic energy

Fig.4 Effect of unsteadiness on damping coefficient ; theory

Fig.5 Effect of unsteadiness on added mass ; theory

Fig.6 Phase angle between pressure drag and harmonic piston velocity

Fig.7 Phase angle between friction drag and harmonic piston velocity

An inspection of Fig.4 and Fig.5 reveals the following interesting facts: The ratio c_p/c_{ps} is a function of λ alone, but the remainders c_f/c_{fs} , C_{Ap}/C_{Aps} and C_{Af}/C_{Afs} depend on A and λ . However these almost depend only on λ provided the diameter of piston is not much smaller than the inner diameter of cylinder. And no particular change due to unsteady flow is recognized within $\lambda < 2$, because the parabolic velocity distribution is kept up to $\lambda = 2$ as shown in Fig.2. In the case of $\lambda > 2$, the damping coefficient increases and the added mass decreases by the effect of unsteady flow.

Fig.6 and Fig.7 show the phase angles ξ and δ of the pressure and the friction drag versus the piston velocity. When the dimensionless frequency λ is small, the angles ξ and δ are nearly equal to zero. But with an increase of λ , ξ approaches 90 deg. and δ approaches 45 deg., asymptotically.

4. Comparison of Theoretical with Experimental Results

Experimental equipment and experimental method for the oil damper are described in detail in the 1st report of this work, so in this report these are described briefly. Dimensions of the oil damper are $d_c = 36$ mm, $d_p = 16 \sim 35.8$ mm and $l = 20$ mm, and four types of silicone oil with different

viscosities are used. Fig.8 shows temperature dependencies of kinematic viscosity of the silicone oils measured by the Saybolt Universal viscometer.

The damping coefficient c of the oil damper can be obtained by measuring the amplitudes a_0 , a_1 , the phase angle $\phi - \gamma$ and the frequency ω of the vibratory system shown in Fig.1(a).⁽¹¹⁾ The experimental values are plotted against the radial clearance h in Fig.9. The theoretical values c of unsteady flow calculated by Eq.[2.6] are shown as solid lines, and the theoretical values c_s of steady flow calculated by Eq.[2.5] are drawn with dashed lines. The experimental values have better coincidence with the theoretical ones based on unsteady flow analysis than those of steady flow analysis. In order to make clear above observation, the experimental values c are divided by the theoretical ones c_s of steady flow. The values of c/c_s versus λ are plotted in Fig.10, and theoretical curves vary slightly with v because c_f/c_{fs} is influenced by A . Fig.10 shows that experimental values are larger than theoretical ones, but they vary with similar inclination. It was made clear from Fig.10 that the ratio c/c_s of damping coefficients with reference to the theoretical values based on the steady flow analysis is almost determined by the factor λ alone, even if the quantities ω , v and h are arbitrarily changed.

Similarly, the experimental values of added mass m_A , which are evaluated from the measurement of undamped natural frequency, are shown in Fig.11 and Fig.12. As another added mass m_A' due to fluid motion outside of the clearance was neglected in the theory, the experimental added mass m_A was corrected by means of subtraction of $m_A' = \rho d_p^3/3$ which is an estimated value based on the potential theory.⁽¹²⁾ However, it is likely that a greater value of kinematic viscosity ν will result in a larger error of m_A , which appears in this figure.

5. Conclusions

Assuming a one dimensional flow, the authors have analysed the oil damper having the piston in simple harmonic motion and the cylinder at rest. The results are tabulated in Tables 1 and 2. Comparing the theoretical values with the experimental ones, the following results are derived.

(1) The drag force acting on piston and the piston velocity are out of phase owing to the inertia force of fluid. So the oil damper has two characteristics ; damping coefficient and added mass.

(2) The ratios of damping coefficient and added mass based on theory of steady flow show that the effect of unsteady flow may depend only on one parameter $\lambda = h\sqrt{\omega/(2\nu)}$. The flow is considered to be steady in the range of $\lambda < 2$. As λ is larger than 2, however, the damping coefficient increases and the added mass decreases by the effect of unsteady flow.

(3) In order to estimate the damping coefficient in this type of oil damper, the following equation has been used up to the present day.⁽⁵⁾⁻⁽⁷⁾

$$c = \frac{12\mu LA_p^2}{\pi d_p h^3} \\ = 12\mu l \frac{\pi d_p}{h} A^2 \quad (39)$$

which is applicable only at low frequency level because

Fig.8 Viscosity-temperature relationship for typical silicone oils

Fig.9 Damping coefficient versus radial clearance in oil damper

Fig.11 Added mass versus radial clearance in oil damper

Fig.10 Effect of unsteadiness on damping coefficient ; theory and experiment

Fig.12 Effect of unsteadiness on added mass ; theory and experiment

of the assumption of steady flow. (It is interesting to compare Eq.(39) with Eq.[2.1] in Table 2, putting $b = \pi d_p$. Eq.(39) can be derived from Eqs.[1.1] and [1.3] by converting the boundary condition [1.5] into $v(0) = v(h) = 0$ in Table 1.) The equation proposed in this report can be used up to higher frequency, and covers the case of a laminar flow through the path. The transition from laminar to turbulent regime may occur when the Reynolds number

$$Re = \frac{(d_c - d_p)v_a}{\nu} = \frac{d_c^2 v_p}{(d_c + d_p)\nu} \quad (40)$$

increases beyond almost 2000, where v_a is average velocity in the clearance. It is easily possible to keep Re well below 2000, provided an oil with very small viscosity is not used.

(4) In using the present equation the piston length l must be sufficiently large compared to the radial clearance h in order to decrease the effect of the inlet length where the flow does not become fully developed and to decrease the non-linearity due to vortices caused in upper and lower regions of the piston.

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