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Fluid Inertia Effects on the Characteristics of Spherical Hydrostatic Bearings (Part 1 Bearing Forces)*

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Based on a first-order perturbation solution in a modified Reynolds number, an analysis is presented to determine the fluid inertia effects on the dynamic characteristics of a spherical hydrostatic bearing that has a continuous spherical surface. This analysis demonstrates that fluid inertia influences the load capacity and dynamic properties. The corrections to the conventional elastic and damping coefficients derived on the assumption that inertia forces are negligible, are found not too small to neglect when inertia parameter $\delta = \rho \omega^2 r_0^2 / p^* > 0.1$ (ρ =lubricant density, ω =rotational angular velocity, r_0 =radius of spherical bearing, p^* =ambient pressure). And the acceleration coefficients may become significant when the displacement of a rotor from the bottom of the bearing is small.

Key Words: Lubrication, Vibration of Rotating Body, Spherical Hydrostatic Bearing, Inertia Effect, Dynamic Characteristics

1. Introduction

Classical lubrication theory assumes a laminar flow and neglects inertia terms in the equation of motion governing a lubricant film flow. Although these assumptions are justified for small values of the Reynolds number, they are valid in the majority of bearing applications. However, inertia effects may become noticeable at high operating speeds without affecting the assumption of laminar flow. So far several investigators have examined the fluid inertia effects on the static characteristics of spherical hydrostatic bearings.^{(1)~(3)} Dowson and Taylor have carried out theoretical and experimental works and concluded that the inertia force alters the pressure distribution considerably while the inertia effects on load capacity are rather small.

This paper describes the inertia effects on the dynamic characteristics of spherical hydrostatic bearings. This analysis, which employs a first order perturbation expansion in Reynolds number,^{(4)~(7)} demonstrates that fluid inertia effects on the dynamic properties appear even for small values of Reynolds number.

2. Nomenclature

a_{xx} : acceleration coefficient
($A_{xx} = a_{xx} c_r \omega^2 / p^* r_0^2$)

$b_{ij}(i, j = x, y)$: damping coefficients
($B_{ij} = b_{ij} c_r \omega / p^* r_0^2$)

c_r : radial clearance

e_r, e_z : radial and axial displacements

($\epsilon_r = e_r / c_r, \epsilon_z = e_z / c_r$)

h : film thickness ($H = h / c_r$)

$k_{ij}(i, j = x, y)$: elastic coefficients

($K_{ij} = k_{ij} c_r / p^* r_0^2$)

p : pressure distribution ($P = p / p^*$)

p_i : pressure at inlet ($P_i = p_i / p^*$)

p^* : ambient pressure

r_0 : bearing radius

$R = r / r_0$

r, θ, φ : spherical coordinates (Fig.1)

$R_e^* = \rho c_r^2 \omega / \mu$

t : time ($\tau = \omega t$)

v_r, v_θ, v_φ : velocity components in r, θ, ϕ directions, respectively

$V_r = v_r / c_r \omega, V_\theta = v_\theta / r_0 \omega, V_\varphi = v_\varphi / r_0 \omega$

Fig.1 Spherical hydrostatic bearing

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$\delta = R_e^*/A$
 $\zeta = (r_0 - r)/c, \quad (0 \leq \zeta \leq H)$
 θ_0, θ_1 : inlet and outlet angles (Fig.1)
 $A = c^2 p^*/\mu \omega r_0^2$
 μ : lubricant viscosity
 ρ : lubricant density
 ω : angular velocity of rotor

3. Analysis

3.1 The governing equations

This paper is concerned with a spherical hydrostatic bearing of the form shown in Fig.1. With the assumptions of usual lubrication theory, the governing equations for an incompressible fluid are written in dimensionless form as:

$$\partial P / \partial \zeta = 0 \quad (1)$$

$$R_e^* \left\{ \frac{\partial V_\theta}{\partial \tau} - V_r \frac{\partial V_\theta}{\partial \zeta} + V_\theta \frac{\partial V_\theta}{\partial \theta} + \frac{V_\theta}{\sin \theta} \frac{\partial V_\theta}{\partial \varphi} - V_\theta^2 \cot \theta \right\} = -A \frac{\partial P}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial \zeta^2} \quad (2)$$

$$R_e^* \left\{ \frac{\partial V_\varphi}{\partial \tau} - V_r \frac{\partial V_\varphi}{\partial \zeta} + V_\theta \frac{\partial V_\varphi}{\partial \theta} + \frac{V_\theta}{\sin \theta} \frac{\partial V_\varphi}{\partial \varphi} + V_\theta V_\varphi \cot \theta \right\} = -\frac{A}{\sin \theta} \frac{\partial P}{\partial \varphi} + \frac{\partial^2 V_\varphi}{\partial \zeta^2} \quad (3)$$

$$\frac{\partial V_r}{\partial \zeta} - \frac{1}{\sin \theta} \left\{ \frac{\partial}{\partial \theta} (V_\theta \sin \theta) + \frac{\partial V_\varphi}{\partial \varphi} \right\} = 0 \quad (4)$$

where $R_e^* = \rho c^2 \omega / \mu$ and $A = c^2 p^* / \mu \omega r_0^2$.

The left hand sides in Eqs.(1) and (3) are the contributions from inertia forces which have been ignored in the classical theory. A parameter Re^* accounts for the inertia forces, and it is generally a small value. Equation (2) only expresses that the pressure does not vary across the film, and is of no interest in the further analysis. To investigate the inertia effects, a first-order perturbation solution in Re^* is carried out by setting⁽⁴⁾,

$$P = P_0 + R_e^* P_1 \quad (5)$$

$$V_r = V_r^{(0)} + R_e^* V_r^{(1)}, \quad V_\theta = V_\theta^{(0)} + R_e^* V_\theta^{(1)}, \quad V_\varphi = V_\varphi^{(0)} + R_e^* V_\varphi^{(1)} \quad (6)$$

Substituting Eqs.(5) and (6) into Eqs.(2) through (4) and rearranging the terms of zeroth and first order in Re^* yields the following equations:

$$(R_e^*)^0: -A \frac{\partial P_0}{\partial \theta} + \frac{\partial^2 V_\theta^{(0)}}{\partial \zeta^2} = 0 \quad (7)$$

$$-A \frac{1}{\sin \theta} \frac{\partial P_0}{\partial \varphi} + \frac{\partial^2 V_\varphi^{(0)}}{\partial \zeta^2} = 0 \quad (8)$$

$$\frac{\partial V_r^{(0)}}{\partial \zeta} - \frac{1}{\sin \theta} \left\{ \frac{\partial}{\partial \theta} (V_\theta^{(0)} \sin \theta) + \frac{\partial V_\varphi^{(0)}}{\partial \varphi} \right\} = 0 \quad (9)$$

$$(R_e^*)^1: \frac{\partial V_\theta^{(0)}}{\partial \tau} - V_r^{(0)} \frac{\partial V_\theta^{(0)}}{\partial \zeta} - V_\theta^{(0)} \frac{\partial V_\theta^{(0)}}{\partial \theta} + \frac{V_\theta^{(0)}}{\sin \theta} \frac{\partial V_\theta^{(0)}}{\partial \varphi} - V_\theta^{(0)2} \cot \theta = -A \frac{\partial P_1}{\partial \theta} - \frac{\partial^2 V_\theta^{(1)}}{\partial \zeta^2} \quad (10)$$

$$\frac{\partial V_\varphi^{(0)}}{\partial \tau} - V_r^{(0)} \frac{\partial V_\varphi^{(0)}}{\partial \zeta} + V_\theta^{(0)} \frac{\partial V_\varphi^{(0)}}{\partial \theta} + \frac{V_\theta^{(0)}}{\sin \theta} \frac{\partial V_\varphi^{(0)}}{\partial \varphi} + V_\theta^{(0)} V_\varphi^{(0)} \cot \theta = -\frac{A}{\sin \theta} \frac{\partial P_1}{\partial \varphi} - \frac{\partial^2 V_\varphi^{(1)}}{\partial \zeta^2} \quad (11)$$

$$\frac{\partial V_r^{(1)}}{\partial \zeta} - \frac{1}{\sin \theta} \left\{ \frac{\partial}{\partial \theta} (V_\theta^{(1)} \sin \theta) + \frac{\partial V_\varphi^{(1)}}{\partial \varphi} \right\} = 0 \quad (12)$$

The velocity boundary conditions are:

$$\zeta = 0: V_r = 0, \quad V_\theta = 0, \quad V_\varphi = 0, \quad \zeta = H: V_\varphi = \sin \theta, \quad V_\theta = 0, \quad V_r = -(\partial H / \partial \tau) - \partial H / \partial \varphi \quad (13)$$

By substituting these expressions into Eqs.(6), the boundary conditions are written as:

$$(R_e^*)^0: \zeta = 0: V_r^{(0)} = V_\theta^{(0)} = V_\varphi^{(0)} = 0, \quad \zeta = H: V_\varphi^{(0)} = \sin \theta, \quad V_\theta^{(0)} = 0, \quad V_r^{(0)} = -\partial H / \partial \tau - \partial H / \partial \varphi \quad (14)$$

$$(R_e^*)^1: \zeta = 0: V_r^{(1)} = V_\theta^{(1)} = V_\varphi^{(1)} = 0, \quad \zeta = H: V_\varphi^{(1)} = V_\theta^{(1)} = V_r^{(1)} = 0 \quad (15)$$

Integrating Eqs.(7) and (8) twice yields the velocity profiles of classical lubrication theory:

$$V_\varphi^{(0)} = \frac{\zeta}{H} \sin \theta + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} (\zeta^2 - H\zeta) \quad (16)$$

$$V_\theta^{(0)} = \frac{A}{2} \frac{\partial P_0}{\partial \theta} (\zeta^2 - H\zeta) \quad (17)$$

Inserting $V_\varphi^{(0)}$ and $V_\theta^{(0)}$ into the continuity equation (9), and applying the boundary condition at $\zeta = H$, we obtain $V_r^{(0)}$ and a differential equation for P_0 as:

$$V_r^{(0)} = -\frac{A}{2} \frac{\zeta^3 - H\zeta^2}{H} \left(\frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \varphi} \right) - \frac{2H\zeta^2 - \zeta^3}{H^3} \frac{\partial H}{\partial \varphi} - \frac{3H\zeta^2 - 2\zeta^3}{H^3} \frac{\partial H}{\partial \tau} \quad (18)$$

$$\frac{A}{12} \left\{ \sin \theta \frac{\partial}{\partial \theta} \left(H^3 \sin \theta \frac{\partial P_0}{\partial \theta} \right) + \frac{\partial}{\partial \varphi} \left(H^3 \frac{\partial P_0}{\partial \varphi} \right) \right\} = \frac{1}{2} \sin^2 \theta \frac{\partial H}{\partial \varphi} + \sin^2 \theta \frac{\partial H}{\partial \tau} \quad (19)$$

Since P_0 is independent of ζ , integrating Eqs.(10) and (11) twice with respect to ζ , and using Eqs.(16) through (18), we obtain $V_\theta^{(1)}$ and $V_\varphi^{(1)}$. Substituting $V_\theta^{(1)}$ and $V_\varphi^{(1)}$ into Eq.(12) and applying the boundary conditions Eqs.(15), we get a differential equation for P_1 as

$$\frac{A}{12} \left\{ \sin \theta \frac{\partial}{\partial \theta} \left(H^3 \sin \theta \frac{\partial P_1}{\partial \theta} \right) + \frac{\partial}{\partial \varphi} \left(H^3 \frac{\partial P_1}{\partial \varphi} \right) \right\} = \sin \theta \left[\frac{\partial S_1}{\partial \theta} + \frac{\partial S_2}{\partial \varphi} \right] \quad (20)$$

where S_1 and S_2 are functions of P_0 and are given in an Appendix to this paper.

3.2 Bearing forces

We will treat radially unloaded bearings. Then a shaft axis coincides with the z-axis (i.e., $\epsilon_r = 0$) in a static equilibrium state. We assume that the pressure distributions induced by small vibrations about z-axis can be written in the following forms:

$$P_0 = P_{00} + \epsilon_r P_{01} + \epsilon_r P_{02} + \epsilon_r P_{03} \quad (21)$$

$$P_1 = P_{10} + \epsilon_r P_{11} + \epsilon_r P_{12} + \epsilon_r P_{13} + \epsilon_r P_{14} + \epsilon_r P_{15} + \epsilon_r P_{16} + \epsilon_r P_{17} \quad (22)$$

where $' = d/d\tau$. Substituting these expressions into Eqs.(19) and (20), together with equations:

$$H = H_0 - \epsilon_r \cos \varphi \sin \theta, \quad H_0 = 1 - \epsilon_r \cos \theta \quad (23)$$

and collecting terms of like order results in zeroth and first order equations for P_{ij} ($i=0,1$, $j=0 \sim 7$).

The differential equations for P_{00} and P_{10} are written as:

$$\sin \theta \frac{\partial}{\partial \theta} \left(H_0^3 \sin \theta \frac{\partial P_{00}}{\partial \theta} \right) = 0 \quad (24)$$

$$\sin \theta \frac{\partial}{\partial \theta} \left(H_0^3 \sin \theta \frac{\partial P_{10}}{\partial \theta} \right) = \frac{12}{A} \sin \theta \frac{\partial}{\partial \theta} (\sin \theta S_{10}) \quad (25)$$

where

$$S_{10} = -\frac{A^2}{120} \left\{ H_0^3 \frac{\partial P_{00}}{\partial \theta} \frac{\partial^2 P_{00}}{\partial \theta^2} + 2H_0^2 \frac{\partial H_0}{\partial \theta} \left(\frac{\partial P_{00}}{\partial \theta} \right)^2 \right\} + \frac{H_0^3}{40} \sin \theta \cos \theta \quad (26)$$

Integrating Eqs.(24) and (25) twice with respect to θ and applying the boundary conditions:

$$\theta = \theta_0 : P_{00} = P_0, \quad P_{10} = 0, \quad \theta = \theta_1 : P_{00} = 1, \quad P_{10} = 0$$

we obtain

$$P_{00} = C, \quad K(\theta) = P_0 \quad (27)$$

$$P_{10} = \frac{3}{20A} \left\{ (\sin^2 \theta - \sin^2 \theta_0) - (\sin^2 \theta_1 - \sin^2 \theta_0) \frac{K(\theta)}{K(\theta_1)} \right\} + \frac{3Ac_1^2}{280} \left\{ F(\theta) - \frac{K(\theta)}{K(\theta_1)} F(\theta_1) \right\} \quad (28)$$

where

$$c_1 = -(P_0 - 1)/K(\theta_1) \quad (29)$$

$$K(\theta) = \int_{\theta_0}^{\theta} \frac{d\theta}{H_0^3 \sin^3 \theta}, \quad F(\theta) = \int_{\theta_0}^{\theta} \frac{\cos \theta - \epsilon_r \cos 2\theta}{H_0^3 \sin^3 \theta} d\theta \quad (30)$$

P_{10} is the pressure distribution due to inertia effects. When we consider only centrifugal effects, the second term in the right hand side of Eq.(28) vanishes.⁽²⁾

The axial load capacity F_z is calculated from the following expressions:

$$F_{z0} = \int_0^{2\pi} \int_{\theta_0}^{\theta_1} (P_{00} - 1) \sin \theta \cos \theta d\theta d\varphi, \quad F_{z1} = \int_0^{2\pi} \int_{\theta_0}^{\theta_1} P_{10} \sin \theta \cos \theta d\theta d\varphi \quad (31)$$

$$F_z = F_{z0} + R_{\epsilon} F_{z1} \quad (32)$$

where

$$F_{z0} = \pi(P_0 - 1) \sin^2 \theta_1 - \frac{2\pi(P_0 - 1)}{K(\theta_1)} \int_{\theta_0}^{\theta_1} K(\theta) \sin \theta \cos \theta d\theta \quad (33)$$

$$F_{z1} = \frac{3\pi}{10A} (\sin^2 \theta_1 - \sin^2 \theta_0) \left\{ \frac{1}{4} (\sin^2 \theta_1 - \sin^2 \theta_0) - \frac{1}{K(\theta_1)} \int_{\theta_0}^{\theta_1} K(\theta) \sin \theta \cos \theta d\theta \right\} + \frac{3\pi Ac_1^2}{140} \left\{ \int_{\theta_0}^{\theta_1} F(\theta) \sin \theta \cos \theta d\theta - \frac{F(\theta_1)}{K(\theta_1)} \int_{\theta_0}^{\theta_1} K(\theta) \sin \theta \cos \theta d\theta \right\} \quad (34)$$

In this paper we set $\theta_0 = 10^\circ$ and $\theta_1 = 90^\circ$. The influence of these values on the load capacity and the dynamic properties will be discussed in the following paper.

The dependence of load capacity F_{z0} on ϵ_r at different values of P_0 is shown in Fig.2. Load capacities are dependent on P_0 and not on A . The load capacity increases with the axial displacement, i.e., $Cx(1 - \epsilon_z)$. Fig.3 shows F_{z1} as a function of

Fig.2 F_{z0} versus ϵ_z

Fig.3 F_{z1} versus ϵ_z

ϵ_z . As the axial displacement increases, F_z becomes positive from negative. Thus, for large values of axial displacement, i.e., $\epsilon_z < 0$, fluid inertia effects increase load capacity F_z since F_z is positive. When the rotor speed is high, the value of Λ is generally low. Under such a condition, the second term in the right hand side of Eq. (34) is much smaller than the first term which is a contribution from the centrifugal force only. In other words, the effect of centrifugal force on load capacity is much greater than that of the other inertial forces.

Considering P_{ij} ($i=0,1, j=1\sim 7$) as functions with a period of 2π with respect to φ , we set

$$P_{ij} = P_{ij}^c(\theta) \cos \varphi + P_{ij}^s(\theta) \sin \varphi \quad \dots\dots\dots (35)$$

Then, the differential equations for P_{ij} are written as

$$L(P_{ij}^k) = R_{ij}^k(\theta) \quad (i=0,1, j=1\sim 7, k=c,s) \quad \dots\dots\dots (36)$$

where

$$L(P) = \frac{d}{d\theta} \left(H_0^3 \sin \theta \frac{dP}{d\theta} \right) - \frac{H_0^3}{\sin \theta} P \quad \dots\dots\dots (37)$$

, and P_{ij}^c and P_{ij}^s are functions of θ . Assuming the solution of Eqs. (36) to be written in the form:

$$P_{ij}^k = \sum_{m=1}^N (a_{ij}^k)_m \sin \left(m\pi \frac{\theta - \theta_0}{\theta_1 - \theta_0} \right) \quad \dots\dots\dots (38)$$

since the boundary conditions are

$$\begin{aligned} \theta = \theta_0 : P_{ij}^c = P_{ij}^s = 0 \\ \theta = \theta_1 : P_{ij}^c = P_{ij}^s = 0 \end{aligned} \quad (i=0,1, j=1\sim 7)$$

we obtain P_{ij}^c and P_{ij}^s by Galerkin's method. The final solution for P is given in the form of Eq. (5).

The radial and tangential bearing forces F_r and F_ϕ are calculated from the following expressions:

$$\begin{aligned} F_r &= \int_0^{2\pi} \int_0^{\theta_1} P \cos \varphi \sin^2 \theta d\theta d\varphi \\ F_\phi &= \int_0^{2\pi} \int_0^{\theta_1} P \sin \varphi \sin^2 \theta d\theta d\varphi \end{aligned} \quad \dots\dots\dots (39)$$

By introducing an x-y coordinate system with origin in the bearing center as shown in Fig. 4, the bearing forces in the x- and y-directions are:

$$F_x = F_r \cos \phi - F_\phi \sin \phi, \quad F_y = F_r \sin \phi + F_\phi \cos \phi \quad \dots\dots\dots (40)$$

Considering the relations:

$$\epsilon_x = \epsilon_r \cos \phi, \quad \epsilon_y = \epsilon_r \sin \phi \quad \dots\dots\dots (41)$$

Fig. 4 Radial and tangential components of lubricant film forces

we obtain F_x and F_y as:

$$\begin{aligned} F_x &= K_{xx}\epsilon_x + K_{xy}\epsilon_y + B_{xx}\dot{\epsilon}_x + B_{xy}\dot{\epsilon}_y + A_{xx}\ddot{\epsilon}_x \\ F_y &= K_{yx}\epsilon_x + K_{yy}\epsilon_y + B_{yx}\dot{\epsilon}_x + B_{yy}\dot{\epsilon}_y + A_{yy}\ddot{\epsilon}_y \end{aligned} \quad \dots\dots\dots (42)$$

where

$$\begin{aligned} K_{xx} &= K_{xx0} + R_e^* K_{xx1}, \quad K_{yy} = K_{yy0} \\ K_{xy} &= K_{xy0} + R_e^* K_{xy1}, \quad K_{yx} = -K_{xy} \end{aligned} \quad \dots\dots\dots (43)$$

$$\begin{aligned} B_{xx} &= B_{xx0} + R_e^* B_{xx1}, \quad B_{yy} = B_{yy0} \\ B_{xy} &= B_{xy0} + R_e^* B_{xy1} = R_e^* B_{yx1} \\ B_{yx} &= -B_{xy} \end{aligned} \quad \dots\dots\dots (44)$$

$$A_{xx} = R_e^* A_{xx1}, \quad A_{yy} = A_{yy0} \quad \dots\dots\dots (45)$$

In Eqs. (43) through (45), the terms multiplied by R_e^* are corrections due to fluid inertia effects. In addition to the usual elastic and damping coefficients, Eq. (42)

Fig. 5 Elastic coefficient K_{xx} ($Ps=2$)

Fig. 6 Elastic coefficient K_{xy} and damping coefficient B_{xx} ($Ps=2$)

implies that the bearing forces also depend on the rotor accelerations. These acceleration coefficients A_{xx} and A_{yy} vanish when fluid inertia effects are neglected.

In Figs.5 through 12 are shown the elastic, damping and acceleration coefficients as functions of ϵ_z and Ps . K_{xx0} depends not on Λ but on Ps . Although K_{xy0} and B_{xx0} depend on Λ , ΛK_{xy0} and ΛB_{xx0} are nearly constant for values of $\Lambda < 0.1$. Similarly the coefficients ΛK_{xx1} , K_{xy1} , B_{xx1} , ΛB_{xy1} and ΛA_{xx1} , which are due to fluid inertia effects, are considered to be independent of Λ for values of $\Lambda < 0.1$. Fig.5 shows that K_{xx0} and K_{xx1} increase with the axial displacement, i.e., $Cr(1-\epsilon_z)$. When the axial displacement is large, K_{xx1} is positive. In other words, the inertia effects increase the radial stiffness K_{xx} for $\epsilon_z < 0$. On the other hand, when the axial displacement is small, K_{xx1} is negative. Thus, the inertia effects decrease

the radial stiffness for $\epsilon_z > 0$. In Fig.6, as the axial displacement increases, $-K_{xy1}$ increases while ΛK_{xy0} decreases. The cross coupling elastic coefficient $K_{xy} = K_{xy0} + Re^* K_{xy1}$ decreases as the axial displacement increases. Fig.7 shows that the cross-coupling damping coefficients $B_{xy} = B_{xy0} + Re^* B_{xy1}$ appears when $Re^* \neq 0$, that is, when the inertia effects are not negligible. The acceleration coefficient ΛA_{xx1} is shown in Fig.8. As the axial displacement increases, ΛA_{xx1} decreases rapidly at first and then slowly at larger axial displacement.

Figs.9 and 10 show that K_{xx0} , $-K_{xy1}$ and B_{xx1} increase linearly with Ps , while the other coefficients, ΛK_{xy0} , ΛB_{xx0} , ΛK_{xx1} and ΛA_{xx1} are independent of Ps as shown in Figs.11 and 12.

Rewriting Eqs.(43), (44) and (45), we get

$$\left. \begin{aligned} B_{zz} &= (\bar{B}_{zz0} + Re^* \Lambda B_{zz1}) / \Lambda, & B_{zy} &= \partial \bar{B}_{zy1} \\ K_{zz} &= K_{zz0} + \partial K_{zz1} \\ K_{zy} &= (\bar{K}_{zy0} + Re^* \Lambda K_{zy1}) / \Lambda, & A_{zz} &= \partial \bar{A}_{zz1} \end{aligned} \right\} \dots\dots\dots (46)$$

Fig.7 Damping coefficient B_{xy}

Fig.9 K_{xx0} versus Ps

Fig.8 Acceleration coefficient A_{xx}

Fig.10 K_{xy1} and B_{xx1} versus Ps

Fig. 11 ΔB_{xx0} , ΔK_{xy0} and ΔB_{xy1} versus P_s Fig. 12 ΔK_{xx1} and ΔA_{xx1} versus P_s

where

$$\delta = R_e^* / \Lambda \quad (47)$$

and $\bar{B}_{xx0} = \Lambda B_{xx0}$, $\bar{B}_{xy1} = \Lambda B_{xy1}$, $\bar{K}_{xx1} = \Lambda K_{xx1}$, $\bar{K}_{xy0} = \Lambda K_{xy0}$, $\bar{A}_{xx1} = \Lambda A_{xx1}$. Λ and Re^* are small values, and as shown in Fig. 6 K_{xy1} and B_{xx1} are much smaller than ΛK_{xy0} and ΛB_{xx0} , respectively. Therefore we make approximations:

$$\bar{B}_{xx1} \approx \bar{B}_{xx0} / \Lambda = B_{xx0}, \quad K_{xy1} \approx K_{xy0} / \Lambda = K_{xy0} \quad (48)$$

Expressions (48) show that the fluid inertia effects on the coefficients B_{xx} and K_{xy} are small.

As mentioned above, $\bar{K}_{xx0}$, $\bar{K}_{xx1} = \Lambda K_{xx1}$, $\bar{B}_{xy1} = \Lambda B_{xy1}$ and $\bar{A}_{xx1} = \Lambda A_{xx1}$ are independent of Λ . Thus, K_{xx} , B_{xy} and A_{xx} vary linearly with δ . Calculations are performed for a bearing whose bearing parameters are: $r_0 = 10\text{mm}$, $Cr = 10\mu\text{m}$. Setting $p^* = 0.1\text{MPa}$, $\rho = 0.9 \times 10^3 \text{kg/m}^3$, and $\mu = 0.01\text{Pa}\cdot\text{s}$, we get $\Lambda = 0.01$ and $Re^* = 0.009$ for $\omega = 1000\text{rad/s}$, and $\Lambda = 0.001$ and $Re^* = 0.09$ for $\omega = 10000\text{rad/s}$. In Table 1, the values of the three coefficients are listed. It can be noted that the values of these coefficients vary considerably even when Re^* is small.

Table 1 Changes of coefficients with δ ($P_s = 2$, $\epsilon_z = -1$)

δ	0	0.9	90
Λ		0.01	0.001
Re^*	0	0.009	0.09
K_{xx}	0.475	0.555	8.50
B_{xy}	0	0.113	11.3
A_{xx}	0	0.227	22.7

4. Conclusions

- 1) The effect of centrifugal force on load capacity is much greater than that of the other inertial forces.
- 2) The coefficients K_{xx} and B_{xy} vary linearly with $\delta = Re^* / \Lambda$.
- 3) The inertial effects on B_{xx} and K_{xy} are quite limited.
- 4) When inertial forces are considered, the bearing forces depend not only on the rotor displacement and velocity but also on its acceleration. The acceleration coefficients A_{xx} and A_{yy} increase linearly with δ .

Appendix

$$S_1 = \sin \theta \{ -(H/2)T_1 + T_2 \}, \quad S_2 = -(H/2)T_3 + T_4$$

$$T_1 = \frac{A}{2} \left(-\frac{H^2}{12} \frac{\partial^2 P_0}{\partial \tau \partial \theta} - \frac{H^3}{6} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \tau} \right) + \frac{A \cdot H^3}{2 \cdot 60} \frac{\partial H}{\partial \varphi} \frac{\partial P_0}{\partial \theta} + \frac{A \cdot H^3}{2 \cdot 60} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \tau} \\ + \left(\frac{A}{2} \right)^2 \frac{\partial P_0}{\partial \theta} \left(\frac{H^3}{60} \frac{\partial^2 P_0}{\partial \theta^2} + \frac{H^3}{30} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} \right) + \frac{A}{2 \sin \theta} \left\{ -\frac{H^4}{30} \sin \theta \frac{\partial^2 P_0}{\partial \varphi \partial \theta} \right. \\ \left. - \frac{H^3}{12} \sin \theta \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \varphi} + \frac{A H^3}{120 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial^2 P_0}{\partial \varphi \partial \theta} + \frac{A H^3}{60 \sin \theta} \frac{\partial H}{\partial \varphi} \frac{\partial P_0}{\partial \varphi} \frac{\partial P_0}{\partial \theta} \right\} \\ - \cot \theta \left\{ \frac{(H \sin \theta)^2}{12} + \frac{H^3}{240} \left(\frac{A}{\sin \theta} \frac{\partial P_0}{\partial \varphi} \right)^2 - \frac{A H^3}{30} \frac{\partial P_0}{\partial \varphi} \right\}$$

$$\begin{aligned}
T_2 = & \frac{A}{2} \left(-\frac{H^5}{40} \frac{\partial^2 P_0}{\partial \tau \partial \theta} - \frac{H^4}{24} \frac{\partial H}{\partial \tau} \frac{\partial P_0}{\partial \theta} \right) + \frac{H^6}{840} \left(\frac{A}{2} \right)^2 \left(\frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \varphi} \right) \frac{\partial P_0}{\partial \theta} \\
& - \frac{AH^4}{1680} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \varphi} - \frac{AH^4}{840} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \tau} + \left(\frac{A}{2} \right)^2 \frac{\partial P_0}{\partial \theta} \left(\frac{H^7}{210} \frac{\partial^2 P_0}{\partial \theta^2} + \frac{H^6}{120} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} \right) \\
& + \frac{A}{2 \sin \theta} \left(-\frac{H^5}{120} \sin \theta \frac{\partial^2 P_0}{\partial \varphi \partial \theta} - \frac{H^4}{60} \sin \theta \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \varphi} + \frac{AH^7}{420 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial^2 P_0}{\partial \varphi \partial \theta} \right. \\
& \left. + \frac{AH^6}{240 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \varphi} \right) - \cot \theta \left\{ \frac{H^5 \sin^2 \theta}{60} + \frac{H^7}{840} \left(\frac{A}{\sin \theta} \frac{\partial P_0}{\partial \varphi} \right)^2 - \frac{AH^5}{120} \frac{\partial P_0}{\partial \varphi} \right\} \\
T_3 = & -\frac{H}{6} \sin \theta \frac{\partial H}{\partial \tau} - \frac{A}{24 \sin \theta} \left(H^4 \frac{\partial^2 P_0}{\partial \tau \partial \varphi} + 2H^3 \frac{\partial H}{\partial \tau} \frac{\partial P_0}{\partial \varphi} \right) + \sin \theta \left\{ -\frac{AH^3}{60} \left(\frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} \right. \right. \\
& \left. \left. + \frac{1}{\sin^2 \theta} \frac{\partial H}{\partial \varphi} \frac{\partial P_0}{\partial \varphi} \right) + \frac{7}{60} H \frac{\partial H}{\partial \varphi} + \frac{3}{20} H \frac{\partial H}{\partial \tau} \right\} + \frac{AH^3}{120 \sin \theta} \frac{\partial P_0}{\partial \varphi} \left(\frac{\partial H}{\partial \varphi} + \frac{\partial H}{\partial \tau} \right) \\
& + \frac{A}{2} \frac{\partial P_0}{\partial \theta} \left\{ \frac{H^5}{30} \left(-\frac{\cos \theta}{H} + \frac{\sin \theta}{H^2} \frac{\partial H}{\partial \theta} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right) + \frac{AH^4}{120} \left(\frac{1}{\sin \theta} \frac{\partial^2 P_0}{\partial \theta \partial \varphi} - \frac{\cos \theta}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \right) \right\} \\
& + \frac{1}{\sin \theta} \left\{ -\frac{AH^4}{60} \frac{\partial^2 P_0}{\partial \varphi^2} - \frac{H^3}{12} \sin \theta \left(\frac{\sin \theta}{H^2} \frac{\partial H}{\partial \varphi} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right) + \frac{H^6}{240} \left(\frac{A}{\sin \theta} \right)^2 \frac{\partial P_0}{\partial \varphi} \frac{\partial^2 P_0}{\partial \varphi^2} \right. \\
& \left. + \frac{AH^3}{60 \sin \theta} \frac{\partial P_0}{\partial \varphi} \left(\frac{\sin \theta}{H^2} \frac{\partial H}{\partial \varphi} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right) \right\} + \frac{A}{2} \frac{\partial P_0}{\partial \theta} \cot \theta \left\{ -\frac{H^4}{30} \sin \theta + \frac{AH^6}{120 \sin \theta} \frac{\partial P_0}{\partial \varphi} \right\} \\
T_4 = & -\frac{H^2}{24} \sin \theta \frac{\partial H}{\partial \tau} - \frac{A}{2 \sin \theta} \left(\frac{H^5}{40} \frac{\partial^2 P_0}{\partial \tau \partial \varphi} + \frac{H^4}{24} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \tau} \right) + \frac{\sin \theta}{H} \left\{ -\frac{AH^5}{240} \left(\frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} \right. \right. \\
& \left. \left. + \frac{1}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \varphi} \right) + \frac{H^3}{40} \frac{\partial H}{\partial \varphi} + \frac{H^3}{30} \frac{\partial H}{\partial \tau} \right\} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \left\{ \frac{AH^5}{1680} \left(\frac{\partial P_0}{\partial \theta} \frac{\partial H}{\partial \theta} \right. \right. \\
& \left. \left. + \frac{1}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \varphi} \right) - \frac{H^4}{840} \frac{\partial H}{\partial \varphi} - \frac{H^4}{420} \frac{\partial H}{\partial \tau} \right\} + \frac{A}{2} \frac{\partial P_0}{\partial \theta} \left\{ -\frac{H^6}{120} \left(\frac{\cos \theta}{H} - \frac{\sin \theta}{H^2} \frac{\partial H}{\partial \theta} \right) \right. \\
& \left. - \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right\} + \frac{AH^7}{420} \left(\frac{1}{\sin \theta} \frac{\partial^2 P_0}{\partial \theta \partial \varphi} - \frac{\cos \theta}{\sin^2 \theta} \frac{\partial P_0}{\partial \varphi} \right) + \frac{1}{\sin \theta} \left\{ -\frac{AH^5}{240} \frac{\partial^2 P_0}{\partial \varphi^2} \right. \\
& \left. - \frac{H^4 \sin \theta}{60} \left(\frac{\sin \theta}{H^2} \frac{\partial H}{\partial \varphi} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right) + \frac{H^7}{840} \left(\frac{A}{\sin \theta} \right)^2 \frac{\partial P_0}{\partial \varphi} \frac{\partial^2 P_0}{\partial \varphi^2} \right. \\
& \left. + \frac{AH^6}{240 \sin \theta} \frac{\partial P_0}{\partial \varphi} \left(\frac{\sin \theta}{H^2} \frac{\partial H}{\partial \varphi} + \frac{A}{2 \sin \theta} \frac{\partial P_0}{\partial \varphi} \frac{\partial H}{\partial \theta} \right) \right\} + \frac{A}{2} \frac{\partial P_0}{\partial \theta} \cot \theta \left\{ -\frac{H^5}{120} \sin \theta + \frac{AH^7}{420 \sin \theta} \frac{\partial P_0}{\partial \varphi} \right\}
\end{aligned}$$

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