539.373.01:539.4.014

Paper No. 217-2

Plastic Behavior of Type 304 Stainless Steel after Superimposing Prestrains*

By Kenji KANEKO**

Stress-strain relations of commercialized metal in reloading in various directions after axial and torsional preloadings of various sequences and magnitudes are superimposed, are experimentally investigated. Experimental results are put in order on the basis of a double surface hardening model which consists of elastic, transitional hardening and steady hardening regions. Utility of the double surface theory for describing the complex plastic deformation behavior of metal is confirmed under combined stress conditions. It is also confirmed that the effect of the past deformation history on plastic properties of metal can not perfectly be liquidated in general by any following loadings. Effective stress-strain relation in the transitional hardening region, which is represented in non-dimensional expressions, can be shown by a specific expression without regard to material and prestrain history. Many other significant conclusions are obtained.

Key Words; Plasticity, Anisotropic Metal, Hardening Model, Superposing Prestrains, Yield Surface, SUS 304

1. Introduction

Commercialized industrial materials (metal) are variously prestressed (rolling, drawing, et.) and put to various heat treatments. For an accurate strength design of plastic worked goods and for an accurate structure design, plastic constitutive equation must be constructed for an initially anisotropic material which suffers various superposed amounts of prestrains in various directions.

We already have a considerable number of papers which report on plastic constitutive equations of metal. But, in most of these papers, materials are assumed initially isotropic and some of these theories are too complex with excessive attention to small strain and the rest are too simple to represent an actual behavior of metal accurately.

In this paper, stress-strain relations of type 304 stainless steel under reloading in various directions after axial and torsional preloadings of various sequences and magnitudes are experimentally investigated. Usefulness of the double-surfaces-hardening-model proposed in the previous report, is verified in application to SUS 304 stainless steel. Some important rules on plastic behavior of metal are found and formulated.

2. Symbols

σ, ϵ : Effective stress and effective strain of Mises type, respectively.
 $\sigma_x, \sigma_y, \sigma_{xy}$: Longitudinal, circumferential and shear stresses of a thin walled tubular specimen, respectively.

$\epsilon_x, \epsilon_y, \epsilon_{xy}$: Longitudinal, circumferential and tensor type shear plastic strains of a thin walled tubular specimen, respectively.
 $\sigma_i; i=1 \sim 5$: Cartesian coordinate system components in a five-dimensional deviatoric stress space.⁽²⁾ In this report, only two components, that is, $\sigma_1 (= \sigma_x)$ and $\sigma_3 (= \sqrt{3}\sigma_{xy})$ are used.
 $\epsilon_i; i=1 \sim 5$: Cartesian coordinate system components in a five-dimensional deviatoric stress space. In this report, only two components, that is, $\epsilon_1 (= \epsilon_x^p)$ and $\epsilon_3 (= (2/\sqrt{3})\epsilon_{xy}^p)$ are used.
 σ_I, σ_{II} : Cartesian plane coordinate system components which are arbitrarily set in a five-dimensional deviatoric stress space with common origin. σ_I -axis agrees with the last prestress direction.
 e, p, t : Subscripts which denote elastic, plastic and total components of strain, respectively.
 $j=I, II, \dots$: Upper subscripts which denote the first prestrain, the second prestrain and so on.
 ϵ^T : Amount of total prestrains ($= \sum \epsilon^j; j=I, II, \dots$).
 θ : Angle of reloading direction to the last prestress direction.
 $\sigma_t^{\theta}(\epsilon^j)$: Effective stress in the transitional hardening region in the j th order preloading of θ -direction.
 $\sigma_{\theta}(\epsilon), \sigma'_{\theta}(\epsilon)$: Effective stress in initial loading of θ -direction and the corresponding stress whose curve is moved so as to be in contact with that of $\sigma_t^{\theta}(\epsilon^j)$, respectively.
 F, G : Boundary surface between elastic region and transitional hardening region and yield surface based on the rigid plasticity approximation, respectively.

* Received 12th June, 1981.

** Associate Professor
Science University of Tokyo
Kagurazaka 1-3, Shinjuku-ku
Tokyo 162.

- ρ_d :Amount of transitional plastic strain which arises in the transitional hardening region.
- X, Y :Nondimensional values with respect to plastic strain and stress, respectively, which are used to formulate stress-strain relation in the transitional hardening region.
- A, B, C :Half lengths of axis in σ_1 -axis direction and in σ_3 -axis direction of surface G approximated to ellipse, and length between its center and the coordinate origin, respectively.
- a, b, c :Half length of axis in σ_1 - axis direction and in σ_{II} -axis direction of surface F approximated to ellipse, and length between its center and the coordinate origin, respectively.
- ψ :Loading angle to σ_1 -axis direction and this also is subscript which denotes variables concerned in direction. 1 and 3 are substituted for $\psi=0^\circ$ and 90° , respectively, for example, A_{31} means value A of the case that the first preloading is torsion(3) and the second preloading is tension(1).
- $\Delta\sigma$:Apparent variation in the amounts of A and B, which is caused by superimposing the second prestrain.
- $\Delta\sigma^*$:(Steady) variation in the amounts of A and B, which arises in initial preloading.
- $\Delta\hat{\sigma}$:Transitional variation in the amounts of A and B, which is caused by superimposing the second prestrain. ($=\Delta\sigma-\Delta\sigma^*$)
- $\Delta\sigma_s$:Saturated value of $\Delta\hat{\sigma}$ after fully superimposing the second prestrain.
- $\alpha, \beta, m, n, \xi, m_i (i=1\sim 5), k_i (i=1\sim 3)$:Material constants.

3. Experiment

3.1 Specimen Material of specimen is SUS 304 stainless steel pipe of which inner diameter is 14.0 mm and wall thickness is 7.0 mm. Fig.1 shows the dimensions of specimen. Table 1 shows the chemical compositions of material. 36 specimens were used

Table 1. Chemical composition of material of specimen

Chemical Composition (%)							
C	Si	Mn	P	S	Ni	Cr	Fe
0.042	0.39	1.82	0.031	0.01	8.82	18.49	Rms.

(JIS G3459 SUS304 TP)

Fig.1 Dimensions of specimen

in this experiment. Scatter band of stress-strain relations in tension and torsion loadings is $\pm 5.0\%$ in stress value. But, it is confirmed that nondimensional stress-strain relations divided by corresponding each stress value to strain value which is determined arbitrarily are within $\pm 1.0\%$ - scatter band for strain range of 0.02 to 0.4 (mm/mm) at least and the same holds for the case that prestrains are superimposed. In this paper, the average values of all experimental data are presented.

3.2 Experimental procedures Tests were performed under combined axial load(tension and compression), internal pressure and torsion by using a multi axial combined stress testing machine, closed loop servo-controlled. Strains of specimen were measured by resistance wire strain gauges. Strain rate was controlled within the effective strain rate range of 80 to 100 μ -strain/sec.

Fig.2(a) shows the typical strain paths used in the experiments. Specimens which had been prestrained up to the values of ϵ^I or ϵ^{II} were unloaded perfectly and were held for about twenty hours at room temperature before reloading. Fig.2(b) shows the stress paths used to determine a yield locus after prestraining, where the numbers denote the loading sequences. Yield point is defined as the 0.02% effective proof stress point. The following amounts of $\epsilon^I, \epsilon^{II}$ are selected.

ϵ^I ; 0.02, 0.08, 0.14, 0.20, 0.26 (mm/mm)

ϵ^{II} ; 0.01, 0.06, 0.12, 0.18, 0.24 (mm/mm)

Fig.2(a) Typical strain paths in experiment

Fig.2(b) Loading paths for determining yield surface

4. Conformable Representation of Experimental Results to the Double Surface Hardening Model

4.1 Introduction Some of the experimental results with regard to the stress-strain relations after various multi-prestrainings are shown in Figs.3(a) and (b). In these figures, thin lines and broken lines are initial stress-strain relations in tension and torsion, respectively. From these results, it is found that the stress-strain curves change variably in a shape and a stress level depending on the magnitude, the orientation and the sequence of prestraining.

Author has proposed a double surfaces hardening model as shown in Figs.4(a) and (b) to obtain the common interpretation with respect to elasto-plastic deformation behaviors and has experimentally verified it by many experimental data on mild steel, brass and Al-alloy. In this report, all of the experimental data obtained were arranged on the basis of this hardening model.

Table 2. Material coefficients in the description of initial stress-strain relations

	α	β	n
σ_1	169.16	0.068	0.6512
σ_3	122.24	0.0765	0.5786

Fig.3(a) Stress-strain relations in multi-prestrainings (1)

Fig.3(b) Stress-strain relations in multi-prestrainings (2)

4.2 Initial stress-strain relations

It is confirmed that initial stress-strain relations of SUS 304 can be accurately formulated for a wide strain range of $\epsilon=0.015$ to 0.4 , at least, by the expression (1).

$$\sigma_i = \alpha_i (\beta_i + \epsilon_i)^{n_i} \quad : i = 1, 3, \psi \quad (1)$$

Table 2 shows the values of coefficients of the expression (1) for SUS 304 in tension and torsion loadings.

4.3 Elastic region (Surface F)

Fig.5 shows the relations between the characteristic coefficients a , b and c and the total prestrain values. The symbols are explained in the table of the figure.

In the previous report, the shapes of surface F obtained for mild steel, brass and Al-alloy did not so extend and were almost stable for various prestrainings.⁽¹⁾ However, the surface F for SUS 304 after multi prestrains extends as large as that after mono-prestraining to the same amount of prestrain as one in multi-prestraining. Precisely observed, the shape and the magnitude of the surface F are slightly different depending on the orientations of prestrain and the surface F after triple prestrainings seems to be circular. It is complex and not so beneficial to take into account these small differences in formula-

Fig.4(a) Modeling of stress-strain curve in reloading

Fig.4(b) Double surfaces hardening model

ting the behaviors of surface F. Then, the coefficients a, b and c are formulated by the expression (2), as a function of only total prestrain ϵ^T .

$$a, b, c = m_1 + m_2 \epsilon^{Tm_3} + m_4 e^{m_5 \epsilon^T} \tag{2}$$

Table 3 shows material constants of the expression (2) for SUS 304. Calculated results by the expression (2) and associated coefficients are shown in Fig.5 as thick lines.

4.4 Transitional strain amplitude ρ_d

Fig.6 shows some experimental results with regards to transitional strain amplitude ρ_d in reloading after various prestrainings. If some scatter caused in the variations of sequences and magnitudes of prestrains is neglected, ρ_d can be formulated

by the expression (3) as a function of only θ .

$$\rho_d \times 10^2 = \xi (\theta/180)^m \tag{3}$$

Each coefficient for SUS 304 is shown in Fig.6.

4.5 Transitional hardening curve $\sigma_t(\epsilon^j)$

Fig.7 shows some nondimensional stress-strain relations in transitional hardening region, on the basis of expressions (4) and (5). Effective yield stress σ_{p0}^0 is defined as 0.05% proof stress, whereas it was defined as 0.02% proof stress in the previous report. But, the same expression (6) is obtained as that in the previous report. It is confirmed that the transitional stress-strain curves in reloading can be represented by specifying only a one nondimensional curve without regard to prestrain history and material.

Fig.5(a) Relations between characteristic values of surface F and total prestrain value

Table 3. Material constants in the characteristic expressions of surface F

	m ₁	m ₂	m ₃	m ₄	m ₅
a	8.5	46.19	0.7841	10.0	24.2
b	18.5	41.06	0.3240	0	—
c	0	25.15	0.4183	0	—

Fig.6 Relation between transitional amounts of strain and direction of reloading

a	b	c	Pre-straining		
			I st	II nd	III rd
⊖	⊞	△	T _e		
⊙	⊞	△	T _e		
⊖(⊖)	⊞(⊞)	△(△)	T _e (T _e)	T _e (C _e)	
⊙(⊙)	⊞(⊞)	△(△)	T _e (T _e)	T _e (R _e)	
⊖(⊖)	⊞(⊞)	△(△)	T _e (T _e)	T _e (T _e)	T _e (R _e)
⊙(⊙)	⊞(⊞)	△(△)	T _e (T _e)	T _e (C _e)	T _e (T _e)
⊖	⊞	△	T _e / T _e		
⊙	⊞	△	T _e / T _e	T _e / R _e	
⊖	⊞	△	T _e / T _e	T _e / R _e	T _e / T _e
T _e : Tensile pre-straining					
T _e : Torsional //					
C _e : Compressive //					
T _e / T _e : Combined Tensile and Torsional //

Fig.5(b) Preloading patterns regarding surface F

Fig.7 Nondimensional transitional hardening curve

$$Y = 1 - \{ \sigma_{\theta}^j(\epsilon) - \sigma_{\theta}^j(\epsilon^j) \} / (\sigma_{p1}^{\theta} - \sigma_{p0}^{\theta}) \quad (4)$$

$$X = \epsilon^j / \rho_d^{\theta} \quad (5)$$

$$X^{0.5} + (1 - Y)^{0.43} = 1 \quad (6)$$

4.6 Yield surface determined by the rigid plasticity approximation concept (Surface G) Surface G nearly equals to a yield surface determined by the so-called extrapolation method.

Figs. 8 and 9 show experimental characteristic values A, B and C. Symbols are explained in the figures. From these experimental results, it is found that work hardening in direction of $\theta=90^\circ$, that is, so-called "cross effect" is remarkably large. Especially in torsional reloading after tensile preloading, the stress level is higher than in monotonous tensile loading.

Each curve of $A_3(\Delta)$ and $B_1(\square)$ is not parallel to the initial stress-strain curve of tension and torsion, respectively, but, crosses with them. Hence, it seems that there is no special correlation between the values A, B and the initial stress-strain curve. However, these values of A and B after mono-prestraining can be accurately formulated by the expression (7) which is the same form as the expression (1). Table 4 shows values of coefficients in the expression (7) for SUS 304.

Table 4. Material constants in the expression (7)

	α	β	n
$A_1(\bigcirc)$	139.60	0.0589	0.5679
$B_1(\square)$	196.56	0.0986	0.8094
$A_3(\Delta)$	145.16	0.0716	0.5618
$B_3(\nabla)$	105.46	0.0621	0.4930

The value C can be calculated from the values A and B using Eq. (8). Some calculated results are compared with the corresponding experimental results in Figs. 8 and 9.

$$S_i = \alpha_i (\beta_i + \epsilon_i)^{n_i} \quad (S=A, B, i=1, 3) \quad (7)$$

$$C_i = \sigma_i(\epsilon_i) - S_i(\epsilon_i) \quad (S=A; i=1, S=B; i=3) \quad (8)$$

As regards to A and B after multi-prestrainings, let us observe the results of B_{31} (Symbol; $\blacksquare$) which can be obtained when the second prestrain of tension ($\epsilon^{II}; 0.01, 0.06, 0.12$ and 0.18) is applied after torsional prestraining ($\epsilon^I; 0.08$), for example. The values of B_{31} increase with an increase of ϵ^{II} rapidly at first, later slightly and stably. The steady increase rate is almost the same as that of B_1 (Symbol; $\square$) which can be obtained after tensile mono-prestraining. Variation of the values A_{13} (Symbol; $\blacktriangle$) seems to have the same tendency as variation of B_{31} . The stress levels of B_{31} and A_{13} do not seem to become consistent with those of B_1 and A_3 , respectively, in the range of large values of ϵ^{II} .

This result gives an important suggestion for constructing a general theory of plasticity. Till now, it has been believed that if a deformation not less than a prescribed value is given in a direction of ψ after some various prestrainings, plastic deformation properties become the same as those obtained after initially prestraining up to the same value of strain in the direction of ψ , without regard to the effect of past prestrainings. This assumption is found not to provide a general law of plasticity on these experimental results.

The values A and B after multi-prestrainings are formulated on the basis of the following thinking. For example, Fig. 10 shows schematically the variation of the value B_{31} (Symbol; $\blacksquare$) which can be obtained when tensile preloading is superposed after torsional preloading. In this figure, B_{31}

Fig. 8. Relations between characteristic values of surface G and total prestrain value

is equal to $B_3(\epsilon^I)$ at $\epsilon^{II}=0$. With an increase of ϵ^{II} , the value of B_{31} rises rapidly at first as shown by the symbols ■ and later: after $\epsilon^{II}=\bar{\epsilon}^{II}$ it shows almost the same rate of increase as that of $B_1(\epsilon)$. At the strain point $\epsilon = \epsilon^I + \epsilon^{II}$ in the transitional region, an apparent increased quantity of B_{31} is $\Delta\sigma$ and a transitional net increased portion $\Delta\hat{\sigma}$ is obtained by subtracting a steady increased one $\Delta\sigma^*$ from $\Delta\sigma$. $\Delta\hat{\sigma}$ takes the largest value $\Delta\sigma_s$ at $\epsilon = \epsilon^I + \bar{\epsilon}^{II}$. Accordingly, the value B_{31} can be formulated by expressing $\Delta\sigma_s$ and $\Delta\hat{\sigma}$ as functions of ϵ^I and ϵ^{II} , respectively.

Fig.11 shows the relations between $\Delta\hat{\sigma}$ and ϵ^{II} for various values of ϵ^I , where the symbols denote the experimental results. $\Delta\hat{\sigma}$ increases with an increase of ϵ^{II} rapidly at first and later seems to saturate. There are slight differences in tendency between these results, which seem to come from differences in sequence of prestrain and magnitude of ϵ^I . However, $\Delta\hat{\sigma}$ is formulated by the expression (9) where $\bar{\epsilon}^{II}$ is assumed to be 0.06 (mm/mm), because, it is anticipated that large errors do not arise even if $\bar{\epsilon}^{II}$ is assumed to be constant regardless of direction and magnitude of ϵ^I . $\Delta\sigma$ in the expression (9) is equal to the value of $\Delta\sigma$ at $\epsilon^{II}=0.06$, which is plotted in relation to ϵ^I in Fig.12. $\Delta\sigma_s$ increases with an increase of ϵ^I rapidly at first, slowly later. There are some differences between the two results in the range of

comparatively small values of ϵ^I , which are mainly caused by sequences of prestrains. Factors which affect the value $\Delta\sigma_s$ are still to be identified. The relations between $\Delta\sigma_s$ and ϵ^I are described by the expression (10). Each value of coefficient in the expression (10) is shown in Fig.12.

The value C which denotes the center position of surface G is assumed to be the same as one which can be obtained when the whole prestrain is added in the final pre-stress direction. The values of A_{31} and B_{13} can be obtained by subtracting the value C from the values A_3 and B_1 , respectively.

4.7 Influence of the definition of yield

on the surface F To apply the above mentioned empirical equations, a yield surface which is determined by using the definition of 0.05% proof stress must be obtained. The whole shape of the yield surface defined as 0.05% proof stress could not be determined experimentally by using only one specimen. Because, loading processes in determining yield points affect the following process seriously. Then, subsequent yield surface of various proof stress definitions after torsional prestraining of ϵ^I ; 0.08 were determined by using three specimens. On the basis of these results, the relations in the characteristic values a, b and c between yield surfaces of 0.05% and 0.02% proof stress definitions were assumed to be prescribed as follows, regardless of prestrain history.

$$a' = 1.4a, b' = 1.1b, c' = 1.2c \quad (11)$$

Fig.9 Relations between center position of surface G and total prestrain value

Fig.11 Transitional amounts of variation in characteristic values of surface G

Fig.10 Variations of characteristic values of surface G in multi-prestrainings

Fig.12 Saturated amounts of transitional variation in characteristic values of surface G

To confirm the total degree of approximation of the above mentioned expressions, author applied them to the corresponding loading paths to the experimental results shown in Figs.3(a) and (b). The calculated results are shown by thick lines in these figures. In Fig.8, some thin broken lines denote the corresponding calculated results to the experimental results with regard to B_{31} and A_{13} .

There is wholly a good agreement between the calculated results and the experimental ones. The error range is ± 5.0 per cent at most. Therefore, these expressions are precise enough for practical use.

5. Conclusions

In this paper, plastic deformation behaviors of commercialized SUS 304 stainless steel pipe after multi-prestrainings under combined axial load and torsion were experimentally investigated. Experimental data obtained were arranged on the basis of the double surfaces hardening model which had been proposed in the previous report.

The conclusions are as follows.

(1) Elastic region can be approximated to an ellipse of which center is in the final prestrain direction, whose distance measured from the coordinate origin and lengths of the major and minor axes can be determined depending almost only on the total prestrain value.

(2) Effective stress-strain relations in the transitional hardening region, which is represented in non-dimensional expressions divided by the transitional stress

amplitude and that of corresponding strain, can be shown by a specific monotonous increasing curve without regard to material (brass, mild steel, Al-alloy and type 304 stainless steel) and prestrain history.

(3) i) Though, the characteristic values A and B of yield surface which is determined on the basis of the rigid plasticity approximation concept and can be approximated to an ellipse, may change depending on types, magnitude and sequences of prestrains in general,

ii) the values A and B obtained after mono-prestraining ϵ^I increase steadily and can be described by an expression of the same type as the initial stress-strain relations;

iii) variations of the values A and B after multi-prestrainings can be separated into two parts, that is, a transitional increasing part and a steady increasing one;

iv) the amount of the transitional part increases and gradually saturates with an increase of the first and second prestrains; and

v) the steady increasing rate is almost the same as that for a mono-preloading in the same direction as for second preloading.

(4) It is confirmed that the effect of the past deformation history on plastic properties of metal can not perfectly be liquidated in general by any following loadings.

(5) Utility of the double surface theory for describing the complex plastic deformation behavior of metal is also confirmed under combined stresses conditions.

Appendix Fig.1 Experimental results with regard to surface G for mild steel after multi-prestrainings

Acknowledgement

The author wishes to thank Mr. Kiyoshi Takahashi and Mr. Tomiaki Hattori for performing the experiments.

Appendix. On movement of the center of the surface G

In the results shown in Fig.3(b), it is assumed that the center of surface G moves from the last prestress direction to the direction of reloading, that is, the direction of ϵ^{II} , even if ϵ^{II} is rather small, then the same effective stress - strain relations are calculated for the third preloadings of tension or compression. Accordingly, the agreement between the experimental and the predicted behaviors in the third preloading is not good.

This poor agreement can be improved by formulating the movement of the center of surface G, more strictly. For example, Appendix Fig.1 shows the experimental results with regard to the surface G and its center after multi-prestrainings for mild steel. Patterns of preloadings are shown in the table in the figure. Symbol $\blacktriangle$ denotes a corresponding yield locus to the surface F.

From this result, we can see that the center of surface G moves almost linearly with an increase of ϵ^{II} and the amount of movement of the center is almost exponential to the amount of ϵ^{II} .

References

1. Kaneko, K. et al.; Trans. of JSME (in Japanese) 45-400(1979), 1564 pp.
(Shiratori, E. et al.; Proc. volume of Colloque International n°295 du CNRS, (1979), 257 pp.)
2. Ilyushin, A.A.; PMM (in Russian) 18-6 (1954), 641 pp.