Sound Radiation from a Housing Containing a Vibration Source

by Kiyohiko UMEZAWA **, Masatoshi FUTAKAWA *** and Haruo HOUJOH ****

The behaviour of the sound radiation from the plates of a housing being excited with each mode frequency has been studied theoretically and experimentally from a view point of directivity and by means of acoustical holography.

The directivity is theoretically determined from the relation between the bending wavelength of a plate ($\lambda_p$) and the sound wavelength in air ($\lambda_{air}$) by the use of the lattice mode of monopole sound sources. When $\lambda_p/\lambda_{air}$ is smaller than 1.0, the sound is radiated along the plate and the acoustic power is reduced. And the reported images of acoustical holography have been explained with the directivity of sound depending on $\lambda_p/\lambda_{air}$.

Key Words: Sound, Housing, Sound radiation, Directivity, Acoustical holography, Mode of vibration, Wavelength ratio, Sound source

1. Introduction

A housing is a final machine element which radiates sound in a machine system. But the sound radiation mechanism has not been revealed satisfactorily. By means of a developed acoustical holography system [1] which is capable of mapping sound sources, we examined the sound radiation mechanism on rectangular enclosures, modeling a housing [2].

It has been made clear that the behavior of the sound radiation depends on the relation between the bending wavelength in the housing plate ($\lambda_p$) and the sound wavelength in air ($\lambda_{air}$). (Hereinafter the “Wavelength ratio”.)

Saito et al. [3] have measured the radiation efficiency of various shaped plates by using a reverberation chamber. The results show a reduction in radiation under the condition $\lambda_p/\lambda_{air}<1$ which agrees with Maidanik’s theory [4]. But their work does not concern the condition that $\lambda_p$ is greater than $\lambda_{air}$. Fukuda et al. [5] have investigated the sound energy radiated from a circular plate which is resonating at lower mode. Hori [6] has calculated the sound fields from a complex shaped structure by locating point sources on the surface.

In this paper, the behavior of the sound radiation from the plates of a housing being excited at each mode frequency has been studied theoretically and experimentally from the view point of directivity and source location.

2. Method of Experiment

2.1 The tested enclosure

Figure 1 shows a tested enclosure, a model of a gear box. Each plate of the enclosure is welded except for the top plate being bolted. A shaker in the enclosure vibrates the plates through an axle fitted to the flanges with 20µm clearance and bolted with M6 bolts from two directions 90° to each other.

Figure 2 shows a model with 20 speakers (4x5) which corresponds to a plate resonating like a checker board. Each speaker (38mm diameter) is placed at the end of a square aluminum pipe. These speakers can be held at any distance from each other. When each speaker is driven out of phase to the adjacent one, it is realized that there are nodes between the speakers.

![Fig.1 Schematic drawing of tested housing.](image-url)
Further, in order to make clear the relation between the wavelength ratio and the behavior of the sound radiation, a linear array of five speakers is examined. Being set up on a baffle of rubber plate, these are held at any distance from each other on the line as shown in Fig. 3. The deviations of the speaker characteristics are within ±1dB in sound pressure and ±5° in phase at measured frequencies.

2.2 The method of experiments
2.2.1 Acoustical holography
Front and top views of the sound sources are observed with an acoustical holography. As shown in Figs. 6-10, blank areas indicate the sound sources and the lines indicate the outlines of the housing.

2.2.2 Measuring the resonant mode of vibration
With the frequency response of sound pressure measured by a microphone close to the front plate, the resonant frequencies are determined. At the resonant frequencies, the modes of vibration are measured with an accelerometer pickup on a magnet base traversing all over the plate surface. The nodes are identified when the amplitude is minimum and the phase changes by 180°.

2.2.3 Measuring the directivity
In order to make clear the relation between the reconstructed images of acoustical holography and wavelength, the directivity is measured with the method shown in Fig. 3. The microphone is fixed and the test object rotates on a turning table. The measurements are performed in the outdoor field where there are no objects within 25m.

The characteristics of the field are measured and shown in Fig. 4. The data satisfy the inverse square law, hence the field can be recognized to be a free field.

3. The Directivity of a Sound Source Constructed of Many Sound Sources
On a linear array of many point sources with equal intervals as shown in Fig. 5, adjacent ones being driven out of phase to each other, the velocity potential $\phi(r_n, \gamma, \nu)$ at the measuring point $P(r_n, \gamma)$ in the far field is calculated by summing each velocity potential $\phi$. [7]. At a point $r_n$ distant from each point source, we have

$$\phi_n = (-1)^{n-1} \frac{\dot{Q}}{4\pi r_n} e^{-(\omega t - k r_n)}$$

where $\dot{Q}$ is the volume velocity of the point source, 
$r_n$ is the distance between measuring point and sound source,
$\omega$ is the circular frequency and $\omega$ is the wave number $= 2\pi/\lambda_{air}$

Fig. 2 Two-dimensional array of point sources.

Fig. 3 Sound pressure measurement of test object for directivity.

Fig. 4 Acoustic characteristic of outdoor field.
The sound pressure on the field from the array of \( n \) point sources is calculated by summing the velocity potential of each point source.

\[
\phi(r, \gamma, R) = \sum_{i=1}^{n} \phi_{i}
\]

\[
\phi_{i} = \frac{Q}{4\pi\rho_{0}} \left( \frac{1}{1 - e^{iR \sin \gamma}} \left( \frac{1 + e^{iR \sin \gamma}}{1 - e^{iR \sin \gamma}} \right) \right)
\]

where \( R \) is the wavelength ratio \( (\equiv \lambda_{p},/\lambda_{air}) \) and \( \gamma \) is the direction from the normal axis to the array source.

The normalized directivity \( D(\gamma, R) \) is calculated from the wavelength ratio \( R \) and the direction \( \gamma \).

\[
D(\gamma, R) = \frac{\phi(r, \gamma, R)}{\phi_{n}} = \begin{cases} 
\sin((m\pi R \sin \gamma) / \cos((\pi R / 2) \sin \gamma)) & \text{when } n \text{ is even, } m = \frac{n}{2} \\
\cos((m+1/2)\pi R \sin \gamma) / \cos((\pi R / 2) \sin \gamma) & \text{when } n \text{ is odd, } m = \frac{n-1}{2}
\end{cases}
\]

Fig. 5 Linear array of point sources having alternate phase difference of 180°.

4. Measured Results and Discussion

4.1 Results on the model of plate sound source with five speakers

4.1.1 The reconstructed images with acoustical holography

In Fig. 6, the reconstructed images at several wavelength ratios are shown under the condition that the distance between the sound sources and hologram plane is fixed at 74 cm.

These are classified into two groups according to the wavelength ratio. When \( \lambda_{p}/\lambda_{air} \) is less than 2.1, only two speakers on both ends are recognized with a decrease of wavelength ratio. On the contrary when \( \lambda_{p}/\lambda_{air} \) is greater than 2.1, all the five speakers are recognized definitely.

Under the condition that the wavelength ratio is constant, the change of reconstructed images with the variation in the distance \( z \) between the sound source and hologram plane is shown in Fig. 7. When \( \lambda_{p}/\lambda_{air} = 2.1 \), these are classified into two groups as aforesaid, that is, over and under 74 cm. These phenomena are synthesized as shown in Fig. 8, the horizontal axis representing the measuring distance \( z \) and the vertical axis the wavelength ratio.

With a decrease in the wavelength ratio and with an increase in the measuring distance, the reconstructed images become such that only two speakers on both ends are recognized.

4.1.2 The directivity

On the speaker model as shown in Fig. 5, the normalized directivity is measured from 0° to 90° at each 10°.
interval as shown in Fig.9. The distance between the microphone and the speakers is 4m.

The measured results are normalized with respective maximum sound pressures. In Fig.9, the solid lines indicate measured results and the dotted lines the values calculated from Eq. (3).

These agree well to each other and the condition of the maximum sound pressure comes down to the plate surface of the sound sources with a decrease in the wavelength ratio.

4.2 Measured results on a rectangular housing.

4.2.1 Mode of vibration and images by acoustical holography

The reconstructed images and the modes of a resonating housing are measured at 7599 Hz, 5006 Hz, 3507 Hz and 2140 Hz which are determined by frequency sweep excitation. The measuring distance z between the front plate of the housing and the hologram plane is 70cm. At 7599 Hz, the modes of vibration of both the top and the front plate are recognized to be like a checkerboard pattern as shown in Fig.10.

The measured wavelength ratios $\lambda_{sp}/\lambda_{air}$ and $\lambda_{tp}/\lambda_{air}$ are over 2.0 on each plate, and the sound sources are recognized all over the plate.

At 5006Hz and 2140 Hz as shown in Figs. 11 and 12, the measured wavelengths $\lambda_{sp}/\lambda_{air}$ is smaller than 2.0 while $\lambda_{tp}/\lambda_{air}$ is over 2.0. The sound sources on the front plate are located only along the vertical edges. On the top plate, however, sources are located along the transverse edges because $\lambda_{sp}/\lambda_{air}$ is greater than 2.0 and $\lambda_{tp}/\lambda_{air}$ is less than 2.0.

The above behaviour on the speaker model is applied to the rectangular housing, that is, under the condition z=70cm, the sound sources appear on the edge if the wavelength ratio is smaller than 2.0, and all over the plate if the wavelength ratio is over 2.0.

4.2.2 The directivity of the housing

In Fig.3, the microphone is set at 1m from the rotary table and 360mm below the top plate. The measured directivities are shown in Fig.13 as solid lines. In the

figure, the dotted lines mean theoretical values of Eq. (3), under the condition that 180°-shifted sound sources are located on each loop of the mode.

With a decrease in the wavelength ratio, the direction of maximum sound pressure comes to be inclined to the plate surface. This characteristic has been observed on the five speaker model. It is clear that the directivity of the housing is calculated from Eq. (3).

4.3 The directivity and the reconstructed images of acoustical holography

The relation between the wavelength ratio and the direction of maximum sound pressure $\theta_m$ which is derived from Eq. (3) is shown in Fig.14.

Regardless of the number of sound sources, the direction becomes 90° with a decrease in the wavelength ratio to 1.0, hence the sound radiates in the direction along the plate surface.

![Fig.9 Directivity on the linear array vs. wavelength ratio.](image)

![Fig.10 The mode of vibration and sound image (7599Hz).](image)
The results of measurement on the speaker model and the rectangular housing agree well to the line of Eq.(3) in Fig.14. So, given the wavelength ratio, the direction $\gamma$ is determined as shown in Fig.14.

With the use of this relation, the change in the reconstructed images with the variation of the wavelength ratio in Fig.8 is explained as follows. As the dimensions of the hologram plane are fixed, and the hologram plane is set far from the sound sources, the major sound transmission passes out of the hologram plane. That is, the reconstructed image from this insufficient hologram which does not analyze the major radiation, cannot show the sound sources as shown in Fig.8.

The effect of the wavelength ratio can be explained in the same way. With a decrease of the wavelength ratio, the directivity comes down to the plate, and the sound passes out of the hologram plane. Thus the reconstructed images cannot either show the sound sources exactly. Instead of using the linear array model of Fig.8, measurements are performed on a plate model which consists of 20 speakers. The reconstructed images are shown in Fig.15. The distance between the sound sources and the hologram plane is set at 70cm.

The wavelength ratios $\lambda_{px}/\lambda_{air}$ and $\lambda_{py}/\lambda_{air}$ are over 2.0 in pattern A, and the images of each speaker are clearly observed in the reconstructed image without the effect of the directivity.

In the pattern B, only $\lambda_{px}/\lambda_{air}$ is under 2.0, and a cancellation in the x direction is observed.

In the pattern C, both $\lambda_{px}/\lambda_{air}$ and $\lambda_{py}/\lambda_{air}$ are under 2.0, and the images are observed at four corners with the cancellation in both directions.

Finally, it has been made clear that the relation between the reconstructed image and the directivity (that is the wavelength ratio) can be indicated in Fig.8.

4.4 Acoustic radiation power

Table 1 shows the relation between the wavelength ratio and the radiation power of the five speaker model that is calculated from the measured sound pressure.

![Fig.13](image)

**Fig.13** The directivity of radiation from the housing.

![Fig.14](image)

**Fig.14** The direction of maximum sound pressure vs. $\lambda_{px}/\lambda_{air}$.

![Fig.11](image)

**Fig.11** The mode of vibration and sound image (5006 Hz).

![Fig.12](image)

**Fig.12** The mode of vibration and sound image (2140 Hz).
pressure as a transmitting power from inside to outside of a sphere of each radius.

When the wavelength ratio is over 1.0, the radiation powers at 1.1 and 1.5 are nearly equal to each other. The value at 0.8 of wavelength ratio decreases by 3dB at 2m and by 6dB at 4m.

At the measuring point $P(r_0, T)$, the radiation power transmitted through an unit area $E(r_0, T; R)$ is calculated from Eq.(2).

$$E(r_0, T; R) = |j\omega \Phi(r_0, T; R)|^2/c$$  \hspace{1cm} (4)

where $\rho$ is the density of acoustic media, and $c$ is the velocity of sound.

The radiation efficiency $\sigma$ is defined as the ratio of the resultant radiation power of the array source to the sum of the power of each point source.

$$\sigma = \frac{\int E(r_0, T; R) ds}{\sum _{j=1}^{N} |j\omega \Phi_{j}|^2/c \rho}$$  \hspace{1cm} (5)

With the use of Eq.(5), the relation between the wavelength ratio and the radiation power is calculated as shown in Fig.16.

Under the condition that $\lambda_{pl}/\lambda_{air}$ is greater than 1.0, $\sigma$ is constantly 1.0, and is independent of the number of point sources. On the other hand, $\sigma$ decreases remarkably as $\lambda_{pl}/\lambda_{air}$ comes lower than 1.0 because of the cancellation of radiation. This feature agrees well with the data given in Table 1.

5. Conclusions

In this paper, we have investigated the mechanism of sound radiation from a rectangular housing containing a mechanical exciting force and the relation between the reconstructed image of acoustical holography and the wavelength ratio.

The results of this study are summarized as follows:

1. The directivity of the sound radiated from the housing depends on the wavelength ratio of the bending wavelength $\lambda_{pl}$ in plate to the sound wavelength $\lambda_{air}$ in air.

2. And the directivity is calculated as the sound field from point sources with a phase difference of 180° from each other, by modeling a resonating housing.

(2) The acoustic power emitted from a housing decreases when $\lambda_{pl}/\lambda_{air}$ is less than 1.0.

(3) The relation between the reconstructed image and the directivity with $\lambda_{pl}/\lambda_{air}$ has been made clear.

References