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A Dynamic Substructuring Method for Elastoiimpact

Stress Analysis Using Finite Element Method *

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A dynamic substructuring method is presented for the efficient calculation of the Finite Element Method. The method is mainly composed of the Runge-Kutta-Gill method, the lumped mass matrix, and the substructuring stiffness matrices.

To verify the validity of the method, it is applied to the longitudinal elastoiimpact analysis of two-dimensional elastic rods having a uniform cross section and a variable one in collision with a rigid wall. The calculated values of the substructuring models agree precisely with those of the whole rod models.

The use of this method makes it effective to decrease computer memory capacity and computing time. If the method is combined with the condensation method, the impact behavior of two-dimensional and/or three-dimensional structures with complicated shapes can be calculated more efficiently using smaller computer memory capacity and shorter computing time.

Key Words: Shock, Impact, Dynamic Contact, Dynamic Stress, FEM, Stress Analysis, Substructure Method, Calculation Method

1. Introduction

Impact contact behavior arises in many machine parts such as meshing gears, rolling bearings, clutches and so on. At present it is very difficult to measure and/or calculate the contact stresses, their distributions, and contact states on impact surfaces. In recent years, the increase of large-scale computational capabilities makes it much easier to apply a finite element method (FEM) especially available for complicated configurations in many areas of structural analysis. Ohte⁽¹⁾, Tsuta and Yamaji⁽²⁾, Okamoto and Nakazawa⁽³⁾, Yagawa et al.⁽⁴⁾ proposed the FEM for elastostatic problems involving contact effects. On the other hand, the author has formulated the elastoiimpact contact theories with frictions,^{(5),(6)} using, respectively, a conventional FEM for small strain, and a new non-incremental FEM⁽⁷⁾ for flexible deformation. Moreover, the author reported on virtual work principles for two elastoiimpact bodies^{(8),(9)} to formulate easily the FEM for elastoiimpact contact structures.^{(5),(8),(10)}

From the above description, the elastoiimpact FEMs^{(5),(6),(8)} make it possible to analyze various impact behaviors on contact surfaces. On account of a great number of small-sized element divisions near the contact surfaces, the use of the FEMs requires a large computer memory capacity and a long computing time. Various practical methods for the efficient calculation of the FEMs have been developed. The author proposed a dynamic zooming method,⁽¹¹⁾ and investigated condensation methods,⁽¹²⁾ to calculate efficiently the impact behaviors of structures. The decrease of the memory capacity and the computing time is very important for an efficient analysis of two-dimensional (2D) and/or three-dimensional (3D) structures with complicated shapes.

This paper presents a dynamic substructuring method for the efficient calculation of elastoiimpact stress problems by using the FEM. This method in the numerical solution of an impact matrix equation is mainly composed of the Runge-Kutta-Gill (RKG) method, the lumped mass matrix (LM) of a whole structure and the stiffness matrix of any substructure. To verify the validity of this method, it is applied to the longitudinal elastoiimpact analysis of 2D elastic rods with a uniform cross section and a variable one in collision with a rigid wall. In addition, this method is combined with the condensation method, to calculate more efficiently using smaller computer memory capacity and shorter computing time.

2. Theory of Calculation

2.1 Outline of Formulation

For the simplification of formulation, let us consider a one-dimension (1D) elastic rod fixed on a rigid wall, as shown in Fig. 1. It is assumed that the alternating load $F(t)$ acts on the free end of a rod without any damping effect. First the rod is divided into n pieces. The LM and the stiffness matrix of each piece can be obtained by the FEM.^{(5),(10)} Figure 2 shows a diagram model of the rod divided into many pieces of Fig. 1. The divided ends between two pieces are subjected to unknown connective forces R_{12} , R_{23} , R_{34} , etc. in the opposite direction.⁽⁵⁾ If we denote the lumped masses on both ends of each piece

Fig. 1. One-dimensional elastic rod fixed on a rigid wall.

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by M_{1L} , M_{1R} , M_{2L} , M_{2R} , etc., these masses can be connected by the spring constants k_1 , k_2 , k_3 and so on. We indicate the displacements on both ends by U_{1L} , U_{1R} , U_{2L} , U_{2R} , etc. Then the equation of motion in Fig. 2 is expressed by

$$\left. \begin{aligned} [M][\ddot{U}] + [K][U] &= [F] + [R], \\ [M] &= \text{diag}(m_{1L}, m_{1R}, m_{2L}, m_{2R}, \dots), \\ [K] &= \text{diag}([K_1], [K_2], \dots), \quad [\ddot{U}] = \frac{d^2}{dt^2} [U], \\ [U] &= \begin{bmatrix} U_{1L} \\ U_{1R} \\ U_{2L} \\ U_{2R} \\ \vdots \end{bmatrix}, \quad [F] = \begin{bmatrix} F(t) \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}, \quad [R] = \begin{bmatrix} 0 \\ -R_{12} \\ R_{12} \\ -R_{23} \\ R_{23} \\ \vdots \end{bmatrix}, \\ [K_\xi] &= \begin{bmatrix} k_\xi & -k_\xi \\ -k_\xi & k_\xi \end{bmatrix}, \quad k_\xi = \frac{A_\xi E_\xi}{l_\xi}, \quad (\xi=1, 2, \dots, n) \end{aligned} \right\} \dots (1)$$

where $\text{diag} (*)$ means a diagonal matrix. Moreover, A_ξ , l_ξ and E_ξ are, respectively, the area, the length, and the modulus of elasticity of the piece (ξ) in Fig. 1. The following relations are obtained by the continuity or divided ends between two pieces.

$$\left. \begin{aligned} U_{1R} &= U_{2L} = U_2, & U_{2R} &= U_{3L} = U_3, \dots \\ \dot{U}_{1R} &= \dot{U}_{2L} = \dot{U}_2, & \dot{U}_{2R} &= \dot{U}_{3L} = \dot{U}_3, \dots \end{aligned} \right\} \dots (2)$$

From the above relations, the transformation matrix $[I^*]$ and $[I^*]$, related to acceleration and displacement, are

$$[I^*] = [I^*] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \dots (3)$$

where $[*]^T$ denotes a transposed matrix.

The matrix $[I^*]$ of connective forces on the divided ends between two pieces is⁽⁵⁾

$$[I'] = [I^*]^T \dots (4)$$

Introducing Eqs. (3) and (4) into Eq. (1), we obtain

Fig. 2. A diagram model of the rod divided into n pieces.

$$\left. \begin{aligned} [I'] [M] [I^*] [\ddot{U}] + [I'] [K] [I^*] [U] &= [I'] [F], \\ [U] &= \frac{d^2}{dt^2} [U], \quad [U] = [U_{1L}, U_{1R}, U_{2L}, U_{2R}, \dots]^T, \\ [I'] [F] &= [F_T] = [F(t), 0, 0, \dots]^T \end{aligned} \right\} \dots (5)$$

Equation (5) expresses the equation of motion for the whole structure formulated by a general FEM. The matrices in Eq. (5) are

$$\begin{aligned} [I'] [M] [I^*] &= [M_T] = \text{diag}(m_{1L}, m_{1R} + m_{2L}, m_{2R} + m_{3L}, \dots), \\ [I'] [K] [I^*] &= [K_T] = \begin{bmatrix} k_1 & -k_1 & 0 & \dots \\ -k_1 & k_1 + k_2 & -k_2 & \dots \\ 0 & -k_2 & k_2 + k_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \dots (6) \end{aligned}$$

The computer memory used in $[K_T]$ is larger than that used in $[M_T]$. The memory of $[K_T]$ increases rapidly due to the number of element divisions. However, $[K_T]$ is composed of superposing the stiffness matrix $[K_\xi]$ of a substructure (ξ). It is also clear from Fig. 1 that the substructure (ξ) is repeatedly applicable to the whole structure. So we can use $[K_\xi]$ repeatedly without constructing $[K_T]$ immediately in order to economize the computer memory. Moreover, the number of matrix calculations decreases, because the sparse property of $[K_\xi]$ is less than that of $[K_T]$ in 2D and/or 3D structural analysis. Hence, the use of the substructuring method makes it effective to decrease the computing time.

2.2 Effect of Coordinate Translation on Lumped Mass Matrix and Stiffness Matrix

To substitute a few substructures for a whole structure, let us investigate the effect of coordinate translation of substructure on its LM. Lumped masses $M_{\xi L}$ and $M_{\xi R}$ on both ends of the substructure (ξ) in Fig. 1 are expressed by^{(5), (7)}

$$\begin{bmatrix} m_{\xi L} \\ m_{\xi R} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ x_{\xi L} & x_{\xi R} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x_{\xi G} \end{bmatrix} m_\xi, \quad m_\xi = \int_{x_{\xi L}}^{x_{\xi R}} \rho_\xi A_\xi dx \dots (7)$$

where ρ_ξ and A_ξ denote, respectively, the density and the area of the substructure. Also, $x_{\xi L}$ and $x_{\xi R}$ indicate, respectively, the x -coordinates on both ends of the substructure, and $x_{\xi G}$ is the coordinate at the center of the substructure. If the substructure shifts to x_i in the x -coordinate, a part of Eq. (7) is expressed as follows:

$$\begin{bmatrix} 1 & 1 \\ x_{\xi L} - x_i & x_{\xi R} - x_i \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x_{\xi G} - x_i \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ x_{\xi L} & x_{\xi R} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x_{\xi G} \end{bmatrix} \dots (8)$$

where $[*]^{-1}$ denotes the inverse matrix. It is clear from Eqs. (7) and (8) that the LM is independent of the coordinate translation of the substructure.

The stiffness matrix $[K_\xi]$ of the substructure (ξ) from Eq. (1) is^{(5), (7)}

$$[K_\xi] = \begin{bmatrix} k_\xi & -k_\xi \\ -k_\xi & k_\xi \end{bmatrix}, \quad k_\xi = \frac{A_\xi E_\xi}{l_\xi} \dots\dots\dots (9)$$

In Eq. (9), $[K_\xi]$ has no effect on the coordinate translation of the substructure.

These facts are also verified in Cartesian coordinates. In the 2D analysis of a round bar, it is necessary for the axisymmetric property to supply different substructures in the coordinates of radius.

2.3 Numerical Solution for Matrix Form of Differential Equation

Various methods are used to obtain numerical solutions for a matrix form of a differential equation. Shimizu, et al.(13) reported that PC-12, Newmark- β (N- β), RKG method, etc., are available to calculate the solution with good phase and gain characteristics, and the PC-12 is the best method. If we use the PC-12 or N- β method, the coefficient matrix is composed of $[K_T]$ and $[M_T]$, and the computer memory increases.

On the contrary, in the RKG the coefficient matrix is composed only of the mass matrix. The use of $[M_T]$ decreases remarkably the computer memory as compared with the consistent mass matrix (CM) and the coefficient matrix of the PC-12 or the N- β . In two elastoimpact structures, we must repeatedly produce and decompose the coefficient matrix whenever the contact state changes at each node. The coefficient matrix becomes especially non-symmetric in the slipping state.(5),(10) Hence, the use of $[M_T]$ makes it much easier to calculate the inverse matrix of the coefficient matrix.

In the manner stated above, we apply the substructuring method to $[K_T]$, as will be expressed in Eq. (10).

2.4 Technique for Calculation

For simplification of treatment, let us consider an elastic rod divided into n pieces in Fig. 2 and substitute one substructure for the whole rod. Let L and R denote the left end and the right one of the substructure. Table 1 shows the corresponding relation between the nodal number of the substructure and that of the whole rod on the basis of Eq. (2) in each piece in Figs.1 and 2. At first, it is necessary to make the corresponding matrix according to Table 1. The same pattern can be applied to the case when the nodes of the substructure increase.

When we use two, or more than two substructures, we must supply the classification matrix denoting the substructures. We must also clarify the relation among the classification matrix, the nodal number of each substructure, and that of the whole rod.

In 2D or 3D analysis, we obtain the corresponding relation as in Fig. 1 including each coordinate component of the substructures and that of the structures. This

method can be applied to the 2D and 3D structures.

Adding the corresponding matrix and the classification one to a general substructuring method, we can virtually determine the matrix $[K_T]$ in Eq. (6) without superposing $[K_\xi]$ in Eq. (9). Then Eq. (5) is rewritten as

$$\left. \begin{aligned} [\ddot{U}] &= [M_T]^{-1} \{ [F_T] - [K_T][U] \} \\ [K_T][U] &= \sum_{\xi=1}^n \{ [K_\xi][U_\xi] \} \end{aligned} \right\} \dots\dots\dots (10)$$

where Σ means the matrix representation, $[U]$ and $[\ddot{U}]$ denote respectively the displacement and the acceleration matrices of the whole rod. $[U_\xi]$ is the partial displacement matrix of ξ -piece in Fig. 1. We can obtain $[\ddot{U}]$ and $[U]$ matrices after integrating numerically $[\ddot{U}]$ of Eq. (10) according to the RKG method.

To decrease the number of substructures having the same shape and different constraint nodes, we replace all the components of the constraint nodes by zero in the computing process of the RKG method. Hence, it becomes unnecessary to eliminate a row and a column in the matrix of the substructures, and computer memory is saved.

3. Investigation of Dynamic Substructuring Method

For simplicity, we treated the 1D elastic rod in chapter 2 to give an explanation of the dynamic substructuring method. In order to check the validity of this method, we apply it to two 2D elastic rods having a uniform sectional area and a variable one, respectively.

3.1 Longitudinal Elastoimpact Behaviors of Two-dimensional Rod with Uniform Cross Section

3.1.1 Models for Analysis

To use the simplest model for elastoimpact, let us

Fig. 3. Models of a two-dimensional rod with a uniform cross section.

Table 1. A corresponding relation between the nodal number of the substructure and that of the whole rod.

		Piece number of divided rod					
		(1)	(2)	(3)	(4)		(n)
Node number of substructure	L	1	2	3	4		n
	R	2	3	4	5		*n+1

1: Node number of load node, *: Node number of constraint node.

investigate the plane stress behavior of a 2D rod with a uniform cross section. The two models in Fig. 3 move longitudinally with the velocity $V = 1$ m/s. and collide with a rigid wall. Model (A) is calculated as a whole structure according to Eq. (6). Model (B) divides a rod into 10 pieces and a substructure is used in each piece according to this dynamic substructuring method based on Eq. (10). In order to realize the collision with a rigid wall, the displacements and the velocities of the impact nodes are replaced by zero. The frictional coefficient on the impact surface is assumed to be zero.

Both models are divided by a constant stress type of elements and integrated by the RKG method of the fourth order. The time interval ΔT for integration is determined as a value less than the time required for the stress wave to propagate through the element with the smallest size. When the models are assumed to be mild steel, the propagation velocity of the stress wave is 5120 m/s, and $\Delta T = 0.976 \mu\text{sec}$. However, a calculation at $T = 0.8 \mu\text{sec}$ is carried out, in order to plot easily the results obtained.

3.1.2 Stress Conditions Based on Theory of One dimensional Stress Wave Propagation

It is assumed that the impact stress based on the theory of 1D elastic stress wave propagation (1D theory) distributes uniformly in any cross-sectional area of rods. When a rod collides with a rigid wall, the stress σ in any cross-sectional area is expressed as

$$\sigma = E \frac{V}{C}, \quad C = \sqrt{\frac{E}{\rho}} \quad \dots\dots\dots (11)$$

where E , ρ , C and V are, respectively, the modulus of elasticity, the density, the velocity of propagation, and the impact velocity.

The compressive stress wave generated at the impact end is reflected at the free end as a tensile stress wave. Let l denote the length of the rod. When the tensile wave returns to the impact end at the time $t = 2l/C$, the stress of the rod disappears and the rod separates from the wall. Then the stress conditions at any point along the rod can be obtained by superposing the compressive wave and the tensile one, as shown in Fig. 4. The stress and the contact time of the 1D theory in Fig. 4 are hereinafter used to be compared with the element stress values of the both models shown in Fig. 3.

3.1.3 Results of Calculation

When both models (A) and (B) at the velocity $V = 1$ m/s collide with a rigid wall, Fig. 5 shows the elastoimpact stresses at the center of the point 35 mm from the impact

Fig. 4. Stress conditions by the theory of propagation of one-dimensional elastic stress wave.

Fig. 5. Elastoimpact stresses at the center of the point 35 mm from the impact end.

Table 2. Computing time and memory capacity for two-dimensional elastic rods with a uniform cross section.

Items	Model	(A)	(B)
Computing time (second)		9.62	9.76
Number of substructures		----	1
Elements of corresponding matrix (number of piece x degree of freedom)		----	10x8
Elements of stiffness matrix (band width x degree of freedom)		5x51	5x8
Elements of lumped mass matrix		51	53
Elements of load matrix		51	53
Elements of constraint matrix		----	2
Total elements		357	228

end of the models, in comparison with the stress by the 1D theory. The calculated values of model (B) agree precisely with those of model (A). Though the longitudinal stress by the FEM fluctuates periodically depending on the deformability of elements due to the lateral effect of inertia, the mean value of the stress agrees well with that of the 1D theory.⁽¹⁰⁾ A response of the stresses by the FEM is slower than that by the 1D theory. Hence, the results by the FEM are more practical than those by the 1D theory idealized.

Table 2 indicates the computing time and the memory capacity used in the typical matrices for both models. We use the number of elements in each matrix as the unit of the memory. The band width of the symmetric stiffness matrix $[K_T]$ of model (A) agrees with that of $[K_E]$ of model (B), as shown clearly in Fig. 3. Model (B) needs the time required for a computer to compose a corresponding matrix like Table 1. For each matrix operation of the substructure, model (B) increase the accessory time required to coincide the substructure with the whole structure by using the corresponding matrix and the constraint one. For these reasons, the computing time of model (B) is slightly longer than that of model (A). However, the total number of elements in model (B) is 1/3 less than those in model (A).

3.2 Longitudinal Elastoimpact Behaviors of Two-dimensional Rod with Variable Cross Section

Figure 6 shows the two models of 2D rod with a variable cross section. These models move longitudinally with the velocity $V = 1 \text{ m/s}$ and collide with a rigid wall. Model (C) is calculated as a whole structure according to Eq. (6). Model (D) divides a rod into 16 pieces, and two substructures are required for the existence of constraint nodes. In order to decrease the number of substructures, we increase the element number of the constraint matrix. Hence, in model (D), a substructure is applied to each piece according to this method based on Eq. (10). The 1D theory cannot be applied to both models because of the complicated shape. However, the validity of the FEM can be verified in Fig. 5.

Figure 7 shows the elastoimpact stresses at the center of three points 5, 15 and 35 mm from the impact end of both models. The calculated values of model (D) agree precisely with those of model (C). As compared with those stresses, the stress fluctuates considerably at the point 35 mm from the impact end and the peak stress is also the highest of all. This is due to the reduction of sectional area at the point. Hence, a part of the elastic stress wave transmits and the rest reflects at the point.

Table 3 indicates the computing time and memory capacity used in the typical matrices for both models. The computing time of model (D) is slightly shorter than that of model (C). Because the band width of $[K_E]$ is remarkably less than that of $[K_T]$, the number of matrix operations decreases. So the time of matrix operation becomes shorter. On the contrary, model (D) increases the access time required for the coincidence of the substructure with the whole structure by using the corresponding matrix and the constraint one. Hence, it is clear that the time of matrix operation is slightly shorter than the access time. By applying the dynamic substructuring method to the 2D and/or 3D analysis of complicated structures which spread the band width of $[K_T]$, we can decrease remarkably computing time and computer memory capacity.

Fig. 6. Models of a two-dimensional rod with a variable cross section.

Fig. 7. Elastoimpact stresses at the center of three points from the impact end.

Table 3. Computing time and memory capacity for two-dimensional elastic rods with a variable cross section.

Items	Model	(C)	(D)
Computing time (second)		11.83	11.74
Number of substructures		----	1
Elements of corresponding matrix (number of piece x degree of freedom)		----	16x10
Elements of stiffness matrix (band width x degree of freedom)		11x80	6x10
Elements of lumped mass matrix		80	94
Elements of load matrix		80	94
Elements of constraint matrix		----	14
Total elements		1040	422

4. Discussions

4.1 Combination of Dynamic Substructuring Method and Condensation Method

A condensation method is used in the conventional substructuring method for a general elastostatic FEM. The combination of both methods is very useful for the more efficient calculation of the FEM, because the degree of freedom for nodes can be reduced by sweeping out a part of nodal points in the condensation method. Especially, the FEM with a static condensation method (SCM) contributes to an efficient calculation of low frequencies for a structure. The SCM is applicable to the elastoimpact stress analysis of a structure, and it is clear that by selecting any condensed points among the nodes apart from the impact surfaces, elastoimpact stresses can be calculated accurately and effectively by the FEM with the SCM.⁽¹²⁾

As the LM of the whole structure is used in this method, we consider the technique of static condensation for mass matrix. For simplification of treatment, we investigate the 1D longitudinal motion of a rod with a uniform section, as shown Fig. 8. Then we obtain the following equation,

$$[M][\ddot{U}] + [K][U] = [F] \dots \dots \dots (12)$$

where various matrices are shown in Fig.8.

Though the LM is expressed by Eq. (7), the LM is composed of summing up the row components or the column ones of a CM. There are two ways in condensing the CM. The first is that the consistent mass matrix $[M_C]$ is changed into the lumped mass matrix $[M_L]$ before $[M_C]$ is condensed by the transformation matrix $[T]$. The second is that $[M_C]$ is condensed by $[T]$ before $[M_C]$ is changed into $[M_L]$. It is clear from Fig. 8 that the mass components condensed by the first way are slightly greater than that by the second way. Hence, the second way is used in the SCM.

The LM becomes a diagonal matrix without coupling elements, and its diagonal elements are larger than those of the CM. In eigenproblems of a structure, the eigenvalues of the LM are slightly lower than those of the CM.⁽¹²⁾ This description is proven in Fig. 8. The stress fluctuation of the LM becomes greater than that of the CM. Though the use of CM is desirable, this method is

composed only of the LM. If the condensed LM is composed using the second way, calculated results may be obtained accurately.

If this method is combined with the SCM, the impact behaviors of 2D and/or 3D structures with complicated shapes can be calculated more efficiently using smaller computer memory capacity and shorter computing time.

4.2 On a Closed Type of Matrix Structure

Let us consider a circular structure such as gears, disk clutches, circular cams, and so on. Since the starting nodes of element divisions coincide with the ending nodes of them, $[K_T]$ of a whole structure is obtained, as shown in Fig. 9. A closed type of $[K_T]$ has a sparse property, because its coupling elements appear. If the coupling elements are separated and arrayed into another memory, the memory capacity for $[K_T]$ decreases greatly. However, a special algorithm is required in matrix operations and its programming becomes more complicated. On the contrary, this method does not require a closed type of matrix structure such as $[K_T]$ in Fig. 9. Special array and algorithm are also unnecessary.

Moreover, the CM of a whole structure is obtained like $[K_T]$. Since the LM is composed of summing up the row components or the column ones of the CM, the coupling elements disappear. Hence, this method is suitable for the 2D and/or 3D structures with closed and complicated shapes.

5. Conclusions

A dynamic substructuring method is presented for an efficient calculation of FEM. The validity of this method is investigated on the longitudinal elastoimpact stress behaviors of various 2D elastic rods in collision with a rigid wall.

The following results are obtained in this paper:

- (1) A whole structure is divided into many pieces in full consideration of their shapes, and all the pieces are replaced by a few substructures. It is proved that the mass matrix and the stiffness one of a substructure are independent of coordinate translations.
- (2) In the RKG method for numerical solution of differential equations, the coefficient matrix is composed only of the mass matrix for a whole structure. We apply the LM to the mass matrix. The stiffness matrices of a few substructures are used repeatedly in those of the whole structure. Hence, the use of the LM and the substructuring stiffness matrices decreases remarkably the computer memory capacity.

$k = \frac{AE}{l}$ $A = \text{area}, \rho = \text{density}$
 $m = \rho A l$ $E = \text{Young's modulus}$

vector & mat. case	displacement vector [U]	Load vector [F]	Transformation matrix [T]	Stiffness matrix [K]	Mass matrix [M]	
					Consistent mass [M _C]	Lumped mass [M _L]
Condensed point displacement	—	$\begin{bmatrix} F(t) \\ 0 \\ 0 \end{bmatrix}$	—	$k \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$	$m \begin{bmatrix} \frac{1}{3} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$	$m \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$
	U_3	$\begin{bmatrix} F(t) \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$	$k \begin{bmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{bmatrix}$	$m \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$	$m \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$
	U_2, U_3	U_1	$F(t)$	$\begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	$\frac{1}{3}k$	m

Fig. 8. Mass and stiffness matrices of a one-dimensional rod with a uniform section.

Fig.9. A closed type of matrix structure.

(3) When many substructures with same shape are required for the existence of different constraint nodes, the use of a constraint matrix can decrease the number of the substructures and economize the computer memory capacity.

(4) This method is applied to the longitudinal elastoimpact analysis of 2D elastic rods with a uniform cross section and a variable one in collision with a rigid wall. The calculated values of the substructuring models agree precisely with those of the whole rod models. The use of this method makes it possible to calculate effectively the impact behavior of structures with complicated shapes.

(5) For the more efficient calculation of FEM, the combination of this method and the SCM is discussed. It is concluded that the calculated results may be obtained accurately by using the LM composed of the condensed CM. If this method is combined with the SCM, the impact behaviors of 2D and/or 3D structures with complicated shapes can be calculated more efficiently using smaller computer memory capacity and shorter computing time.

(6) A closed type of matrix structure has a sparse matrix, because of the existence of coupling elements. Special array and algorithm are required to decrease the memory capacity. This method does not need special array and algorithm. So this method is suitable for the 2D and 3D structures with closed and complicated shapes.

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