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On a Method of the Concentration Measurement by the Use of Light Absorption Law*

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In order to detect the concentration of some material in a field, use is made of the light absorption method. The most important problem to be solved is the deviation from Lambert-Beer's law in the relation between photomultiplier output $\bar{E}_p$ and concentration value $\bar{C}$ which appears under use of a broad band light source. An analysis is made of the deviation from Lambert-Beer's law, in consideration of various spectral characteristics of each optical element in the measuring system. A matching method is shown to make $\bar{E}_p - \bar{C}$ relation follow a simple power law over a broad range of the values of the concentration.

Key words: Flow Measurement, Multiphase Flow, Concentration, Light Absorption Method, Lambert-Beer's Law, Turbulent Diffusion

1. Introduction

In order to investigate the diffusion mechanism of a matter in a turbulent flow, the concentration fluctuations as well as the mean concentration of the matter must be measured. Up to now, as the methods to measure the concentration fluctuations the conductivity probe method (1)~(4), light-scattering method (5)~(10), light-absorption method (11)~(14), laser-Raman scattering method (15)~(18), etc. have been tried. Although at present the most extensively studied method seems to be the light-scattering method or the laser-Raman scattering method, in these methods the measuring equipment tends to become very complicated and the cost is expensive. On the other hand, in the light-absorption method the whole experimental facilities can be pretty simple and its light probe has a high movability so that the system is appropriate to measure the three-dimensional concentration field. Although Lambert-Beer's law which is the basic principle in the light-absorption method holds valid strictly for each monochromatic light wave component, in the case of using such a monochromatic light the kind of available light source is restricted; the sort of a light absorptive matter fitting the properties of light sources is also limited. Further,

due to an insufficient amount of the light emitting from the ordinary monochromatic light source, a low value of signal to noise ratio poses a problem (11), (12). Therefore in measuring the concentrations of the matter accurately, the light sources with broad band spectral characteristics to some degree have been used so far. For instance, Lee & Brodkey (11) firstly made an optical measuring system with the sodium lamp as a light source but for the above mentioned reason, finally they used an intense white light source. And also Nye & Brodkey (12), (13), Batt (14) used the white light source. In detecting the concentrations by using such a broad band light source, the most important problem is the deviation from Lambert-Beer's law in the relation between the photomultiplier output $\bar{E}_p$ and the concentration $\bar{C}$. Because in this case the range of concentrations for the $\bar{E}_p - \bar{C}$ relation to obey a simple power law which can be easily dealt with is restricted to relatively small concentration values, and so the detection of the concentration fluctuations in a wide range becomes difficult (11)~(14).

So as to explain this deviation, the spectral characteristics of other optical elements in the measuring system besides the light source and the light absorptive matter should be also considered. If on the basis of such consideration the deviation from Lambert-Beer's law in the $\bar{E}_p - \bar{C}$ relation could be corrected, it would be exceedingly easy to detect the concentration fluctuations in a wide range because the $\bar{E}_p - \bar{C}$ relation could be easily transformed to a linear relation by using an up-to-date high efficiency logarithmic amplifier.

In general, an investigation of the light absorptive characteristics of the matter is important for a spectrochemical analysis. Since in the spectrophotometer an almost monochromatic light with a narrow spectral band is usually used, only the deviation from Lambert-Beer's law

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caused by the mixture, chemical reaction, scattering etc. becomes serious in spectroscopy (19). At present, a definite consideration mentioned earlier in the case of using the broad band light source seems not to have yet been reported within the author's knowledge. In this study, so as to develop a more useful measuring system which is capable of detecting a wide range of concentrations by the light-absorption method, the $\bar{E}_p$ - $\bar{I}$ relation is derived in consideration of various spectral characteristics of each optical element in the measuring system and a quantitative analysis of the deviation from Lambert-Beer's law is presented. An investigation of the effect of the boundary layer produced at the surfaces of the probe ends on the time response characteristic is also made.

Nomenclature

- λ : wave length of the light (m or nm)
 $\tilde{I}_0(\lambda)$: radiation intensity distribution of the light source ($\text{W/m}^2 \cdot \text{m} = \text{W/m}^3$)
 $\tilde{I}(\lambda)$: transmitted light intensity distribution (W/m^3)
 $\bar{I}$: concentration of matter (g/l or mg/l)
 R_L : load resistance of the photomultiplier
 $\bar{E}_p$: voltage output of the photomultiplier
 $\bar{E}_{p,0}$: voltage output of the photomultiplier when $\bar{I}=0$
 $S_p(\lambda)$: cathode emission sensitivity of the photomultiplier (mA/W)
 $T_i(\lambda)$: transmissivity of each optical element
 l : path length of the light in a sampling volume (the thickness of a light absorptive matter)
 $\beta(\lambda)$: coefficient representing the light absorption characteristic of the matter (defined by Eq.(1))
 $A(\lambda)$: absorbance $= \beta(\lambda)l\bar{I}/\ln 10$
 d_f : bundle diameter of glass fibers

Other notations will be defined each time.

2. The Measuring Principle and System

2.1 Measuring principle

Figure 1 shows a skelton diagram of the concentration measuring system by the light-absorption method. For simplicity, firstly a case will be considered where the light losses of various optical elements A and B in Fig. 1 can be negligible. Let $\tilde{I}_0(\lambda)$ be the radiation intensity distribution of the light source. If the amount $\tilde{I}_0(\lambda)\Delta\lambda$ of the light emitted from the light source is guided into a light absorptive matter with the concentration $\bar{I}$ and the thickness l , and after transmission it attenuates to the amount $\tilde{I}(\lambda)\Delta\lambda$, when the spectral band width $\Delta\lambda$ of the light is small enough compared with the light absorptive band width of matter, Lambert-Beer's law states that

Fig. 1 A diagram for deriving the $\bar{E}_p$ - $\bar{I}$ relation

Fig. 2 Combinations of each spectral characteristics for the light source, light absorptive matter and the photomultiplier

$$-\ln\{\tilde{I}(\lambda)\Delta\lambda/\tilde{I}_0(\lambda)\Delta\lambda\}=\beta(\lambda)l\bar{I} \quad (1)$$

or

$$\tilde{I}(\lambda)\Delta\lambda=\tilde{I}_0(\lambda)\Delta\lambda\exp\{-\beta(\lambda)l\bar{I}\} \quad (2)$$

where it should be noticed that $\Delta\lambda$ appears in Eqs. (1) and (2). Ordinary Lambert-Beer's law holds strictly for each monochromatic light wave component but here the law is extended and adapted to the change in the amount of light within a narrow wave length width $\Delta\lambda$. Notice that these equations are applicable to the attenuation of the light amount per unit area. Therefore, the total light amount transferred is $\int \tilde{I}(\lambda)\Delta\lambda$ [W], where s is the area of the cross section of the light path.

The photomultiplier output $\Delta\bar{E}_p$ by $s\tilde{I}(\lambda)\Delta\lambda$ is given as follows:

$$\Delta\bar{E}_p=\mu R_L S_p(\lambda)s\tilde{I}(\lambda)\Delta\lambda \quad (3)$$

where $S_p(\lambda)$ and μ denote the cathode emission sensitivity and the current multiplication factor of the photomultiplier, respectively. The output $\bar{E}_p$ for all the wavelength components is given in an integral form as follows:

$$\bar{E}_p=\int_0^\infty \mu R_L S_p(\lambda)s\tilde{I}_0(\lambda)\exp\{-\beta(\lambda)l\bar{I}\}d\lambda \quad (4)$$

On the other hand, the output $\bar{E}_p$ when the light absorptive matter does not exist is easily obtained from Eq. (4), putting $\bar{I}=0$. Then the non-dimensional output of the photomultiplier is expressed as follows:

$$\bar{E}_p/\bar{E}_{p,0} = \int_0^\infty S_p(\lambda) \bar{I}_0(\lambda) \exp\{-\beta(\lambda) l \bar{I}^{\bar{I}}\} d\lambda / \int_0^\infty S_p(\lambda) \bar{I}_0(\lambda) d\lambda \quad (5)$$

where s, μ, R_L are cancelled, since all of them are independent of the wavelength. In the actual measuring system, there exist the light losses by other optical attenuation materials such as the lens, optical filter and glass fibers. When the effects of these materials are considered, the right side of Eq. (3) should be multiplied by each transmissivity $T_i(\lambda)$ of these materials, and then the non-dimensional output of the photomultiplier can be described by

$$\frac{\bar{E}_p}{\bar{E}_{p,0}} = \frac{\int_0^\infty S_p(\lambda) \bar{I}_0(\lambda) \exp\{-\beta(\lambda) l \bar{I}^{\bar{I}}\} \prod_i T_i(\lambda) d\lambda}{\int_0^\infty S_p(\lambda) \bar{I}_0(\lambda) \prod_i T_i(\lambda) d\lambda} \quad (6)$$

Therefore, it becomes evident that the $\bar{E}_p$ - $\bar{I}^{\bar{I}}$ relation does not obey a simple power law such as Lambert-Beer's law in general; this relation can be described in an integral form i.e. Eq. (6) in consideration of the effects of various optical elements in the measuring system.

2.2 A consideration about the deviation of the $\bar{E}_p$ - $\bar{I}^{\bar{I}}$ relation from the simple power law

In order to clarify the meaning of Eq. (5) or Eq. (6), the following three typical combinations concerning $\bar{I}_0(\lambda)$, $\beta(\lambda)$ and $S_p(\lambda)$ will be investigated (see Fig. 2), because these combinations are very simple and can become also the models of actual measuring system.

Case 1): the combination of a monochromatic light source, rectangular light absorption spectral distribution of matter and rectangular photomultiplier sensitivity (this combination is denoted by (1) in Fig. 2)

In this case, each optical characteristic is given as follows:

the light source:

$$\bar{I}_0(\lambda) = A\delta(\lambda - \lambda_a) \quad (7a)$$

the light absorptive matter:

$$\beta(\lambda) = B\{U(\lambda - \lambda_a) - U(\lambda - \lambda_e)\} = BH(\lambda; \lambda_a, \lambda_e) \quad (7b)$$

the photomultiplier:

$$S_p(\lambda) = CH(\lambda; \lambda_f, \lambda_g) \quad (7c)$$

where $\delta(\lambda)$; Dirac's δ function, $U(\lambda)$; unit step function and $H(\lambda; \lambda_1, \lambda_2)$ is equivalent to 1 or 0 according as $\lambda_1 \leq \lambda \leq \lambda_2$ or the other range of λ . Then $\bar{E}_p/\bar{E}_{p,0}$ is described from Eq. (5) as follows:

$$\frac{\bar{E}_p}{\bar{E}_{p,0}} = \frac{\int_0^\infty CH(\lambda; \lambda_f, \lambda_g) A\delta(\lambda - \lambda_a) \exp\{-BH(\lambda; \lambda_a, \lambda_e) l \bar{I}^{\bar{I}}\} d\lambda}{\int_0^\infty CH(\lambda; \lambda_f, \lambda_g) A\delta(\lambda - \lambda_a) d\lambda} \quad (8)$$

In the case of $\lambda_a < \lambda_a < \lambda_e$ and $\lambda_f < \lambda_a < \lambda_g$, i.e. when the wavelength emitted from the light source is contained in both spectral bands of the light absorption characteristic of matter and the photomultiplier sensitivity, Eq. (8) reduces to

$$\bar{E}_p/\bar{E}_{p,0} = \exp(-Bl \bar{I}^{\bar{I}}) \quad (9)$$

From the above equation, it is found that the $\bar{E}_p$ - $\bar{I}^{\bar{I}}$ relation obeys a simple power law like Lambert-Beer's law. Even if the light absorption characteristic of matter and photomultiplier sensitivity have any spectral distributions other than rectangular ones, the same results can be obtained as far as the wavelength of the emitted light lies within both distributions mentioned above.

Case 2); the combination of a white light source, rectangular light absorption spectral distribution of matter and rectangular photomultiplier sensitivity (this combination is denoted by (2) in Fig. 2)

In this case, $\bar{I}_0(\lambda)$, $\beta(\lambda)$ and $S_p(\lambda)$ are given as follows:

$$\bar{I}_0(\lambda) = EU(\lambda) \quad (10a)$$

$$\beta(\lambda) = BH(\lambda; \lambda_a, \lambda_e) \quad (10b)$$

$$S_p(\lambda) = CH(\lambda; \lambda_f, \lambda_g) \quad (10c)$$

Then, $\bar{E}_p/\bar{E}_{p,0}$ can be described by

$$\begin{aligned} \bar{E}_p/\bar{E}_{p,0} = & \int_{\lambda_f}^{\lambda_g} \exp\{-Bl \bar{I}^{\bar{I}} H(\lambda; \lambda_a, \lambda_e)\} \\ & \times d\lambda / (\lambda_g - \lambda_f) \quad (11) \end{aligned}$$

when $\lambda_f \leq \lambda_d \leq \lambda_e \leq \lambda_g$ i.e. when the light absorption band of matter is contained in the photomultiplier sensitivity, Eq. (11) reduces to

$$\bar{E}_p/\bar{E}_{p,0} = K \exp(-B\bar{I}) + 1 - K \quad \dots\dots\dots(12)$$

where $K = (\lambda_e - \lambda_d)/(\lambda_g - \lambda_f)$, $0 \leq K \leq 1$. Taking the natural logarithm of Eq. (12) yields

$$\ln(\bar{E}_p/\bar{E}_{p,0}) = \ln\{K \exp(-B\bar{I}) + 1 - K\} \quad \dots\dots\dots(13)$$

In Fig. 3, the qualitative effects of parameter K on the $\ln(\bar{E}_p/\bar{E}_{p,0})-\bar{I}$ relation i.e. Eq. (13) are investigated, where $K=0$ means that the light absorption matter does not exist and $K=1$ means that the light absorption band of matter has the same width as the photomultiplier sensitivity band. It is found from Fig. 3 that when $K=1$, the $\bar{E}_p-\bar{I}$ relation obeys a simple power law and deviation from the power law increases as the parameter K decreases, that is, as the light absorption band width $\lambda_e-\lambda_d$ gets narrower in comparison with the photomultiplier sensitivity band width $\lambda_g-\lambda_f$. This situation means that in the case of using a broad band light source (for instance, tungsten lamp), the deviation increases as the light absorption band width of matter narrows.

Case 3): the combination of a light source with a rectangular spectral distribution, rectangular light absorption spectral distribution of matter and uniform photomultiplier sensitivity (this combination is denoted by (3) in Fig. 2)

In this case, $\bar{I}_0(\lambda)$, $\beta(\lambda)$, $S_p(\lambda)$ and $\bar{E}_p/\bar{E}_{p,0}$ are given by

$$\bar{I}_0(\lambda) = DH(\lambda; \lambda_b, \lambda_c) \quad \dots\dots\dots(14.a)$$

$$\beta(\lambda) = BH(\lambda; \lambda_d, \lambda_e) \quad \dots\dots\dots(14.b)$$

$$S_p(\lambda) = FU(\lambda) \quad \dots\dots\dots(14.c)$$

$$\bar{E}_p/\bar{E}_{p,0} = K' \exp(-B\bar{I}) + 1 - K' \quad \dots\dots\dots(15)$$

where $K' = \Lambda/(\lambda_c - \lambda_d)$, $0 \leq K' \leq 1$ and Λ denotes the width of the wavelength range in which $H(\lambda; \lambda_b, \lambda_c)H(\lambda; \lambda_d, \lambda_e) = 1$. It should be noticed here that $K'=0$ means that there exist no common parts to both spectral distributions of the light source and the light absorptive matter, and $K'=1$ means that the radiation intensity distribution of the light source is contained in the light absorption band of matter or has the same width as the latter. This situation suggests that it is possible to make the $\bar{E}_p-\bar{I}$ relation follow a simple power law if the light is picked out selectively by the optical filter from the broad band light source.

2.3 Measuring system

Figure 4 shows a whole diagram of the measuring system. The light emitted from the tungsten lamp is introduced into the test section through the light

guide after passing the lens system and the optical filter. The intensity of the light introduced into the test section attenuates strictly according to Lambert-Beer's law for each wavelength component due to the effect of the light absorption matter. The light modified according

Fig. 3 The $\bar{E}_p-\bar{I}$ characteristics curves in the cases of (2) and (3) in Fig. 2

Fig. 4 A schematic diagram of the measuring system

Fig. 5 The relative radiation intensity distribution of the tungsten lamp and the cathode emission sensitivity of the photomultiplier R 268

to the information of the concentration fluctuations at the test section goes again through the light guide and lens system, and then it is converted to a photoelectric current proportional to light intensity. We can finally detect the photomultiplier voltage output $\bar{E}_p$ which is imposed on the resistance R_i .

Figure 5 shows the relative radiation intensity distribution of the tungsten lamp and the cathode emission sensitivity of the photomultiplier (a head-on type of photomultiplier R 268, Hamamatsu T.V. Inc.). And Fig. 6 shows the transmissivity of the glass fiber (GF-LG, Sumita Optics Inc.) of the present measuring system. These characteristics will become useful for a later analysis.

3. The Light Absorptive Characteristics of Various Sample Matters

In this section, the $\bar{E}_p$ - $\bar{I}$ characteristics of various inks or dyes will be investigated and the verification of the discussions given in Chapter 2 will be made. In these investigations, two kinds of glass fibers with the bundle diameters of $\phi 0.3$ and $\phi 0.5$ were used. The length of light path in the sampling part was measured by a universal projector.

3.1 The $\bar{E}_p$ - $\bar{I}$ characteristics of various sample matters (without the optical filter)

Figure 7 shows the $\bar{E}_p$ - $\bar{I}$ characteristic curves of blue inks B_1 , B_2 and a red ink R commercially available when the light emitted from the tungsten lamp is directly introduced into the sampling part without the optical filter. Fig. 8 shows one result which was obtained about the direct dye D.F.Orange in the same way. The results of ink B_1 ,

B_2 were obtained when $d_f = \phi 0.3$, $l = 0.56$ mm, and the investigation of ink R was made under two different conditions that $d_f = \phi 0.3$, $l = 0.56$ mm and $d_f = \phi 0.5$, $l = 0.83$ mm. On the other hand, D.F.Orange was investigated under one condition of $d_f = \phi 0.5$, $l = 0.83$ mm. From these figures, it can be seen that characteristics of all the samples do not obey a simple power law. Concerning the ink R, the attenuation rate of the light intensity in the case of $l = 0.83$ mm is greater than the one in the case of $l = 0.56$ mm as we can predict from Eq. (2), where it should be noticed that the bundle diameter d_f of the optical glass fiber lines has no relation to the output $\bar{E}_p$ of the photomultiplier. Next, in order to explain such deviations, the $\bar{E}_p$ - $\bar{I}$ characteristics of D.F.Orange and ink B_2 as typical examples will be considered in the light of Eq. (6) and the discussions given in Chapter 2. For this purpose, precise investigations of light absorption characteristics for D.F.Orange and ink B_2 are required. Fig. 9 shows the absorbance curves of

Fig. 6 The transmissivity of fiber glass lines (GF-LG)

Fig. 7 The $\bar{E}_p$ - $\bar{I}$ characteristics of various inks

Fig. 8 The $\bar{E}_p$ - $\bar{I}$ characteristic of D.F.Orange

Fig. 9 D.F.Orange: absorbance curves at various concentration values

D.F.Orange which were measured by a two beams digital spectrophotometer (its monochromator resolving power was adjusted to 2 nm and the light path length at the sampling part l was fixed at 10 mm). Since the absorbance A is defined by $A = \beta(\lambda) \tilde{I} / 2.303$, when the light absorptive matter obeys Lambert-Beer's law,

Fig. 10 D.F.Orange: absorbance vs. concentration at various wavelengths

Fig. 11 Ink B2: absorbance curves at various concentration values

Fig. 12 Ink B2: absorbance vs. concentration at $\lambda = 604$ nm and $\lambda = 650$ nm

absorbance A should be proportional to the concentration $\tilde{I}$ at each wavelength of the working light. Fig. 10 shows changes in the absorbances at each wavelength of $\lambda = 422, 470, 496, 542$ nm. As shown in Fig. 10, it can be said that the changes in absorbances of D.F.Orange have a good linear relationship to the concentration $\tilde{I}$ as a whole. Fig. 11 and Fig. 12 show the same characteristics of ink B₂ as ones of D.F.Orange in Fig. 9 and Fig. 10. From these figures, it is found that absorbances of ink B₂ are also in proportion to the concentration $\tilde{I}$.

It should be here noted that the maximum wavelength end of absorbance curves in Fig. 11 is about 820 nm (the measuring limit of the spectrophotometer); the high wavelength range in the absorbance curves over 800 nm does not have almost any influence on the photomultiplier output $\tilde{E}_p$, for the photomultiplier sensitivity $S_p(\lambda)$ is very small in this range (see Fig. 5). And one of the very interesting applications in relation to the absorbance curves is as follows: when the light absorption bands of two kinds of substance are separate from each other, it is possible to detect simultaneously both concentrations of two kinds of substances by picking out separately two light beams with the spectral characteristics fit to each separate band.

Fig. 13 The changing process of the light intensity in the measuring system

3.2 An examination of the $\bar{E}_p\text{-}\bar{I}$ characteristics

Figure 13 shows the changing process of the light intensity which occurs in the measuring system. To facilitate the analysis, it is assumed that the attenuation of the light intensity at the sampling section occurs only due to the effect of the light absorption of matter and the effects of the reflection, scattering etc. are negligibly small. Further, it is supposed that the attenuations due to the lens system can be also negligible, i.e. $T_1(\lambda)=T_2(\lambda)=1.0$ and the transmissivities of glass fibers 1 and 2 have the profiles given in Fig. 6.

$\bar{I}_0(\lambda)$ and $S_p(\lambda)$ have been already presented in Fig. 5. Now, in consideration of these characteristics, the non-dimensional photomultiplier output $\bar{E}_p/\bar{E}_{p,0}$ can be calculated theoretically on the basis of Eq. (6).

In Figs. 14 and 15, the results of calculations with the parameter l are compared with the measured values in each case of D.F.Orange and ink B₂. The measured values of D.F.Orange in Fig. 14 indicate the results for $l=0.83$ mm and the values of ink B₂ in Fig. 15 are the ones for $l=0.55$ mm or $l=0.56$ mm. It can be seen from these figures that in the case of D.F.Orange, the measured values are almost in agreement with the calculated results, whereas the measured values of ink B₂ for $l=0.55$ or $l=0.56$ mm agree with the calculated results of $l=0.7 \sim 0.8$ mm; these values shows larger light absorption characteristics

Fig. 14 The comparison of the calculated results with the measured values in the case of D.F.Orange

Fig. 15 The comparison of the calculated results with the measured values in the case of ink B₂

than the calculated results of $l=0.55$ or $l=0.56$ mm. The reasons for this phenomenon can be considered as follows: firstly, both ends of a pair of fiber glass lines which construct the sampling volume do not have smooth surfaces and they are not parallel with each other, and secondly the lights transferred through the fiber glass lines are not perfectly parallel rays so that the incident light into the sampling volume has the some spatial intensity distribution. For the reason mentioned above, it is rather difficult to determine an accurate length of light pass l in the sampling volume. Further more, the inaccuracy of the spectral characteristics of each optical element in the measuring system, especially the inaccuracy of the radiation intensity distribution $\bar{I}_0(\lambda)$ of the tungsten lamp seems to be also an important cause.

According to the considerations in Section 2.2; case (2), when the light source has a broad band characteristic such as the tungsten lamp, the absorbed light amount increases as the value of parameter K which indicates the ratio of the light absorption band width to

Fig. 16 Absorbance spectrum of D.F.Orange and the transmissivities of various optical filters

Fig. 17 The change in the $\bar{E}_p\text{-}\bar{I}$ characteristic due to various combinations of D.F.Orange and optical filters

the photomultiplier sensitivity width increases. Then the $\tilde{E}_p\text{-}\tilde{I}$ relation comes closer to a simple power law as the parameter K increases. In the light of the situation mentioned above, it can be said that in the present investigation the effect of an error of the light path length l on the $\tilde{E}_p\text{-}\tilde{I}$ relation is greater in the case of ink B₂ with a relatively broad light absorption band than in the case of D.F.Orange with a narrow band.

3.3 The improvement of the $\tilde{E}_p\text{-}\tilde{I}$ characteristic by the optical matching technique

In this section, an actual example is given on the basis of the consideration of the case (3) in Section 2.2, how it becomes possible to correct the deviation of the $\tilde{E}_p\text{-}\tilde{I}$ relation from the simple power law by using the optical filter in the case of D.F.Orange. However, in this case there exists an inevitable decline of the S.N. ratio because of a decrease in the light amount of which can be concentrated on the sampling volume, so that the optical filters must be selected within an allowable range of the S.N. ratios in actual measurement. In the present measurements, there was not any problem of the S.N. ratio even with

Fig. 18 Two-dimensional laminar flow formed at the sampling part

Fig. 19 The changing process of $S(t)$

the use of various optical filters given in the following. The absorbance distribution of D.F.Orange and the transmissivities of various optical filters are shown together in Fig. 16. The optical matching conditions of D.F.Orange to the various optical filters are better in the order of BPB-42, BPB-45 and BPB-50. The $\tilde{E}_p\text{-}\tilde{I}$ characteristics which correspond to each combination are shown in Fig. 17. From Fig. 17, it is found that the $\tilde{E}_p\text{-}\tilde{I}$ characteristic approaches the simple power law as the optical matching condition becomes better.

4. The Effect of Viscosity on the Time Response of the Measuring System

In practice, one of the most important problems in detecting the concentration fluctuations is the time response characteristics of the measuring system. When the probe is set up in the actual flow field, the boundary layer which inevitably develops on the surfaces of the probe ends has influence on the value of concentration detected and frequency response of the system. If the optical response and electric response of the measuring system are quick enough in comparison with the frequency of concentration fluctuations, it can be inferred that the effect of the boundary layer becomes the most severe constraint to the time response. However, since it is impossible to deal with the effect of the boundary layer exactly, an idealized analysis will be made here on the assumption that a laminar two-dimensional flow as shown in Fig. 18 is formed at the sampling part. In Fig. 18, l is the distance between both ends of the probe and h denotes the reference thickness of the boundary layer. It should be noted that the actual flow is three-dimensional and there exists also a stirring effect by turbulence, so the situation here dealt with corresponds to the case of a relatively unfavorable condition. Let us assume that a light absorptive matter appears suddenly at $t = 0$ just upstream of the probe surface and the effect of molecular diffusivity is negligibly small, then the volume $S(t)$ of the light absorptive matter existing in the sampling part would change as shown in Fig. 19. Here, $l = 0.825$ mm, $U_m = 50$ cm/s are selected as the average condition of the actual flow and it is assumed that $b = d_f/2$, $h = 2.5\sqrt{(v d_f/2)/U_m}$, where h indicates a half value of the thickness $\delta(20)$ for a Blasius type of laminar boundary layer at the center of the probe surface. The changing process of $S(t)$ is investigated in three cases of $d_f = 0.5, 0.3$ and 0.1 mm. From this figure, it is found that the time response approaches an ideal one as d_f becomes small, but as a whole the effect of the boundary layer is not serious.

Now, in order to examine the propriety of the assumption of negligible molecular diffusion, the order estimation to

the molecular diffusion will be made. It is known in general that, in the case of short time diffusion, the matter sheet diffusing into the turbulent field changes its thickness solely as a results of molecular diffusion, and the effect of the turbulent diffusion is to produce only undulations of the sheet (24). Since the flow is assumed to be laminar here, it is enough to estimate only a spread of b by the molecular diffusion; the amount of change δb of the width b of the matter sheet is estimated as follows:
 $\delta b \approx \sqrt{2Dt}$ (D : molecular diffusivity)
 When $D \approx 1.0 \times 10^{-6}$ cm²/s, $t = d_r/U_m = 0.5 \times 10^{-1}/50 = 1.0 \times 10^{-3}$ s, we estimate that
 $\delta b \approx \sqrt{2Dt} = 4.47 \times 10^{-5}$ cm. Thus, it is concluded that the effect of the molecular diffusion is almost negligible.

5. Conclusions

In order to work out a measuring system which can detect a wide range of concentration fluctuations, an analysis is made of the deviation of the $\tilde{E}_p - \tilde{I}$ relation from a simple power law. The following conclusions are obtained.

(1) The deviation of the relation between the photomultiplier output $\tilde{E}_p$ and the concentration $\tilde{I}$ from a simple power law can be explained quantitatively fairly well by introducing Eq. (6) i.e. the $\tilde{E}_p - \tilde{I}$ relation which is described in an integral form in consideration of the spectral characteristics of each optical element in the measuring system.

(2) It is possible to make the $\tilde{E}_p - \tilde{I}$ relation obey the simple power law by matching the spectral distribution of the incident light to the light absorption spectral characteristic of matter by means of an optical filter.

By using this measuring system which was made on the basis of the above study, authors actually measured the concentration fluctuation in a round free jet and proved its availability (25).

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