Liquid Films Flowing Concurrently with Air in Horizontal Duct*

(3rd Report, The relation between occurrence of
viscous wave and breakdown of liquid film)

By Tohru FUKANO**, Akihiko ITOH***,
Tessho KURIWAKI**** and Yasuo TAKAMATSU**

A thin liquid film flowing concurrently with an air stream was investigated with regard to waves generated when superficial velocity of air was less than about 10 m/s and water flow rate was considerably low. It was made clear theoretically and verified by an experiment that a special wave, named viscous wave by the authors, is generated on the thin liquid film when film thickness was less than a definite value which depended on liquid properties, i.e., 0.255 mm at 20°C for water. Theory shows that the occurrence of a viscous wave depends more on liquid viscosity than on surface tension, and that the wave velocity is lower than liquid surface velocity. The viscous wave occurs always in the vicinity of the nonwetting region, signifying that the occurrence of it reduces the minimum film thickness, which causes a breakdown of liquid film.

Key Words: Multiphase Flow, Film Flow, Neutral Curve, Film Breakdown, Film Thickness, Instability

1. Introduction

Two-dimensional wave flow is one of the flow patterns observed on a liquid film flowing concurrently with a gas flow with relatively low velocity in a rectangular duct⁽¹⁾. According to a previous experiment made by the present authors, the so-called two-dimensional wave flow was divided further into T1 - and T2 - regimes⁽²⁾. Waves in T2 - regime are observed when mean film thickness is comparatively thick. On the other hand, waves in T1 - regime originate when the film thickness is quite small, (0.1~0.2) mm for example, that is, when liquid flow rate is very low. The wave has such a characteristic that its velocity is smaller than liquid surface velocity.

Craik⁽³⁾ discussed both theoretically and experimentally the origination of this kind of wave. In the theory, however, Navier - Stokes equation was applied only to a liquid film flow and the shear force acting on wavy solid wall by gas flow⁽⁴⁾ was used as a boundary condition at gas-liquid interface. Therefore, it is quite difficult to compare the theoretical results with experimental data. In this paper, simultaneous equations obtained by applying Navier-Stokes equation to both

liquid and gas phases are solved. The solution is more general, because boundary conditions adopted in this paper are reasonable in physical sense; i.e., all the balance conditions of stresses are satisfied at gas liquid interface. The theoretical results are easy to compare with the experiments.

Reduction of liquid flow rate with gas flow kept constant while this kind of wave exists causes a breakdown of liquid film. Then, the role of the wave in liquid film breakdown will be briefly discussed.

2. Nomenclature

a : Amplitude of wave mm
 B : Duct width mm
 c : Nondimensional wave velocity [$\equiv C/U_i$]
 C : Wave velocity m/s
 F : Froude number [$\equiv U_i/\sqrt{gt_{fm}}$]
 g : Acceleration of gravity m/s²
 h : Dimensionless thickness of gas layer [$\equiv H/t_{fm} - 1$]
 H : Duct height mm
 j_g : Volumetric flux density of gas m³/m²s
 m : Viscosity ratio [$\equiv \mu/\mu_l$]
 N : Wave passing frequency 1/s
 P : Static pressure Pa
 q : Heat flux W/m²
 r : Density ratio [$\equiv \rho/\rho_l$]
 R : Reynolds number
 t_{fm} : Time averaged film thickness mm
 T : Time s
 U : Velocity in the direction of flow m/s
 U_i : Liquid surface velocity m/s
 V : Velocity perpendicular to flow direction m/s
 W : Weber number [$\equiv \rho_l U_i^2 t_{fm}/\sigma$]
 X : Coordinate in flow direction m

* Received 14th August, 1981.

** Professor, Faculty of Engineering, Kyushu University 36, Hakozaki, Fukuoka, 812 JAPAN.

*** Associate Professor, Faculty of Engineering, Ohita University, Ohita, 870-11 JAPAN.

**** Engineer, IHI, 1, Shinnakahara, Isogoku, Yokohama, 235 JAPAN.

- Y : Coordinate perpendicular to X axis
 α : Dimensionless wave number [$\equiv 2\pi t_{fm}/\lambda$]
 Γ : Liquid volumetric flow rate per unit width of duct m^3/sm
 δ : Dimensionless amplitude [$\equiv a/t_{fm}$]
 λ : Wave length mm or m
 μ : Viscosity $Pa \cdot s$
 ρ : Density kg/m^3
 σ : Surface tension N/m
 ψ : Stream function
 Flow pattern symbols
 NW : Nonwetting regime
 P : Pebble wave regime
 S : Smooth surface regime
 T : Two-dimensional wave regime
 V : Viscous wave regime
 Subscripts
 c : Center of the duct width
 g : Gas phase
 l : Liquid phase
 min : Minimum value
 $-$: Time averaged value

3. Theoretical Analysis

3.1 Basic equations

Fig.1 shows a flow model considered and the nomenclature used. Let the coordinates in the direction of and perpendicular to the flow be X and Y , the velocity and static pressure be $\bar{U}(Y)$ and $\bar{P}(X, Y)$ and their fluctuating components be $U^*(X, Y, T)$, $V^*(X, Y, T)$ and $P^*(X, Y, T)$, then continuity and momentum equations are expressed in dimensionless form as;

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t} + \bar{u} \frac{\partial u^*}{\partial x} + v^* \frac{\partial \bar{u}}{\partial y} = \left(-\frac{1}{r} \frac{\partial \bar{p}}{\partial x} + \frac{m}{rR} \frac{\partial^2 \bar{u}}{\partial y^2} \right) - \frac{1}{r} \frac{\partial p^*}{\partial x} + \frac{m}{rR} \left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v^*}{\partial t} + \bar{u} \frac{\partial v^*}{\partial x} = \left(-\frac{1}{r} \frac{\partial \bar{p}}{\partial y} - \frac{1}{F^2} \right) - \frac{1}{r} \frac{\partial p^*}{\partial y} + \frac{m}{rR} \left(\frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right) \quad (3)$$

where in general small letters are used for dimensionless variables, i.e., $x=X/t_{fm}$, $y=Y/t_{fm}$, $t=T/(t_{fm}/U_i)$, $u=(\bar{U}+U^*)/U_i$, $\bar{u}=U_i$, $v=V^*/U_i$, $p=P/(\rho_l U_i^2) \equiv \bar{p}+p^*$, $R=\rho_l t_{fm} U_i/\mu_l$ (Reynolds number), $F=U_i/\sqrt{g t_{fm}}$ (Froude number), $r=\rho/\rho_l$ and $m=\mu/\mu_l$. r and m are unity in liquid phase, and $r_g(\equiv \rho_g/\rho_l)$ and $m_g(\equiv \mu_g/\mu_l)$ in gas phase. Superscript * means fluctuating component.

3.2 Steady state solution

Solutions for time averaged components (steady state solutions) are obtained by solving the following equations

$$-\frac{1}{r} \frac{\partial \bar{p}}{\partial x} + \frac{m}{rR} \frac{\partial^2 \bar{u}}{\partial y^2} = 0 \quad (4)$$

$$-\frac{1}{r} \frac{\partial \bar{p}}{\partial y} - \frac{1}{F^2} = 0 \quad (5)$$

with boundary conditions of $\bar{u}_g(h)=0$ (upper wall), $\bar{u}_l(-1)=0$ (bottom wall), $\bar{u}_g(0)=\bar{u}_l(0)$ and $m_g \partial \bar{u}_g(0)/\partial y = \partial \bar{u}_l(0)/\partial y$ (at gas liquid interface) as follows;

$$\left. \begin{aligned} \bar{u}_l &= -\frac{1+m_g}{h(1+h)} y^2 + \frac{h^2-m_g}{1+h} y + 1 \\ \bar{u}_g &= -\frac{h+m_g}{m_g h(1+h)} y^2 + \frac{h^2-m_g}{m_g h(1+h)} y + 1 \end{aligned} \right\} \quad (6)$$

where $h \equiv H/t_{fm}-1$ is the thickness of gas layer. By integrating Eq.(6) respectively in gas and liquid phases the volumetric flux density of gas, j_{gc} , and the volumetric flow rate of liquid per unit width of duct, Γ_c , are obtained as,

$$j_{gc} = \frac{U_i(h^3+4m_g h^2+3m_g h)}{6m_g(1+h)^2} \quad (7)$$

$$\Gamma_c = \frac{U_i h(3h^2+4h+m_g)}{6h(1+h)^2} \quad (8)$$

Eliminating U_i from Eqs.(7) and (8) gives

$$h^4 + 4m_g h^3 + 3m_g \left(1 - \frac{1}{f}\right) h^2 - 4\left(\frac{m_g}{f}\right) h - \frac{m_g^2}{f} = 0 \quad (9)$$

where

$$f = \Gamma_c / (j_{gc} H) \quad (10)$$

Since m_g is the property of fluids, film thickness, t_{fm} , is determined by Eq.(9) if only flow conditions Γ_c and j_{gc} are given. Furthermore, substitution of t_{fm} into Eq.(7) or (8) gives liquid surface velocity U_i .

3.3 Basic equation for liquid film stability

Assuming that fluctuating components are two-dimensional, like steady state flow, and introducing a stream function, we obtain

$$u^* = \partial \psi^* / \partial y, \quad v^* = -\partial \psi^* / \partial x \quad (11)$$

Further assuming that a two-dimensional wave-like perturbation of the liquid surface is represented by

$$\eta(x, t) = \delta \exp i\alpha(x-ct) \quad (12)$$

and that the fluctuating components in the fluids are in phase with the wave-like perturbation, we obtain

$$\left. \begin{aligned} \psi^*(x, y, t) &= \phi(y) \eta(x, t) \\ p^*(x, y, t) &= \bar{p}(y) \eta(x, t) \end{aligned} \right\} \quad (13)$$

where $\delta \equiv a/t_{fm}$, $\alpha \equiv 2\pi t_{fm}/\lambda$ and $c=c_r+ic_i \equiv C/U_i$ are respectively the dimensionless amplitude, wave number and complex wave velocity. Substitution of Eq.(13) into Eqs.(2) and (3) gives the Orr-Sommerfeld equation as a basic equation for the present problem.

$$\begin{aligned} \phi^{iv} - 2\alpha^2 \phi'' + \alpha^4 \phi \\ = \frac{i\alpha r R}{m} [(\bar{u}-c)(\phi''-\alpha^2 \phi) - \bar{u}'' \phi] \end{aligned} \quad (14)$$

where prime denotes differentiation with respect to y . Eq.(14) is applied to both the gas and liquid phases and the resulting equations are solved simultaneously. Boundary conditions are: (1) perturbation velocity components must vanish at solid wall and continue at gas-liquid interface, and (2) tangential and normal stresses in gas and liquid phases should be in equilibrium with each other in respective directions. They are expressed as,

$$\phi_g(h) = \phi_l'(h) = 0 \quad (\text{upper wall}) \quad (15)$$

Fig.1 Flow model and nomenclature

$$\phi_{\ell}(-1) = \phi'_{\ell}(-1) = 0 \quad (\text{bottom wall}) \quad (16)$$

$$\phi_g(0) = \phi'_{\ell}(0) \quad (\text{interface}) \quad (17)$$

$$m_g[\phi''_g(0) + \alpha^2 \phi_g(0)] = \phi''_{\ell}(0) + \alpha^2 \phi_{\ell}(0) \quad (18)$$

$$\begin{aligned} \mathbb{H}_g(0) - \frac{2i\alpha m_g}{R} \phi'_g(0) \\ = \mathbb{H}_{\ell}(0) - \frac{2i\alpha}{R} \phi'_{\ell}(0) + \frac{1}{F^2} + \frac{\alpha^2}{W} \end{aligned} \quad (19)$$

$$\phi_g(0) = \phi_{\ell}(0) = c - 1 \quad (20)$$

where Eqs.(17) ~ (20) are linearized by expanding in Taylor's series. $W(\equiv \rho_{\ell} U_i^2 t_{fm} / \sigma)$ and σ are Weber number and surface tension, respectively. Since an analytical solution to the simultaneous equations is not easy to obtain, these are solved by using a series.

3.4 Approximate solution for perturbed quantities in case of $\alpha R < 1$

In a thin liquid film flow in question where α is assumed to be much smaller than unity and R is sufficiently small in general, $\alpha R < 1$ holds. Under such a condition the solutions of Eq.(14), ϕ and c can be obtained in the following forms of series which converge rapidly with power n . (5)

$$\phi(y) = \sum_{n=0}^{\infty} (i\alpha R)^n \phi_n(y) \quad (21)$$

$$c = c_r + ic_i = \sum_{n=0}^{\infty} (i\alpha R)^n c_n \quad (22)$$

Substitution of Eqs.(21) and (22) into Eq.(14) yields the following set of equations;

$$\left. \begin{aligned} \phi_0^{iv} &= 0 \\ \phi_1^{iv} &= \frac{rg}{m_g} [(\bar{u}-c_0)\phi_0'' - \bar{u}''\phi_0] \\ \phi_2^{iv} &= \frac{rg}{m_g} [(\bar{u}-c_0)\phi_1'' - c_1\phi_0'' - \bar{u}''\phi_1] - \frac{2}{R^2}\phi_0'' \\ \phi_3^{iv} &= \frac{rg}{m_g} [(\bar{u}-c_0)\phi_2'' - c_1\phi_1'' - c_2\phi_0'' - \bar{u}''\phi_2 \\ &\quad + \frac{1}{R^2}(\bar{u}-c_0)\phi_0] - \frac{2}{R^2}\phi_1'' \end{aligned} \right\} \quad (23)$$

In the same way, the boundary conditions give the following equations;

$$\left. \begin{aligned} \phi_{gn}(h) &= \phi'_{gn}(h) = 0 \quad (n=0,1,2,\dots) \quad (24) \\ \phi_{\ell n}(-1) &= \phi'_{\ell n}(-1) = 0 \quad (n=0,1,2,\dots) \quad (25) \\ m_g\phi_{g0}(0) &= \phi_{\ell 0}(0), \quad m_g\phi'_{g1}(0) = \phi'_{\ell 1}(0) \\ m_g[\phi_{gn}(0) - \frac{1}{R^2}\phi_{gn-2}(0)] &= \phi_{\ell n}(0) \\ &\quad - \frac{1}{R^2}\phi_{\ell n-2}(0) \quad (n=2,3,\dots) \end{aligned} \right\} \quad (26)$$

$$\begin{aligned} m_g\phi_{g0}''(0) &= \phi_{\ell 0}''(0), \\ r_g[(1-c_0)\phi_{g0}'(0) - \bar{u}'_g(0)\phi_{g0}(0)] &- m_g\phi_{g1}'''(0) \\ &= [(1-c_0)\phi_{\ell 0}'(0) - \bar{u}'_{\ell}(0)\phi_{\ell 0}(0)] - \phi_{\ell 1}'''(0) + \frac{1}{F^2}, \\ r_g[(1-c_0)\phi_{g1}'(0) - c_1\phi_{g0}'(0) - \bar{u}'_g(0)\phi_{g1}(0)] &- m_g\phi_{g2}'''(0) - \frac{3m_g}{R^2}\phi_{g0}'(0) \\ &= [(1-c_0)\phi_{\ell 1}'(0) - c_1\phi_{\ell 0}'(0) - \bar{u}'_{\ell}(0)\phi_{\ell 1}(0)] - \phi_{\ell 2}'''(0) \\ &\quad - \frac{3}{R^2}\phi_{\ell 0}'(0), \\ r_g[(1-c_0)\phi_{g2}'(0) - c_1\phi_{g1}'(0) - c_2\phi_{g0}'(0) &- \bar{u}'_g(0)\phi_{g2}(0)] - m_g\phi_{g3}'''(0) - \frac{3m_g}{R^2}\phi_{g1}'(0) \\ &= [(1-c_0)\phi_{\ell 2}'(0) - c_1\phi_{\ell 1}'(0) - c_2\phi_{\ell 0}'(0) \\ &\quad - \bar{u}'_{\ell}(0)\phi_{\ell 2}(0)] - \phi_{\ell 3}'''(0) - \frac{3}{R^2}\phi_{\ell 1}'(0) - \frac{1}{WR^2}, \\ &\quad \vdots \quad \dots \quad (27) \end{aligned}$$

$$\phi'_{gn}(0) = \phi'_{\ell n}(0) \quad (n=0,1,2,\dots) \quad (28)$$

$$\phi_{g0}(0) = \phi_{\ell 0}(0) = c_0 - 1$$

$$\phi_{gn}(0) = \phi_{\ell n}(0) = c_n \quad (n=1,2,3,\dots) \quad (29)$$

Since a power in the right-hand side of Eq.(23) is at least by one order smaller than that in the left, ϕ_i is obtained by solving Eq.(23) successively starting from $\phi_0^{iv}=0$. Under such a condition as $\alpha < 1$ and $\alpha R|c| < 1$, both ϕ_n and c_n converge uniformly, then the first four terms are obtained in this paper. The fourth term c_3 is the first term which has an information of the stability of wave occurring on a thin film. They are expressed as follows,

$$\left. \begin{aligned} c_0 &= 1, \quad c_1 = -\frac{1}{3F^2}, \quad c_2 = \frac{1}{2520} \frac{1}{F^2} \frac{\beta_{20}}{(1+h)} \\ \beta_{20} &= \frac{rg}{m_g} (3h^3 - 20h^2 + 108h) - (245h + 114) \\ c_3 &= \frac{1}{2520} \frac{1}{F^2} \frac{1}{(1+h)^2} \frac{1}{3} \sum_{n=0}^7 Q^{(n)} h^n \end{aligned} \right\} \quad (30)$$

$$\begin{aligned} Q^{(7)} &= -0.013 \frac{rg^2}{m_g^3}, \quad Q^{(6)} = 0.082 \frac{rg^2}{m_g^3} \\ &+ 0.6 \frac{rg^2}{m_g^2}, \quad Q^{(5)} = 0.032 \frac{rg^2}{m_g^3} - 2.35 \frac{rg^2}{m_g^2}, \end{aligned}$$

$$Q^{(4)} = -48.5 \frac{rg^2}{m_g^2} + 5.1 \frac{rg}{m_g},$$

$$Q^{(3)} = -100.4 \frac{rg^2}{m_g^2} - 29.5 \frac{rg}{m_g} + 84mK_2$$

$$Q^{(2)} = -108 \frac{rg^2}{m_g^2} + 174.5 \frac{rg}{m_g} - 211.7 - 84K_1$$

$$+ 672m_gK_2 - 2520K_3, \quad Q^{(1)} = 87.9 \frac{rg}{m_g} - 199$$

$$+ (630m_g - 168)K_1 + 1428m_gK_2 - 5040K_3$$

$$Q^{(0)} = -47.9 + (1890m_g - 84)K + 1176m_gK_2 - 2520K_3$$

$$K_1 = \frac{1}{F^2} - \frac{6}{R^2}, \quad K_2 = \frac{rg}{m_g} \frac{1}{F^2} - \frac{6}{R^2}$$

$$K_3 = \frac{2}{R^2} - \frac{F^2}{WR^2} - \frac{1}{6F^2}$$

3.5 Analytical results

Stability of waves is examined by the imaginary part of wave velocity [Eq.(22)], ic_i , and the actual wave velocity by the real part, c_r .

3.5.1 Criterion for wave generation

Wave-like perturbation will grow if the imaginary part of c is positive, or, $c_i = \alpha R c_1 - \alpha^3 R^3 c_3 > 0$, which is reduced to as

$$\begin{aligned} \theta_1(h)R^2 + \theta_3(h) - \theta_4(h)\frac{1}{\alpha^2} &> \frac{R^2}{F^2}\theta_2(h) \\ &+ \frac{F^2}{W}\theta_4(h) \quad \dots \quad (31) \end{aligned}$$

where $\theta_1 \sim \theta_4$ which are functions of h or mean film thickness, t_{fm} , are expressed as follows,

$$\begin{aligned} \theta_1(h) &= -0.013 \frac{rg^2}{m_g^3} h^7 + (0.082 \frac{rg^2}{m_g^3} \\ &+ 0.6 \frac{rg^2}{m_g^2})h^6 + (0.032 \frac{rg^2}{m_g^3} - 2.35 \frac{rg^2}{m_g^2})h^5 \\ &+ (-48.5 \frac{rg^2}{m_g^2} + 5.1 \frac{rg}{m_g})h^4 + (-100.4 \frac{rg^2}{m_g^2} \\ &- 29.5 \frac{rg}{m_g})h^3 + (-108 \frac{rg^2}{m_g^2} + 174.5 \frac{rg}{m_g} \\ &- 211.7)h^2 + (87.9 \frac{rg}{m_g} - 199)h - 47.9 \end{aligned}$$

$$\begin{aligned}\theta_2(h) &= 84x_g h^3 + (336 + 672x_g)h^2 + (630m_g \\ &\quad + 1428x_g + 67.2)h + (336 + 1176x_g + 1890m_g) \\ \theta_3(h) &= (-504m_g)h^3 + (-4536 - 4032m_g)h^2 \\ &\quad + (-9072 - 12348m_g)h + (-18396m_g - 4536) \\ \theta_4(h) &= 2520(h+1)^2\end{aligned}$$

It must be noticed that although the left-hand side of Eq.(31) is determined if only t_{fm} and the properties of both phases are given, gravity and surface tension terms are included in the right of it.

Neutral stability curve which is defined by $c_i=0$ and obtained by equating the terms on both sides of Eq.(31) is plotted in Fig.2, t_{fm} being the parameter. With an increasing α , each neutral curve asymptotically approaches a specific value of Reynolds number, R_c , which varies with t_{fm} . These values are shown by one-dotted-chain lines in the figure. R_c corresponds to $\alpha=\infty$ which is beyond the limitation of the analysis. R , however, approaches R_c sufficiently close even though α is smaller than unity. Therefore, R_c is useful as a criterion for stability of the perturbation. As shown in the figure, a small disturbance grows into a wave provided that the flow condition is on the right side of each neutral curve and does not grow on the opposite side. In short, liquid film is absolutely stable if $R < R_c$.

Fig.2 Definition of neutral curve

Fig.3 Neutral curve ($\alpha=\infty$) and the region of the viscous wave

In this sense, R_c is referred to as a critical Reynolds number. The neutral curves determined by R_c are plotted by one-dotted-chain lines in Fig.3, temperature $t^\circ\text{C}$ being the parameter. Wave will occur on the right side of each curve. Inspection of this figure reveals that the higher the gas flow rate and the liquid temperature, the easier the generation of wave.

Thick solid lines in Fig.4 show the effects of α ($\equiv 2\pi t_{fm}/\lambda$) on the neutral curves which are rewritten from Fig.2. Thin solid lines which express isopleth for film thickness obtained by the theory are straight. This is inferred from Eqs. (9) and (10). A shift of the neutral curve toward the right with an increase in α means that an incipient wave which appears at a larger gas flow rate has a larger wave length.

3.5.2 Wave velocity

Wave velocity c_r is expressed as

$$c_r \approx c_0 - \alpha^2 R^2 c_2 = 1 + \frac{\alpha^2}{2520} \cdot \frac{R^2}{F^2} \frac{\beta_{20}}{1+h} \quad (32)$$

$$\beta_{20} = -\frac{rg}{mg} (3h^3 - 20h^3 + 108h) + 245h + 114 \quad \dots\dots (33)$$

In Fig.5 c_r is plotted against t_{fm} , wave length being the parameter. It shows clearly the case where c_r is smaller than unity if t_{fm} is below a certain value, t_{fv} , which is independent of λ and is 0.255 mm, for example, when water temperature is 20 °C. This means that wave velocity is less

Fig.4 Neutral curve (effect of α) and isopleth curve for film thickness

Fig.5 Wave velocity (Theory)

than liquid surface velocity.

Eq.(32) is rewritten as,

$$c_r \approx \frac{\beta_{20} 4\pi^2 t_{fv}^4}{2520\lambda^2 \nu_l^2} \left\{ \frac{2520\lambda^2 \nu_l^2}{\beta_{20} 4\pi^2 t_{fv}^4} + \frac{gt_{fv}^2}{H} \right\} \dots (34)$$

The first and the second terms in the parenthesis express respectively a viscous and a gravitational effect. If film thickness is small, in particular in the case where a special wave with $c_r < 1$ takes place the first term, i.e., viscous term, dominates c_r . Therefore, the wave is referred to as the viscous wave in this paper. As seen from Eqs.(32) and (33) a demarcation line for existence of the viscous wave is determined from $c_r = 1$ or $\beta_{20} = 0$ which is a function of film thickness besides fluid properties, signifying that the lines are parallel with isopleth lines for film thickness shown by solid lines in Fig.3. Furthermore, it is revealed in Fig.6 that the film thickness at which the viscous wave occurs, t_{fv} , becomes smaller as liquid temperature increases. Meanwhile, in the analysis made by Craik(3) a turbulent gas flow was assumed and the shear force exerted on liquid film by gas flow was estimated as that on a wavy solid wall analyzed by Benjamin(4). Therefore, comparison of the present analytical results with those of Craik could not be made in this paper.

4. Experimental Apparatus and Procedure

The horizontal test section with a height of 10 mm, a width of 80 mm and a total length of 7.1 m is made of transparent acrylic resin to permit observation of flow pattern. Working fluids are air and water. Air supplied from a compressor and water supplied from a constant head tank meet at the air-water joining section. Its cross-section decreases downstream so that air is accelerated gradually. Air and water flow separately through the measuring sections for film thickness and velocity profile of gas phase about 4 m downstream (225 times the equivalent diameter of 17.8 mm) of the joining section and go into an air-water separator, where air is released into atmosphere and water flow rate per unit width of duct at the center of the duct width is measured.

At the measuring stations, film

Fig.6 Effect of temperature on t_{fv}

thicknesses averaged over the duct width and the local value at the center of the duct width were measured by a constant current method(6). Velocity profile of gas phase was also measured upwards at the center of duct width by a total pressure tube and a wall pressure hole. Furthermore, liquid surface velocity was measured using aluminium powder as a tracer between two definite points with a separation distance of 0.8 m, the starting point being at about 0.7 m downstream of the measuring stations.

Temperature of liquid and system pressure were 20°C and atmospheric, $(9.81 \times 9.85) \times 10^4$ Pa, respectively. Gas Reynolds number ($\equiv j_g D_e / \nu_g$, D_e : equivalent diameter) ranged from 2200 to 11,000 and liquid Reynolds number ($\equiv \Gamma / \nu_l$) from 7 to 100.

5. Comparison of analytical Results with Experiments

5.1 Viscous wave regime

Fig.7 shows the range of the experiment and the marks ○ and ● indicate the experimental points. The shadowed portion is a transition region of gas flow on the right side of which air flow is turbulent and on the left is laminar. In the region named V in the figure, wave velocity is lower than liquid surface velocity (refer to Fig.16). The V-region corresponds exactly to T1-region defined in the previous paper(2). Fig.8 shows a typical example for time variation of film thickness measured under flow condition marked ● in Fig.7. The larger the output, the thinner the film thickness. The length of arrow and numeral written on the left side of each trace indicates the magnitude of wave height. Capital letter written on the right side expresses flow pattern. The figure shows that pebble wave flow (P-regime) has a considerably irregular appearance. With a decreasing water flow rate, Γ , the pebble wave tends to become a two-dimensional wave (T-regime) which has a regular pattern followed by a smooth surface flow (S-regime) where no wave is observed. With further decreasing of Γ , a

Fig.7 Experimental range and flow patterns

wave again occurs. It is a viscous wave. As clearly seen in the figure, the wave number is much lower than those for P- and T-waves.

Fig.9 shows a flow condition for occurrence of the viscous wave, the fluid Reynolds number and the maximum velocity of gas phase in a cross-section being the coordinates. Experimental data obtained by Craik⁽³⁾ are also cited for comparison. Water Reynolds number at which the viscous wave begins to appear is about 20 at the most in both cases of Craik and the present authors. The demarcation line for the occurrence of the viscous wave by Craik which is expressed by a dotted line, however, shifts to the left from a solid line determined experimentally by the present authors. This difference may be due to the air flow being turbulent in case of Craik (height $\times$ width of the duct: 6 in $\times$ 11.4 in) but laminar in our case. However there is a possibility that velocity gradient near liquid surface does not differ so much from each other.

Fig.10 shows the comparison of the theory (broken line) with the present experiment (thick solid line). It must be noticed that both coordinate variables are defined at the center of the duct width because the effect of duct side walls on the averaged value of the variables is tremendous especially when film thickness is small⁽⁶⁾. Short oblique lines attached to data points mean that they are obtained in a turbulent air flow. Furthermore, letters attached to some data express flow pattern observed, for example S-V means that the viscous wave exists near both side walls of duct, but smooth surface flow near the center.

In the present experiment, no viscous wave was observed at any flow rate of water if air flow rate was less than 2 m/s as shown by shadowed portion in the figure. The present theory predicts it with sufficient accuracy, i.e., water film is always stable on the left side of the neutral curve expressed by a one-dotted line. Furthermore, comparison of the dotted line with a thick solid line in Fig.10 reveals that the theory (dotted line) predicts flow condition, (j_{gc} , Γ_c), for the onset of a viscous wave determined by the experiment very closely. The solid line runs parallel with the isopleth lines for measured film thickness shown by a thin solid line, signifying that the viscous wave oc-

Fig.8 Typical examples of the film thickness variation with time

curs when film thickness is between 0.25 mm and 0.3 mm in this experimental condition. This also agrees quite well with a theoretical value of 0.255 mm calculated at the same temperature as in the experiment.

As already shown in Fig.7 the viscous wave appears in the region adjacent to the NW-region, signifying that there exists a strong relation between the generation of the viscous wave and the breakdown of liquid film. As seen from Fig.11 a decreasing of Γ_c with j_{gc} kept constant causes the minimum film thickness to decrease drastically in the transition region from S to V region. This is due to the liquid film becoming much thinner than the one without wave at the trough of the viscous wave once it appears, which causes a breakdown of liquid film. That is, the viscous wave plays an important role in the film breakdown.

According to the experiment, the minimum film thickness at the onset of the viscous wave is about 0.15 mm when water

Fig.9 The region of viscous wave (experiment)

Fig.10 Demarcation for incipience of the viscous wave, comparison between the theory and experiment

temperature is 20°C. If such a thin film is heated, it will disappear in a short time through evaporation of water. The time required for such a thin liquid film to disappear by evaporation is determined, although approximately, by assuming for simplicity that the liquid has a saturation temperature and the heat added is dissipated by vapor as a latent heat. From heat balance, the period T_d is expressed as,

$$T_d = \rho_l L (t_{fmin}) c / q \quad (35)$$

where L is the latent heat of evaporation and q is the heat flux. Substitution of 0.15 mm for $(t_{fmin})_c$ and 10^5 W/m^2 for q , for example, gives $T_d = 3.4$ seconds. As shown in Fig.6 the higher the liquid temperature, the thinner the film thickness of the incipient viscous wave, and the smaller the T_d . These examinations suggest that a film breakdown takes place in a short time after the viscous wave occurs.

5.2 Characteristics of viscous wave

In the experiment, wave velocity was determined from the time delay of the cross-correlation function between two signals for the time varying film thickness detected at two axially different locations. Passing frequency N was determined from the spectral density distribution of film thickness. From these two quantities, the wave length λ is calculated from a relation $\lambda = C/N$. Direct measurements for C and N were made in the experiment also, but the difference was sufficiently small.

Figs.12~14 show these quantities. Dotted and solid lines are respectively for turbulent- and for laminar-air flow. Inspection of Fig.12 reveals that (1) wave velocity remains approximately constant in the range where P- and T- waves prevail; and (2) on the other hand the wave velocity of the viscous wave the data of which are distinguished by short oblique line decreases sharply and takes a very small value at $j_g \approx 7 \text{ m/s}$. As shown in Fig.13 the passing frequency of the viscous wave is much smaller than that in P- and T-regions, signifying that even the viscous wave takes nearly a constant value of wave length (15~25 mm as shown in Fig.14) if its flow condition is close to that in T-

Fig.11 Minimum film thickness (Experiment)

Fig.12 Wave velocity (Experiment)

Fig.13 Wave passing frequency (Experiment)

Fig.14 Wave length (Experiment)

Fig.15 Liquid surface velocity (Experiment)

Fig.16 Comparison of c_r between the theory and experiment

region. If the viscous wave occurs in the region adjacent to S-region, however, its wave length tends to be large. This is inferred from the fact that the onset of the viscous wave which has a larger wave length comes in the region of small Γ_c and large j_{gc} as is already shown in Fig.4.

Fig.15 shows liquid surface velocity, U_i . Of course, the liquid surface velocity is not uniform. In Fig.15 the data collected at the center of the duct width are shown. U_i generally increases with Γ_c while it takes a larger value for the larger air velocity.

Fig.16 shows a comparison of velocity ratio, $c_r = C/U_i$, between the experiment and the theory in which λ is assumed to be 20 mm. It is unavoidable that there are cases where c_r is less than unity, as clearly shown by data marked by short oblique lines. Excepting the case for small Γ_c the theoretical value of c_r is larger than the experimental value in general, which is more evident in the case of the larger Γ and smaller j_{gc} . This is due to two facts; that under such a flow condition film thickness, t_{fm} , or αR becomes larger, and that the experimental value for U_i is always larger than that predicted by the theory⁽⁶⁾. In Fig.17 the relation between c_r and t_{fm} is shown, where a dotted line is for $\alpha R = 1$. This figure shows that experimental values agree closely with the theoretical results within the range of small t_{fm} ; the difference, however, becomes larger when t_{fm} exceeds about 0.5 mm, because in that case αR approaches unity and the assumption for the theory does not hold.

6. Conclusions

A wave which occurs on the surface of a quite thin liquid film flowing concurrently with a gas flow in a horizontal rectangular duct is investigated both experimentally and theoretically. The results are summarized as follows:

(1) If air flow rate is less than 10 m/s with liquid flow rate kept very low, the film is quite thin and a special wave is observed to occur which has a different

Fig.17 $c_r - t_{fm}$

mechanism of wave propagation from those of two-dimensional wave and pebble wave. According to the present theory the generation of this kind of wave is controlled mainly by liquid viscous term. Therefore, the wave is referred to as the viscous wave in this paper. Its velocity is less than the liquid surface velocity.

(2) Decreasing of liquid flow rate or increasing of gas flow rate from a flow condition for S-region causes a decrease in film thickness and a viscous wave occurs. Once it appears, the breakdown of liquid film is easy to take place because the minimum film thickness in the viscous wave flow is much less than that of a film flow without wave. In other words, the fluid dynamic condition which causes a breakdown of liquid film is the occurrence of a viscous wave in the range of this experiment. The incipient viscous wave does not appear until the film thickness is reduced to a definite value which depends only on the density- and viscosity-ratios of gas and liquid phases.

(3) In an absolutely stable region which is predicted by the theory no viscous wave was observed in the experiment regardless of liquid flow rate.

(4) In the range of the experiment, the wave length of the viscous wave was about 20 mm. It, however, tends to become greater if a viscous wave appears in the range adjacent to S-region. The passing frequency of the viscous wave is much less than that of T- or P-wave, because the viscous wave moves quite slow.

References

- (1) Fukano, T., et al., Bull. of JSME., Vol.22, NO.172 (1979-10), pp. 1374.
- (2) Fukano, T., et al., Bull. of JSME., Vol.24, NO.191 (1981-5), pp. 781.
- (3) Craik, A.D.D., J. Fluid Mech., Vol.26, NO.2 (1966), pp. 369.
- (4) Benjamin, T.B., J. Fluid Mech., Vol.6, NO.2 (1959), pp. 161.
- (5) Tatsumi, T and Gotoh, K, Instability of Fluid Flow, (1976), Sangyo Tosho, pp. 108.
- (6) Fukano, T., et al., Preprint of JSME, NO.804-10 (1980-11), pp. 33.