An Experimental Study on Strain Distributions in the Near-tip Field under the General Yielding and Crack Growth*

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The object of this paper is to experimentally assess the applicability of HRR solutions for the strain distribution in the near-tip field under the general yielding and stable crack growth. The measurement of strain components of a 1.2mm thick SUS 430 stainless steel compact tension specimen was performed by using the Moiré method. The phenomena of the formation of the "blunting strain gradient zone (BSGZ)" defined by the authors are discussed by the comparison between the experimental results and analytical ones obtained by HRR formula.

It is clear that the HRR solutions, except those in the BSGZ, are valid to evaluate the strains in the near-tip field of a ductile thin metal sheet under the general yielding and crack growth.

Key Words: Fracture, Experimental Stress Analysis, General Yielding, Stable Crack Growth, J-Integral Crack Tip Strain Singularity, Crack Tip Opening Displacement

1. Introduction

The J-contour integral proposed by Rice¹⁾ has been recognized as one of the useful fracture parameters for the onset of ductile crack extension, since Begley and Landes²⁾ developed an experimental method of measuring J from load displacement records for slightly different crack lengths in otherwise identical specimens. Recently a number of investigators have attempted to establish the applicability of J-integral analysis to evaluate the unstable ductile fracture³⁾⁴⁾, fatigue crack extension⁵⁾ and creep cracking⁶⁾.

To assess the validity of J-integral approach, Hutchinson⁷⁾, Rice and Rosengren⁸⁾ made it clear by obtaining HRR solutions that J was the intensity of a strain-stress field surrounding the crack tip. Aizawa, Yagawa and Ando⁹⁾ and Atluri, Nakagaki and Chen¹⁰⁾ tried to numerically analyse the accuracy of HRR solutions by using the finite element method (FEM), and Taira and Tanaka¹¹⁾ tried to make a measurement of the strains in the near-tip field by means of a profile-meter. But, we have not yet had sufficient results to verify the accuracy of HRR solutions. In order to assess the applicability of HRR solutions for the ductile metals, it must be made clear whether or not the strain distributions in the near-tip field are dominated by the J-integral under the

general yielding and stable crack growth.

On the other hand, since the J-contour integral values (J_R) shown in the HRR formula can not be obtained experimentally, those values are estimated by the potential energy release rate (J_E), the so called equivalent energy method. Therefore, in order to assess the applicability of HRR solutions by the experiments, we must see that the J_E value coincides with the J_R one.

The first thing the authors attempted in this study was to check the equality of the J_E value to the J_R one by using the FEM results under the general yielding. Moreover, these J-values are compared with the experimental ones about 1.2mm thick SUS 430 stainless steel sheet specimens.

After the examination of the relation between J_E and J_R , a measurement of strain components in the near-tip field was performed by using Moiré method under the fully-plastic situation and stable crack growth. And the experimental results are compared with the analytical ones obtained by HRR solutions.

2. Experimental Procedure

The material used in this study is a 1.2mm thick SUS 430 stainless steel sheet which was annealed at 830°C after cold-rolling. The chemical compositions and mechanical properties are given in Tables 1 and 2.

A compact tension specimen as shown in Fig.1 was examined in this investigation. The specimen was notched and fatigue-pre-cracked with the crack running in the rolling direction. Moiré grid sheet, which had a cross grating with density of 20 lines/mm, was glued to the specimen surface in the vicinity of the crack tip in order to make a measurement of strains.

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A schematic illustration of the test procedure is shown in Fig.2. The test material was sandwiched between 20mm thick plates whose outer surfaces were clamped by hand-vises to prevent upsetting of the holes and buckling of the sheet. The load point velocity was kept at about 6mm/h and in-plane strains under the general yielding and the growing crack were measured by Moiré rotational mismatching method in testing. Figure 3 shows the typical Moiré fringes.

The mean strains ϵ_z in the thickness direction were evaluated through a measurement of the shrinkage in the necking cross-sections perpendicular to the crack running direction as shown in Fig.4.

Table 1. Chemical compositions of SUS 430, (%)

C	Si	Mn	P	S
0.052	0.51	0.36	0.023	0.006
Ni	Cr	Al	N	
0.09	16.45	0.066	0.0097	

Table 2. Mechanical properties of SUS 430

Yield Stress	Ultimate Strength	Elongation
307 MPa	463 MPa	40.4 %

Fig.1 Specimen geometries

Fig.2 Schematic illustration of test apparatus

3. Correlation between J-contour Integral (J_R) and Potential Energy Release Rate (J_E)

According to the small-strain finite element analysis based on the incremental strain theory, the J_R -values were computed under the conditions of plane stress and general yielding.

Figure 5 shows the mesh division for a half specimen, of which the total number of nodes and elements are 412 and 747, respectively. The fat lines are contours of J-integral (J_R) shown by the following equation proposed by Rice¹⁾

$$J_R = \int_P (W dy - T \frac{\partial u}{\partial x}) dS,$$

$$W = \int \sigma_{ij} d\epsilon_{ij},$$

where the coordinates are defined as indicated in Fig.4. The load P versus displacement δ curve of calculation using the FEM, a dotted line, is in good agreement with the experimental result as shown in Fig.6. Therefore, it may be declared that the effect of the coverplates on the specimen stiffness can be neglected and the experimental results are in the condition of plane stress. At the onset of the stable crack growth, the critical displacement δ_{crit} ($\otimes$) is about 2.4mm. This occurs after the general yielding as shown by the extent of yield zone in Fig.6.

Figure 7 shows the plots of J_R , computed from the finite element results, versus R which is the representative distance of the contour from the crack tip. Γ_i s indicate the contours on which J_R is calculated. The J_R values for $\Gamma_1 \dots \Gamma_4$,

Fig.3 Moiré fringes

Fig.4 Schematic illustration of the coordinate for the cracked plate

Fig.5 Finite-element idealization for crack configuration

Fig.6 Comparison between experimental and calculated P versus δ curve diagram

except for Γ_5 , are within a deviation of 5% from the mean values of those in fully-plastic situation ($\delta \geq 1.66\text{mm}$). It seems that a reasonable accuracy of calculation for Γ_5 will be obtained by subdividing the adjacent mesh inside of this contour.

The J versus δ curves are shown in Fig.8. J_R indicates the mean value for four different paths, $\Gamma_1 \dots \Gamma_4$. And J_E values are evaluated from P - δ curves by using the following empirical relation of Merkle and Corten¹²⁾

$$J_E = (\eta_A A + \eta_B D) / Bb,$$

where B : thickness, b : uncracked ligament length, A : strain energy, D : complementary energy, η_A , η_B : constant values owing to a_0 and b .

Fig.7 Plots of J_R -integral, computed from the finite element results, versus the distance from the notch tip of contour

Fig.8 Comparison between J_E and J_R

An excellent correlation is established between the J_E values and the J_R ones, though the material used in this study has high ductility such as that of SUS 430 stainless steel sheet. Merkle-Corten's formula to evaluate the J-integral from a single specimen test data is found valid for a thin compact tension specimen under the fully-plastic conditions.

4. Experimental Results and Discussion

4.1 Distribution of strains in the vicinity of the crack tip under the general yielding

Figure 9 shows iso-strain contours of ϵ_y , which are the normal strains parallel to the y-axis, in the x-y plane at $J=0.196 \text{ MJ/m}^2$. The coordinate axes, x, y have the origin at the undeformed precracked tip. The solid lines are Hutchinson's solutions obtained by the following equation.

$$\left. \begin{aligned} \epsilon_{ij} &= K_E r^{-n/(n+1)} \tilde{\epsilon}_{ij}(\theta), \\ K_E &= \epsilon_0 \alpha (J/\alpha I \sigma_0 \epsilon_0)^{n/(n+1)}, \end{aligned} \right\} (1)$$

Fig.9 Iso-strain contours of the y-directional strain, ϵ_y , in the x-y plane

Fig.10 Strain, ϵ_y , distribution along the x axis under the fully plastic situation

where (r, θ) are the polar coordinates and σ_0 , ϵ_0 are the yield stress and the corresponding yield strain, respectively. α is a material constant and the term $1/n$ is the hardening exponent in power law hardening relationship of the form

$$(\epsilon/\epsilon_0) = (\sigma/\sigma_0) + \alpha (\sigma/\sigma_0)^n \quad (2)$$

There is a good approximation for the σ - ϵ curve obtained by the uniaxial tension test by taking 6.55 and 0.2 for n and α in Eq.(2), respectively.

As indicated in Fig.9, the experimental results are in fairly good agreement with Hutchinson's solutions.

Figures 10, 11 and 12 show the strain distributions of ϵ_y , the x-directional strain ϵ_x , and the in-thickness directional strain ϵ_z along the x-axis at $J_E = 0.196 \text{ MJ/m}^2$. There is a blunted crack tip at about $x = 0.1 \text{ mm}$.

For ϵ_y , the experimental results (O) and finite element results (Δ) are found to agree with Hutchinson's solutions at $x > 0.3 \text{ mm}$. However, both results are smaller than the analytical solutions at $x < 0.3 \text{ mm}$. We define this feature as "strain gradient

Fig.11 Strain, ϵ_x , distribution along the x axis under the fully plastic situation

Fig.12 Strain, ϵ_z , distribution along the x axis under the fully plastic situation

blunting (SGB)", which will be discussed later.

On the other hand, the experimental results of ϵ_x extremely decrease with a decreasing x in the near-tip field and turn into negative values at about $x = 0.5\text{mm}$. Meanwhile analytical solutions show always positive values with strain singularity of $-n/(n+1)$ th power of x , and the finite element solutions ($\blacktriangle$) have the same trend as Hutchinson's ones indicated in Fig.11.

The above difference in the strain distributions is explained by the following considerations ; the strain singularity will disappear with the crack tip blunting owing to the formation of CTOD (Crack Tip Opening Displacement) with an increasing J value. Therefore, if we assume that the situation at the crack tip is similar to that of uniaxial tension, it is likely that the strain ϵ_x is negative near the crack tip. Assuming the initial crack tip radius $\rho_0 = 0.05\text{mm}$, the finite element results have the same tendency as the experimental ones. Then, it seems that " the strain gradient blunting " is caused by the disappearance of strain singularity at the crack tip.

Figure 12 shows $|\epsilon_z|$ versus x . The finite element results and Hutchinson's solutions are obtained under the assumption of no volume change in the plastic situation, i.e. $\epsilon_z = -(\epsilon_x + \epsilon_y)$. The experimental results ($\circ$) are in fairly good agreement with the finite element results ($\square$) and there is a close agreement between the analytical results indicated by a solid line and the experimental ones at $x > 0.3\text{mm}$. In the region of $x < 0.3\text{mm}$, the experimental result $|\epsilon_z|$ shows a maximum value, which is about 0.3, but $|\epsilon_z|$ decreases towards the crack tip. This phenomenon of a decreasing $|\epsilon_z|$ will be explained by considering the effects on $|\epsilon_z|$ of the plastic stress triaxiality and the disappearance of strain singularity caused by CTOD. Namely, from the observation of flat fracture at the precracked tip as shown in Fig.13, we can conclude that the crack tip is not in the plane stress state but in the triaxial stress state in spite of a very thin sheet specimen being used which is 1.2mm thick. From the above consideration, it may be concluded that HRR solutions, except those in the "blunting stress gradient zone (BSGZ)", are valid to evaluate the strain in the near-tip field of a ductile thin metal sheet beyond the general yielding, though the HRR solutions are based on the conventional small strain assumption.

Fig.13 Photomicrograph of specimen cross section

4.2 Strain distributions under the crack extension

By substituting $\bar{r}$, $\bar{\theta}$ into Eq.(1), we analytically obtain the strain components under the crack growth, where $\bar{r}$ and $\bar{\theta}$ are polar coordinates centered at the current crack tip.

Figure 14 shows the iso-strain contours of ϵ_y in the x - y plane under the crack extension of which the length Δa is about 0.42mm. Though the crack is growing, the experimental results are found to agree with the analytical ones.

The strain components ϵ_{ij} along the x -axis are calculated by the following equation deduced from Eq.(1).

$$\epsilon_{ij} = \epsilon_0 \alpha (J/\alpha I \epsilon_0 \sigma_0)^{n/(n+1)} \bar{r}^{-n/(n+1)} \epsilon_{ij\theta=0}, \quad \text{--- (3)}$$

where $\bar{x} = (\bar{r})_{\bar{\theta}=0} = x - \Delta a$

Figure 15 shows the relation between ϵ_y and $\bar{x}$ at $\Delta a = 0.42$ and 1mm. The experimental results are remarkably smaller than the analytical ones with a decreasing $\bar{x}$ in the region of $\bar{x} < 1\text{mm}$, but there is close association between both results at $\bar{x} > 1\text{mm}$.

The distributions of ϵ_x and ϵ_z along the x -axis are shown in Figs.16 and 17, respectively. The same tendencies are recognized as in the case of ϵ_y . Therefore, it is clear that Hutchinson's solutions are useful to evaluate the strain components in the near-tip field under the stable crack growth and plane stress condition, except those in the BSGZ. In the previous section, it was considered that the "strain gradient blunting" was caused by the formation of CTOD, that is, the disappearance of strain singularity. Under the crack

Fig.14 Iso-strain contours of ϵ_y in the x - y plane

growth, however, we must give another consideration to the formation of BSGZ, because the blunting of a growing crack tip did not occur in this test.

Now, it is assumed that the strain components ϵ_{ij} along the x-axis are equal to the summation of $(\epsilon_{ij})_c$ and the incremental strain $(\Delta\epsilon_{ij})_m$ as shown in Fig.18.

$$\epsilon_{ij} = (\epsilon_{ij})_c + \sum_{m=1}^n (\Delta\epsilon_{ij})_m, \quad \text{--- (4)}$$

where $(\epsilon_{ij})_c$ is the strain at the onset of the stable crack extension. Furthermore, it will be adequate to say that $(\Delta\epsilon_{ij})_m$ can be obtained from Eq.(3) as follows,

Fig.15 Strain, ϵ_y , distribution along the x axis under the stable crack extension

Fig.16 Strain, ϵ_x , distribution along the x axis under the stable crack extension

Fig.17 Strain, $|\epsilon_z|$, distribution along the axis under the stable crack extension

$$\left. \begin{aligned} (\Delta\epsilon_{ij})_m &= R_m (\Delta J_m)^{n/(n+1)} (\chi - \Delta a_m)^{-n/(n+1)} \tilde{\epsilon}_{ij}, \\ \Delta J_m &= (dJ/da) \Delta a_m (\Delta a_m - \Delta a_{m-1}), \quad \chi > \Delta a_m. \end{aligned} \right\} (5)$$

At $x=A$, the total strain $\overline{AB}$ is expressed by the summation of $\overline{AA}_c$ and $\overline{AA}_m$ for each crack length Δa_m as indicated in Fig.18.

Namely

$$\begin{aligned} \overline{AB} &= \overline{AA}_c + \overline{AA}_1 + \overline{AA}_2 + \dots + \overline{AA}_m + \dots + \overline{AA}_n \\ &= (\epsilon_{ij})_c + \sum_{m=1}^n (\Delta\epsilon_{ij})_m. \end{aligned}$$

The results of calculation by using Eq.(4) are in fairly good agreement with the experimental ones in a wide region including the BSDZ as shown in Figs.14 and 16. Thereby the calculation was performed assuming $(\Delta a_m - \Delta a_{m-1}) = 0.02 \text{ mm}$ and using the value $J_c = 0.24 \text{ MJ/m}^2$ obtained by this test as shown in Fig.19.

Fig.18 Schematic illustration of incremental strain components under the stable crack extension

Fig.19 J_E versus Δa

5. Conclusions

The results obtained in this investigation are summarized as follows:

(1) Before the initiation of crack extension, the experimental results for ϵ_y and ϵ_z are found to agree with Hutchinson's solutions, except those in the "blunting strain gradient zone (BSGZ)", under the fully-plastic condition. For the distribution of ϵ_x , however, the experimental results are extremely different from the analytical ones, i.e. ϵ_x shows negative value towards the crack tip. It may be considered that this difference and the formation of BSDZ are owing to the disappearance of strain singularity caused by the CTOD.

(2) Under the crack extension, the experimental results are in close agreement with the analytical solutions except those in the BSGZ. However, assuming that the strains at the BSGZ are caused by the summation of the incremental strains due to the transition of strain singularity, we have good results.

(3) An excellent correlation is found between the J_E value obtained by using Merkle-Corten's formula and the J-contour integral value proposed by Rice. Therefore, Merkle-Corten's formula to estimate the J-integral from a single specimen test data is found to be valid for a thin ductile metal sheet in fully-plastic condition.

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